Size Effect in Notched Split-Tension Square Prismatic Mortar Specimens

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***Abstract*— The splitting concrete specimens exhibit several advantages, e.g., compactness and lightness, and the weight of the specimen can be disregarded in the calculation of fracture parameters. Existing design codes contain formulas that do not consider size effect in calculations of the tensile strength of concrete. However, it is well known that strength of concrete structures generally decreases with increasing structure size. Size effect in concrete structures can be well explained by fracture mechanics. The experimental investigations on fracture mechanics of cement-based materials until 1970s indicated that classical linear elastic fracture mechanics (LEFM) is no longer valid for quasi-brittle materials such as concrete. This inapplicability of LEFM is due to the existence of a relatively large inelastic zone in front and around the tip of the main cracks in concrete. This so-called fracture process zone (FPZ) is ignored by LEFM. Consequently, several investigators have developed deterministic fracture-mechanics approaches to describe fracture-dominated failure of concrete structures. The size effect law and the modified size effect law suggest that size effect is primarily related to a relatively large FPZ in concrete. In this study, two series of different sizes of geometrically similar square prismatic mortar specimens with maximum aggregate size=4 mm are tested by side and diagonal splitting loading. Based on 21 test results, maximum loads obtained from the test results are analyzed using both the size effect law (SEL) and the modified size effect law (MSEL). Approximate formulas based on the SEL and MSEL are developed for predicting the split-tension strength of square prismatic mortar members. Consequently, the results of the approximate formulas look viable and very promising.**

***Index Terms*— concrete, modified size effect law, splitting strength**

I. Introduction

The split-tension specimens are frequently used to determine the tensile strength of materials, such as concrete and rock [1]. The split-cylinder test, which is also named the Brazilian split test, was proposed by Carneiro and Barcellos (1949) [2]. This test was successfully applied to cubes by Nilsson (1961) [3]. However, split-tension test specimens, namely, cylinders, cubes, diagonal cubes and strip-tension specimens have also been successfully utilized in concrete fractures over the last decade [4-11] because they have certain advantages, such as compactness and lightness, compared to beams. Additionally, cubical and cylindrical test specimens have the following advantages [12].

a) These specimens are easy to handle, and there is no risk of breaking them during handling.
b) The same molds can be used to cast specimens for both fracture and strength tests.
c) In determining the fracture parameters of cement-based materials, the contribution of the weight of the specimen can be ignored, unlike notched beams.

Several design codes [13,14] for size effect, for determining tensile strength capacity of concrete. However, it is well known that strength of concrete structures generally tends to decrease with increasing structure size. Size effect in concrete/reinforced concrete structures can be well explained by fracture mechanics. The experimental investigations on fracture mechanics of cementitious materials until 1970s indicated that classical linear elastic fracture mechanics (LEFM) is invalid for quasi-brittle materials such as concrete. Consequently, several investigators have developed non-linear fracture mechanics approaches based on size effect to describe failure of concrete structures. The effect of specimen size on strength of concrete has been investigated by means of the size effect law (SEL) and the modified size effect law (MSEL) by previous investigators [15,19].

In this study, two series of different sizes of geometrically similar square prismatic mortar specimens with maximum aggregate size=4 mm are tested by side and diagonal splitting loading. Based on 21 test results, maximum loads obtained from the test results are analyzed using both SEL and MSEL. Approximate formulas based on the SEL and MSEL are developed for predicting the split-tension strength of the notched square prismatic mortar members. Consequently, the results of the approximate formulas look viable and very promising.

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II. Size Effect in Concrete Structures

Experiments on concrete member indicated that the strength is expected to decrease because the probability of larger flaws (weak links) increases. This concept explains the size effect based on Weibull's theory (1939) [16], which states that if tensile tests are performed on two geometrically similar specimens with different volumes, the corresponding ultimate strengths will differ. Weibull's approach has been extensively used to estimate the safety factor for brittle structures. However, in the early 1980s, a few scholars realized that neither LEFM nor Weibull's approach were adequate for predicting the size effect in cement-based materials [15]. For this reason, deterministic size-effect theories based on non-linear fracture mechanics were developed.

This "fracture-type" size effect in concrete fractures was initially described by Bazant (1984) [15], who formulated the "size effect law" (SEL). Bazant derived the SEL on the basis of the energy balance at the onset of crack propagation and the dimensional analysis applied to geometrically similar specimens. The formulation of the SEL is expressed as

$$\sigma_{Nc} = B\left(1+\frac{d}{d_0}\right)^{-1/2} \qquad (1)$$

where σ_{Nc} is the nominal strength at failure, d is a characteristic dimension of the specimen, and B and d_0 are empirical constants that are determined by fitting the test results of geometrically similar specimens. For small test specimens, the size effect does not exist and the strength at failure is proportional to the material strength. For large specimens, the maximum possible size effect plays a significant role. The material strength at failure, which is proportional to a characteristic dimension and corresponds to classical LEFM, is expressed by the inclined line with slope -1/2, as shown in Fig. 1.

The size effect on concrete behavior has been experimentally and theoretically examined with considerable success. The results of some published experiment were different than that of predicted by Bazant's SEL. Large concrete members without initial cracks, such as Hasegawa's (1985) [17] split-cylinder test specimens and Shioya's (1989) [18] test beams, can resist to occurred stress. Consequently, Kim and Eo (1990) [19] developed a modified size effect law (MSEL) based on the concept of dissimilar cracks, in which an empirical constant of size-independent strength σ_0 is added to Equation 1 as follows:

$$\sigma_{Nc} = B\left(1+\frac{d}{l_0}\right)^{-1/2} + \sigma_0 \qquad (2)$$

in which l_0 is the width of the crack band. Equation 2 is also named as the extended SEL in a different approach by Bazant et al. (1991). According to the MSEL, the size effect becomes insignificant for extremely small and extremely large characteristic dimensions, as shown in Fig. 1. In practice, the width of the crack band $l_0 = \lambda_0 d_{max}$ in Eq. (2) is chosen approximately $2–3d_{max}$, in which d_{max} is the maximum aggregate size.

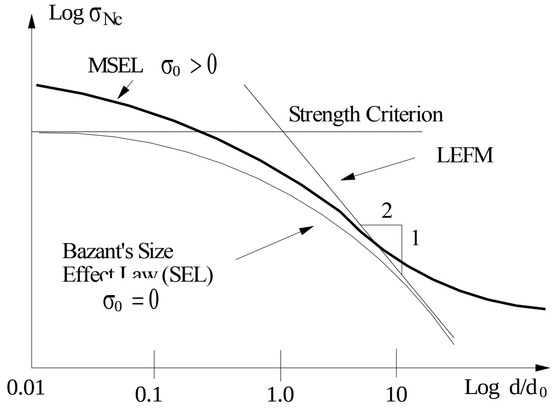

Figure 1. Deterministic size effect laws

III. An Overview of Splitting Tests

As shown in Fig. 1a, the split-tension specimen is placed between the platens of the test machine and the load is subsequently applied until failure, which is caused by splitting along the vertical diameter due to the lateral tensile stress occurred [1]. According to elasticity theory (Timoshenko and Godier, 1970) [20], the nominal tensile strength of split-tension specimens is defined as

$$\sigma_{Nc} = \frac{2P_c}{\pi bd} \qquad (3)$$

where P_c is the ultimate load, b is the specimen width and d is the specimen depth. However, Equation 1 is only valid for the concentrated loading condition shown in Fig. 2a. In practice, the applied load is distributed on the specimens over a finite width (t) using soft materials, such as plywood and hard cardboard, as indicated in Fig. 2c [1, 13]. Tang et al. (1992) investigated the effect of distributed load in three-point bending beams and split-tension cylinders. The nominal strength decreased with increasing width of the distributed load in the split-tension cylinder, whereas this effect was not significant in the bending specimens. According to Tang (1996), the maximum tensile stress value of the un-notched cylinder specimens at the plane of loading can be calculated as

$$\sigma_{max} = \frac{2P}{\pi bd}\left(1-\beta^2\right)^{3/2} \qquad (4)$$

in which P is the total compressive load and $\beta = t/d$ is the ratio of the distributed-load width to the specimen depth, as depicted in Fig. 1. Rocco *et al.* (1995) [21] examined the cylinder and cube specimens and proposed that the maximum tensile stress can be calculated for the un-notched cube specimens at the plane of loading as follows:

$$\sigma_{max} = \frac{2P}{\pi bd}\left[\left(1-\beta^2\right)^{5/3} - 0.0115\right], \quad \beta \le 0.20 \qquad (5)$$

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Using the cohesive crack model, Rocco *et al.* (1999) [22] simulated the Brazilian test by considering the effect of specimen size, specimen shape (cubical/cylindrical), and distributed-load width. The investigators performed a series of experimental studies on mortar, with d_{max}=5 mm, and granite materials and concluded that the developed formulas were in agreement with the experimental results.

Using the boundary element method, a formula for the maximum tensile strength of concrete was similarly derived for un-notched diagonal cubes in the plane of loading by Ince (2012a) as follows:

$$\sigma_{\max} = \frac{2P}{\pi bd}\left(\frac{1}{0.931+38.931\beta^{4.778}}\right), \quad \beta \le 0.25 \tag{6}$$

One of the advantages of diagonal splitting-cube specimens is the nearly constant=1/0.931=1.074 maximum stress in the plane of loading for $\beta \le 0.15$, which differs from other splitting specimens. Meanwhile, the number of theoretical and experimental studies with cube and especially diagonal cube specimens are limited than that of cylinders (1,7,10,11). For instance, a closed-form analytical formula including size-dependent responses for estimating the split-tensile strength of the notched diagonal cube specimens has not been developed. Therefore, in this research, notched diagonal cubes were initially analyzed using the MSEL.

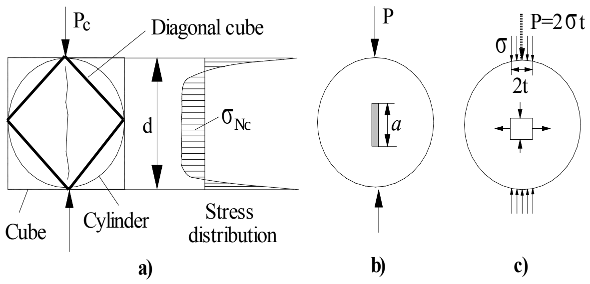

Figure 2. Splitting tension test:

- **a)** geometry and stress distribution
- **b)** notched specimen
- **c)** distributed loading case.

IV. Experimental Program

The test specimen was a square prism with specimen width b=50 mm. To determine size effect, specimens with cube sides: 50, 100, 200 and 300 mm in splitting cube tests and with cube sides: 50, 100 and 200 in splitting diagonal cube tests were tested. All specimens were cast from the same batch of concrete. Three identical cube specimens with 150 mm edge length were also cast from each batch of concrete to determine compressive strength of concrete. The notches, which have the relative crack length $\alpha=a/d$=0.35, were pre-cast in all specimens. The maximum sand grain size was 4 mm. Mineralogically, the aggregate consisted of river sand. The sand was air-dried prior to mixing. CEM I 42.5 R cement was used in the mix. The cement content was 450 kg/m^3 and the ratios of sand: cement: water was 3.34: 1: 0.6. All specimens and identical cubes were removed from the mold after 1 day and were subsequently cured at approximately 20 ℃ in water until testing at 28 days.

Values for the ratio of the distributed-load width to the specimen depth $\beta=t/d$ of 0.1 was selected for the splitting cube specimens, the compression tests and the splitting-tension tests were performed using a digital compression machine with a capacity of 2000 kN. The plywood loading strips, with thicknesses of 3 mm and lengths 10 mm greater than the specimen width, were used in the splitting tests. They were attached on the specimens in the correct positions. The specimens were loaded monotonically until final failure and care was taken to apply a constant loading rate. In diagonal cube tests, a smooth bearing head of stainless steel with a chevron notch was used in the diagonal splitting tests. The steel plates did not indicate any flexural or other type of deformation after testing. Typically, approximately 2 min (±30 sec) elapsed before the maximum load capacity for each specimen was reached. Identical cubes were tested at an age similar to the other specimens.

V. Test Results and Analysis

The compressive strengths of the mix f_c are 37.7 MPa Table I summarizes the characteristic dimension d and the observed failure load P_c. The nominal strength σ_{Nc} were calculated according to Eq. (3) for each of the 28 specimens tested. Typically, it observed that as the load is gradually increased, the first crack occurred along the center line in the vertical direction (Fig. 1). When the maximum load is reached, two diamond-shaped wedges under the bearing plates were formed in the diagonal cube test (Fig. 3b). Similarly, in the cube test, wedges under the bearing plates were formed. Split-cube specimens, the similar rupture modes were also found in during the tests by Ince [6, 7].

TABLE I. Experimental Results

Test	d (mm)	P_c (kN)		
		1	2	3
Cube	50	7.98	8.39	8.32
	100	14.12	14.16	13.63
	200	24.75	24.9	24.9
	300	31.79	30.85	32.38
Diagonal cube	70.7	9.66	9.73	9.61
	141.4	16.54	16.61	16.9
	282.8	30.1	29.6	30.53

For cube and diagonal cube specimens, empirical constants in Eq. (1) based on the size effect law (SEL) can be calculated as $B=1/\sqrt{C}$ and $d_0=C/A$ from the linear regression made on $y=Ax+C$ with $y=1/\sigma_N^2$, $x=d$. Fig. 3 shows results of the linear regression analysis, and the size effect law analysis in the bilogarithmic plane, respectively. The the correlation coefficient values (r) are also given in Fig. 3 for splitting test.

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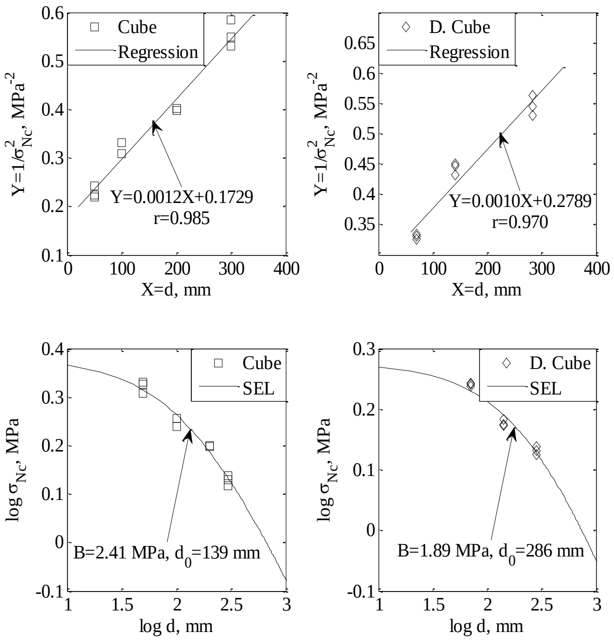

Figure 3. Size effect law in splitting test.

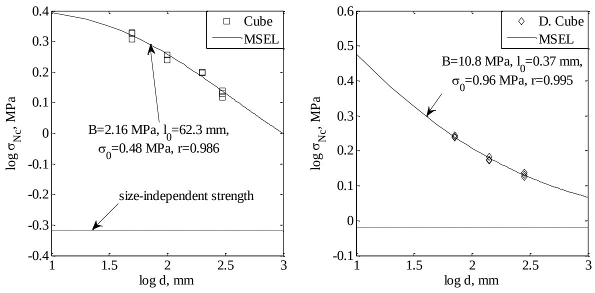

Figure 4. Modified size effect law in splitting test.

For the analysis based on the modified size effect law (MSEL), the empirical constants in Eq. (2) were computed by the nonlinear regression based on the Levenberg-Marquardt algorithm. In Fig. 4, the results of the modified size effect law analysis and the r values were indicated in the bilogarithmic plane.

Cubical and cylindrical members with a specimen size of 100 mm or 150 mm have recommended as the standard test specimen by the design codes (Wu *et al.*, 2013). Specimens with a characteristic size of 100 mm have used as a reference specimen by some investigators in the size effect studies (Kim and Eo, 1990; Ince *et al.*, 2015, 2016). For this reason, the generalized modified size effect laws were derived by using the normalized values of $\sigma_{Nc} / f_{sp}^{100}$ (in which f_{sp}^{100} is the mean splitting- tensile strength for a cube with an edge length of 100 mm in Table I) and the empirical constants in Figs. 3 and 4, as follows:

$$\sigma_{Nc} = \frac{1.35 f_{sp}^{100}}{\sqrt{1+\dfrac{d}{139}}} \quad \text{for cubes} \qquad (7)$$

$$\sigma_{Nc} = \frac{1.26 f_{sp}^{100}}{\sqrt{1+\dfrac{d}{286}}} \quad \text{for diagonal cubes} \qquad (8)$$

$$\sigma_{Nc} = \frac{1.22 f_{sp}^{100}}{\sqrt{1+\dfrac{d}{62.3}}} + 0.27 f_{sp}^{100} \quad \text{for cubes} \qquad (9)$$

$$\sigma_{Nc} = \frac{7.17 f_{sp}^{100}}{\sqrt{1+\dfrac{d}{0.37}}} + 0.64 f_{sp}^{100} \quad \text{for diagonal cubes} \qquad (10)$$

VI. Conclusions

Recently, split-tension specimens such as cylinders and cubes have been commonly used to determine the tensile strength of cement-based materials. Both the split-tension cubes and the diagonal split-tension cubes have been used to determine the fracture behavior of mortar in this article.

- The maximum loads obtained from the test results were analysed by both the size effect law (SEL) by Bazant and the modified size effect law (MSEL) by Kim and Eo. Consequently, the present experimental data indicate that the nominal strength at failure decreases as the specimen size increases. Consequently, the present test results are in a good agreement with both SEL and MSEL.
- According to the SEL and MSEL based on the experimental results, the split-tension strength of mortar can be computed with Equations (7-10) using the strength of the reference geometry (f_{sp}^{100}). The same reference geometry (f_{sp}^{100}) has already been used by the previous researches. However, further research will be required for more reliable findings.
- The experimental studies have shown that fracture behavior of concrete is particularly influenced by the four material parameters: compressive strength, maximum aggregate size, water-cement ratio, and aggregate type. It is noted that fracture resistance of concrete can also be affected by other material parameters such as type of cement and curing conditions, etc. However, this study revealed that when considering shape effect on splitting strength of mortar specimens, the diagonal cube specimens indicate more a brittle behavior than the cube specimens (d_0=286 mm for diagonal cubes > d_0=139 mm for cubes).
- When splitting specimens are produced with molds of the same size, the uncracked ligament length of the diagonal splitting cube specimens is √2 times greater than the lengths of other splitting specimens with the same size. This is another advantage of diagonal splitting cubes, and it is especially useful when studying the size effect.

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Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Cenk FENERLİ conducted the research; Ragıp İNCE analyzed the data; Ragıp İNCE and Cenk FENERLİ wrote the paper. All authors had approved the final version

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