

# Study on Probabilistic Demand Models of Tunnel Linings Subjected to Transverse Seismic Load

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**Abstract**—Since there have been cases of severe damage or even collapse of tunnel structures in recent major earthquakes, the seismic safety of tunnel structures has attracted widespread attention from scholars. In the performance-based seismic design, it is essential to establish a universal and practical demand model. In this paper, to facilitate the use in practice, the probabilistic demand models are developed by adding linear correction term and random term to the commonly used deterministic models. Two types of demand measures, the bending moment and the axial force of the lining to transverse seismic load are considered. The uniform design method is used to generate the samples to calibrate the model parameters, and the uncertainties of ground motions, site properties, and tunnel dimensions are considered. The parameters of the demand models are estimated by the least square method. The probabilistic demand models established in this paper can accurately and reliably evaluate the seismic demand of the tunnel and obtain the probabilistic distribution of the demand, which is of great significance for the seismic vulnerability analysis of tunnel structures.

**Index Terms**—tunnel, demand model, quasi-static analysis, uniform design, least square method

## I. INTRODUCTION

It is generally believed that the seismic performance of underground structures is better than that of surface structures. Therefore, the seismic resistance of underground structures has not received sufficient attention for a long time [1]-[2]. However, many tunnel structures have suffered severe damage [3]-[4] in recent earthquakes. In order to improve the seismic safety performance of tunnels, it is necessary to evaluate the seismic vulnerability of tunnels. The seismic demand is one of the main contents in the research of seismic vulnerability and structural reliability. In the performance-based earthquake engineering (PBEE), system models should incorporate not only modeling uncertainties but also the inherent uncertainties in geotechnical and structural material, component and system properties [5].

Wang [2] and Penzien [6] established the analytical models for the transverse seismic response of tunnels through different analysis procedures, which are called

deterministic models in the field of reliability. However, the prediction results of such models are conservative. Nguyen et al. [7] took the ratio of bending moment demand to bending moment capacity as the damage index and studied the vulnerability of rectangular open-cut subway tunnels through quasi-static numerical simulation. Huang et al. [8] considered four uncertain factors of rock tunnels, including ground motion, tunnel buried depth, surrounding rock and lining, and used the uniform design method to generate the numerical simulation samples. Through numerical simulation, a probabilistic seismic demand model was established. Qiu et al. [9] adopted uniform design method to consider various uncertainties, generated test samples, and established probabilistic seismic demand model of mountain tunnels under transverse seismic load through dynamic finite element analysis. The feasibility was proved by comparing the established model with the empirical model. The above tunnel demand model is obtained through unitary linear regression with the natural logarithm of seismic intensity index (PSA) as the independent variable, so it is only applicable to specific tunnel structures. In 2003, Gardoni et al. [10], using existing deterministic models and observed data, proposed a Bayesian method for constructing probabilistic seismic demand models of reinforced concrete bridge columns to estimate the seismic vulnerability of bridge components and systems. Todorov et al. [11] used a fiber based nonlinear finite element model of a bridge pier, which is designed following the performance-based seismic design demands, to evaluate the damage potential of different types of ground motions. Lu et al. [12] revisited the famous Cornell's intensity measure (IM) and displacement-based formulations for seismic risk. Then they chose a five-storey RC frame as a case study to apply Cornell's formulations to assess the seismic performance of Chinese code-conforming buildings and to investigate the effects of the derived fragility parameters on seismic performance. At present, there are few researches on seismic demand model of tunnel structures, and a universal probabilistic demand model of tunnels is still lacking.

In this paper, probabilistic demand models of underground tunnel lining under transverse seismic load are established by adding correction term and error term on the basis of deterministic models. The various uncertainties affecting the demands are considered and

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the samples are generated by uniform design. The seismic demands are estimated by the quasi-static method, and the parameters of the probabilistic model are calibrated by the least square method. Comparing the probabilistic demand model with the deterministic model, it is found that the model established in this paper is accurate and unbiased. Finally, the probabilistic demand model is used to evaluate the seismic demand of a circular tunnel, and the probabilistic distribution of the tunnel demand is obtained.

## II. DEMAND MODEL

### A. Engineering Demand Parameters

This paper focuses on the study of circular tunnels. For circular tunnel lining, the maximum bending moment and maximum axial force between sections are generally selected as the engineering demand parameters, as shown in Fig. 1. In this paper, both the two indexes are selected as the tunnel demand parameters to establish the demand models.

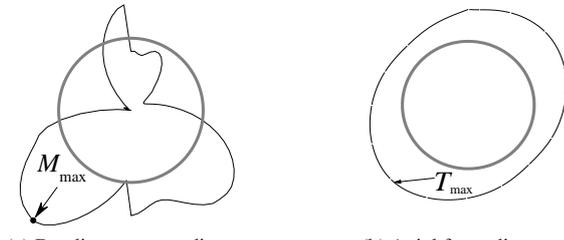


Figure 1. Engineering demand parameters

### B. Model Form

#### 1) Probabilistic demand model

In this paper, the probabilistic demand model is based on the formula proposed by Gardoni et al. [13], the model form is as follows:

$$\ln[D_k(\mathbf{x}, \boldsymbol{\theta}_k, \sigma_k)] = \ln[d_k(\mathbf{x})] + \gamma_k(\mathbf{x}_D, \boldsymbol{\theta}_k) + \sigma_k \varepsilon_k \quad (1)$$

where  $D_k$  is the  $k$ th demand measure;  $d_k(\mathbf{x}_D)$  is the selected deterministic demand model;  $\gamma_k(\mathbf{x}_D, \boldsymbol{\theta}_k)$  is the correction term of the bias inherent in the deterministic model, which is expressed as a function of the variables  $\mathbf{x}$  and parameters  $\boldsymbol{\theta}$ ;  $\mathbf{x}_D$  denotes a series of variables;  $\boldsymbol{\theta}_k$  denotes a set of parameters used to fit the model;  $\sigma_k$  represents the standard deviation of the model error;  $\varepsilon_k$  is the random variable with zero mean and unit variance; through logarithmic transformation of the demand parameters, the model meets the hypothetical demands, namely: homogeneity of variance ( $\sigma_k$  is a constant, independent of  $\mathbf{x}$ ), normality ( $\varepsilon_k$  is normal distribution) and additivity ( $\sigma_k \varepsilon_k$  can be added to the model). The model correction term is in linear form, and the expression can be written as:

$$\gamma_k(\mathbf{x}, \boldsymbol{\theta}_k) = \sum_{i=1}^p \theta_{ki} h_{ki}(\mathbf{x}) \quad (2)$$

where  $h_{ki}(\mathbf{x})$  = the selected explanatory function,  $i=1, \dots, p$ ;  $\theta_{ki}$  denotes the unknown parameters to be fitted.

#### 2) Deterministic model

In this paper, the method proposed by Wang [2] is selected as the deterministic model in the probabilistic demand model. The calculation formulas for axial force and bending moment demand under the assumption of a full slip interface between the soil and the tunnel lining are as follows:

$$T_{\max} = \pm \frac{1}{6} K_1 \frac{E_m}{1 + \nu_m} r R_{\max} \quad (3)$$

$$M_{\max} = \pm \frac{1}{6} K_1 \frac{E_m}{1 + \nu_m} r^2 R_{\max} \quad (4)$$

$$K_1 = \frac{12(1 - \nu_m)}{2F + 5 - 6\nu_m} \quad (5)$$

where  $E_m$  and  $\nu_m$ , respectively, denote the modulus of elasticity and Poisson's Ratio of medium,  $r$  is the radius of the tunnel lining,  $R_{\max}$  = maximum free-field shear strain.  $K_1$  is defined herein as lining response coefficient. The earthquake loading parameter is represented by the maximum shear strain,  $R_{\max}$ , which may be obtained through a simplified approach, or by performing a site-response analysis. It should be noted that the solutions provided here are based on the full-slip interface assumption.

The maximum axial force,  $T_{\max}$ , calculated by (6), however, may be significantly underestimated under the seismic simple shear condition. The full-slip assumption along the interface is the cause. Therefore, it is recommended that the non-slip interface assumption be used in assessing the lining axial force response. The resulting expressions, after modifications based on work of Hoeg [14], are:

$$T_{\max} = K_2 \tau_{\max} r = \pm K_2 \frac{E_m}{2(1 + \nu_m)} r R_{\max} \quad (6)$$

where the lining axial force response coefficient,  $K_2$ , is defined as:

$$K_2 = 1 + \frac{F[(1 - 2\nu_m) - (1 - 2\nu_m)C] - \frac{1}{2}(1 - 2\nu_m)^2 + 2}{F[(3 - 2\nu_m) + (1 - 2\nu_m)C] + C[\frac{5}{2} - 8\nu_m + 6\nu_m^2] + 6 - 8\nu_m} \quad (7)$$

$$C = \text{Compressibility ratio}, C = \frac{E_m(1 - \nu_m^2)r}{E_t t(1 + \nu_m)(1 - 2\nu_m)} \quad (8)$$

where  $F$  is the flexibility ratio,  $E_m$  and  $\nu_m$ , respectively, denote the modulus of elasticity and Poisson's Ratio of medium,  $r$  is the radius of the tunnel lining,  $R_{\max}$  and  $\tau_{\max}$ , respectively, denote the maximum free-field shear strain and maximum free-field shear stress. To avoid the underestimate of the seismic response of the lining, the full-slip interface assumption is generally used to

calculate the bending moment, and the non-slip interface assumption is used to calculate the axial force [1].

### III. QUASI-STATIC NUMERICAL ANALYSIS

#### A. Model Instance

This section introduces the process of the quasi-static analysis used to estimate the lining response by using an example. For the example tunnel: the outer radius is 3 m, the lining thickness is 0.5 m, the burial depth (from the surface to the top of the lining) is 5 m, the elastic modulus of concrete is 30100 MPa, and the Poisson's ratio is 0.2. The soil drilling test data from the University of California, Los Angeles School of Engineering website contains typical types of sites and is suitable for calibrate and further develop existing nonlinear geotechnical models for ground response analysis [15]. Hence, this paper adopts part of site parameters of the data to investigate the tunnel model. The selected site is located at the Jensen Generator Bldg measurement site in Sylmar, with a soil depth of 88.2 m. No. 3 soil is selected on the website under the assumption that the soil of the site is clay, silty clay, and loam with considering the nonlinearity of the soil. The ground motion records of the Pacoima Dam (upper left abut) station in San Fernando in 1971 are selected, which was with the magnitude of 6.6, epicenter distance of 1.81 km, equivalent shear wave velocity of the upper 30 m soil layer was 2016.1 m/s, and the duration of the strong earthquake was mainly distributed within 3~10 s. The acceleration time history curve is shown in Fig. 2, and its peak acceleration amplitude will be modulated to 1 g for analysis and calculation.

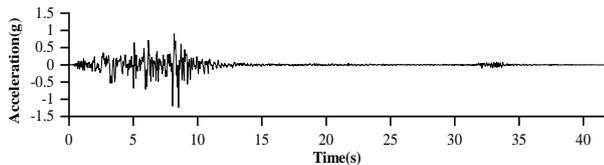


Figure 2. Seismic acceleration time history curve

#### B. Site Response Analysis

The equivalent linearization method is a method to analyze the seismic response of the site by performing equivalent linear calculation based on the soil test data. In this paper, the DEEPSOIL software is used to conduct one-dimensional site response analysis through the equivalent linearization method to calculate the maximum shear strain in the free field and determine the horizontal displacement of the site caused by the earthquake [16]. The relationship between soil shear modulus  $G/G_{max}$  damping ratio  $D$  and shear strain  $R$  is shown in Fig. 3.

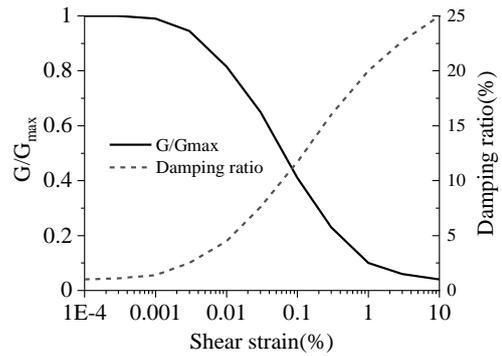


Figure 3. Models of modulus and damping for soil

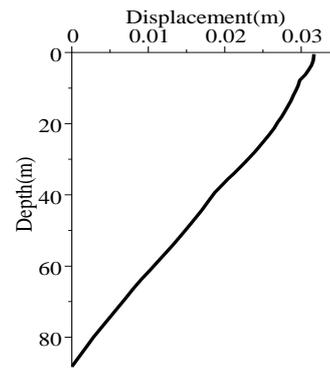


Figure 4. Maximum relative displacement of soil

The soil profile is established and ground motion is input to obtain the soil displacement time history of each layer. The relative displacement of soil at each layer at the time of maximum shear strain at the tunnel is determined as the target displacement. In this example, the maximum shear strain is 0.000226, and the maximum relative displacement of the site is shown in Fig.4. It can be seen that the maximum relative displacement of soil increases with the burial depth decreases, and that of bottom of the soil is zero while that of surface is 0.032m.

#### C. Finite Element Modeling

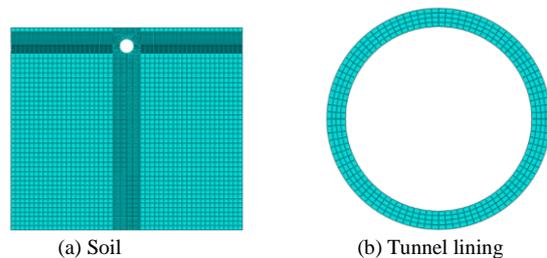


Figure 5. Mesh division

This paper adopts finite element modeling through ABAQUS, the tunnel is equivalent to a homogeneous concrete ring, and the soil is assumed to be an isotropic homogeneous medium. Since this paper designs to research influence of seismic loads on tunnel linings, only seismic loads are considered. In terms of model size, some studies have shown that the width of soil should be greater than 6 times of tunnel diameter in quasi-static

analysis [17], so 100 m is adopted uniformly in this paper. Two-dimensional plane strain elements are selected for both soil and tunnel lining, and more than 5,000 elements are divided into the model. The grid division of soil and tunnel lining is shown in Fig. 5.

It is assumed that the tunnel lining is linearly elastic and the Mohr-Coulomb model is adopted for soil. The interface between tunnel lining and soil is simulated using a finite slip hard contact model, which allows for potential slip between interacting elements during seismic processes. The soil is fixed in the vertical direction and can move freely in the horizontal direction. The nodes at the bottom of the mesh are fixed in both directions. According to the idea of quasi-static analysis, the maximum relative displacement obtained by the one-dimensional equivalent linear field response analysis is applied to the finite element model to calculate the bending moment and axial force demands of the lining.

D. Result Analysis

The demand parameters calculated by the quasi-static numerical simulation and the deterministic method of Wang [2] are compared as follows: the bending moment and axial force calculated by the deterministic method under the full slip assumption are: 81972 N m, 27324 N respectively; the calculated axial force assuming no slip is: 392647 N; the bending moment and axial force obtained by numerical simulation are: 55349 N m, 39806 N.

For the deterministic method, it is considered that the axial forces calculated under the assumption of full slip interface will be underestimated. Therefore, to be conservative, the axial force calculated based on the assumption of no-slip interface is generally used. The results show that the deterministic method produces a conservative result, especially for the axial force demand, and the calculated value of the deterministic method under the assumption of non-slip is about 10 times as much as that of the numerical simulation result.

IV. SAMPLE GENERATION

A. Ground Motion and Site

The uniform design method proposed by Fang et al. [18], which has been widely used in the seismic fragility and reliability analysis of tunnels [19]-[21] is adopted to reduce the amount of samples. The uncertainty of ground motion, soil and structure is considered to design an effective experiment. Twelve typical ground motion records are selected from the Pacific Seismological Research Center (PEER) of the United States and the magnitudes range from 5.6 to 7.9. The amplitude of the selected ground motion is modulated to obtain the time history curve of different ground motion intensities to evaluate the impact of the increment in ground motion intensity on demand.

The soil layer drilling data of 15 measurement sites provided by the University of California, Los Angeles Engineering College website [15] are selected, the shear wave velocities of which are range from 187.1 m/s to 526.2 m/s. The curves of the 15 selected soil shear modulus ratios  $G/G_{max}$  and damping ratio  $D$  with shear strain  $R$  are shown in Fig. 6.

B. Tunnel Structure

In terms of geometric dimensions, underground circular tunnels are mostly small-diameter tunnels with a radius of 3 meters and large-diameter tunnels with a radius of 5 meters. For the lining thickness, this paper chooses the conventional 0.3 m and 0.5 m, and refers to the present concrete strength of general tunnel. The lining concrete has 4 levels: C30, C40, C50, and C60, and the burial depth is selected according to the height of the sites. According to the uniform design Table  $U_4 (4^3)$ , the test plan shown in Table I is obtained. According to the test plan, 4 tests are conducted on each site, and a total of 60 finite element models are established in 15 sites. The lateral displacement obtained from site seismic response analysis is applied to the model to obtain the demand data.

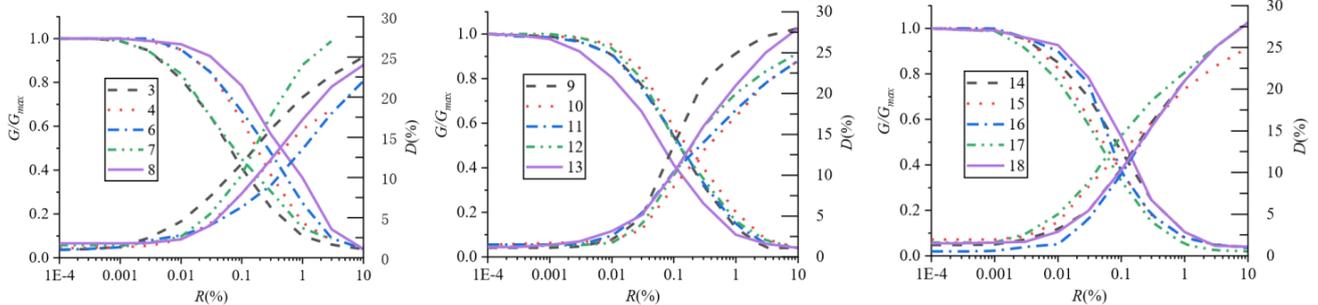


Figure 6. Models of modulus and damping for soils

TABLE I. UNIFORM DESIGN TABLE:  $U_4 (4^3)$

Test Number	Tunnel Depth	Tunnel Radius and Lining Thickness (m)	Concrete Strength Grade
1	(1)	3,0.5 (4)	C40 (2)
2	(2)	3,0.3 (3)	C60 (4)
3	(3)	5,0.5 (2)	C30 (1)
4	(4)	5,0.3 (1)	C50 (3)

V. ESTABLISHMENT OF SEISMIC DEMAND MODEL

A. Selection of Candidate Interpretation Functions

The modification term of the demand model is shown in (2). In the process of regression modeling, the scope of candidate explanatory functions of each model should be determined first. The free-field maximum shear strain model is mainly affected by two factors, ground motion and soil, in the meantime, the structure itself also needs to be considered in the tunnel demand model. Therefore, the explanatory function of the maximum shear strain of the free field is selected as  $h_1(\mathbf{x}) = 1$ , the peak acceleration of ground motion  $h_2(\mathbf{x}) = \ln(PGA)$ , the peak velocity of ground motion  $h_3(\mathbf{x}) = \ln(PGV)$ , the tunnel burial depth  $h_4(\mathbf{x}) = \ln(depth)$ , the equivalent shear wave velocity  $h_5(\mathbf{x}) = \ln(Vs30)$  of 30m soil layer on the surface, and the shear wave velocity  $h_6(\mathbf{x}) = \ln(C_s)$  at the tunnel depth. In addition to the above factors, the tunnel demand model includes the following candidate explanatory functions: deflection ratio  $h_7(\mathbf{x}) = \ln(F)$ , compression ratio  $h_8(\mathbf{x}) = \ln(C)$ , lining response coefficient  $h_9(\mathbf{x}) = \ln(K_1)$  and  $h_{10}(\mathbf{x}) = \ln(K_2)$ , tunnel radius  $h_{11}(\mathbf{x}) = \ln(r)$ , lining thickness  $h_{12}(\mathbf{x}) = \ln(t)$ , and the maximum shear strain  $h_{13}(\mathbf{x}) = \ln(R_{max})$ . In order to make models established in this paper meet the demands of the hypothesis, a logarithmic transformation on the candidate variables is carried out in this paper [22].

B. Bending Moment Demand Model

1) Establishment of model

According to the deterministic bending moment model, (5), the stepwise regression [23] is adopted to select an appropriate interpretation function for the correction term of the bending moment probabilistic demand model, and the least square method is applied to estimate the parameters  $(\theta_{M,1}, \dots, \theta_{M,p}, \sigma_M)$ . Table II shows the stepwise regression process. In the first step, the maximum sum of partial regression squares  $h_6(\mathbf{x})$  is introduced on the basis of the constant term. At this time, the error standard deviation is estimated to be 0.3372. The second step, in order to improve the accuracy of the model, the partial regression square sum of the maximum explanatory function  $h_{12}(\mathbf{x})$  is introduced on the basis of Model 1. The number of parameters in Model 2 has increased to three. At this time, the error standard deviation is estimated to be 0.3352, indicating that the accuracy of the model can be further improved to some extent. After continuing to introduce the explanatory function and repeating the above operation, the error standard deviation of Model 5 is reduced to 0.3256. When  $h_7(\mathbf{x})$  is introduced, it is found that the model collinearity is considerably obvious, indicating that further introduction is not necessary.

TABLE II. STEPWISE REGRESSION PROCESS OF MOMENT DEMAND MODEL

Model	$h_1(\mathbf{x})$	$h_2(\mathbf{x})$	$h_6(\mathbf{x})$	$h_7(\mathbf{x})$	$h_9(\mathbf{x})$	$h_{12}(\mathbf{x})$	$h_{13}(\mathbf{x})$	$E[\sigma_M]$
1	√	--	√	--	--	--	--	0.3372
2	√	--	√	--	--	√	--	0.3352
3	√	√	√	--	--	√	--	0.3335
4	√	√	√	--	--	√	√	0.3292
Final 5	√	√	√	--	√	√	√	0.3256
6	√	√	√	√	√	√	√	0.3155

TABLE III. STEPWISE REGRESSION PROCESS OF AXIAL FORCE DEMAND MODEL

Model	$h_1(\mathbf{x})$	$h_2(\mathbf{x})$	$h_6(\mathbf{x})$	$h_7(\mathbf{x})$	$h_9(\mathbf{x})$	$h_{12}(\mathbf{x})$	$h_{13}(\mathbf{x})$	$E[\sigma_{Add}]$
1	√	--	--	--	√	--	--	0.5962
2	√	--	--	--	√	--	√	0.5604
3	√	--	--	--	√	√	√	0.5493
4	√	√	--	--	√	√	√	0.5451
Final 5	√	√	--	√	√	√	√	0.5342
6	√	√	√	√	√	√	√	0.5323

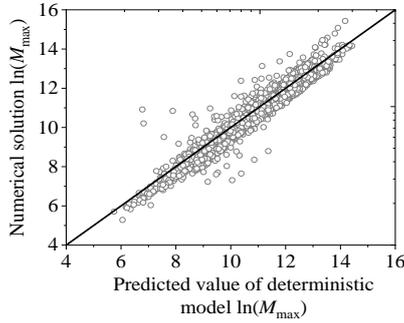
The final bending moment demand model is shown in (9). It can be seen that the bending moment demand model is mainly influenced by the structure of tunnels, intensity of ground motion and site situation. The stepwise regression process shows that the shear wave velocity at the depth of the tunnel has the greatest effect on the correction of the bending moment deterministic model.

$$\ln(D_M) = \ln(\hat{d}_M) + 2.134 + 0.163 \ln(PGA) - 0.538 \ln(C_s) - 0.08 \ln(K_1) + 0.234 \ln(t) - 0.124 \ln(R_{max}) + 0.3256\varepsilon \quad (9)$$

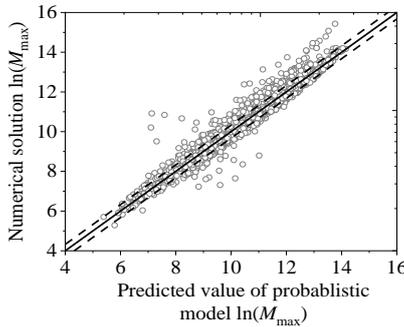
2) Comparison of models

Fig. 7 shows the comparison between the predicted value of the bending moment demand and the numerical

solution. The prediction result of the probabilistic model in the figure corrects the deviation of the deterministic model. From Fig. 7 (b), it can be seen that the data points are more concentrated around the 1:1 line, the model is unbiased, and most of the data points are within a standard deviation range.



(a) Numerical solution and predicted value of deterministic model



(b) Numerical solution and predicted value of probabilistic model

Figure 7. Comparison of moment deterministic model and probabilistic model

### C. Axial Force Demand Model

#### 1) Establishment of model

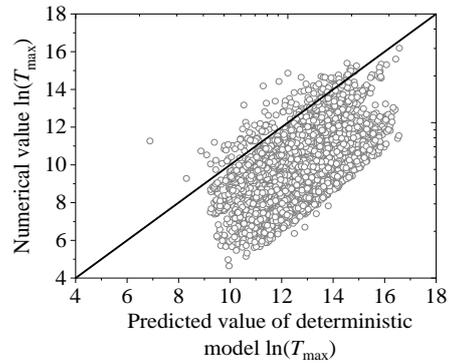
The deterministic model of axial force is shown in (6). Table III shows the stepwise regression process of the axial force probabilistic demand model. Each introduction of an explanatory function corresponds to the posterior mean estimation of the standard deviation of the model error. The order of introducing explanatory functions is determined by the sum of squares (contribution) of partial regression. In the first step, on the basis of the constant term, the maximum sum of partial regression squares  $h_7(\mathbf{x})$  is introduced. At this time, the error standard deviation is estimated to be 0.5962. The second step, in order to improve the accuracy of the model, the partial regression square sum of the maximum explanatory function  $h_{13}(\mathbf{x})$  is introduced on the basis of Model 1. The number of parameters in Model 2 has increased to three. At this time, the error standard deviation is estimated to be 0.5604, indicating that the accuracy of the model can be further improved to some extent. After continuing to introduce the explanatory function and repeating the above operation, the error standard deviation of Model 5 is reduced to 0.5342. When  $h_3(\mathbf{x})$  is introduced, it is found that the model collinearity is considerably obvious, indicating that further introduction is not necessary.

The probabilistic demand model of axial force is shown in (10). The stepwise regression process shows that the deflection ratio has the greatest effect on the correction of the deterministic model of axial force.

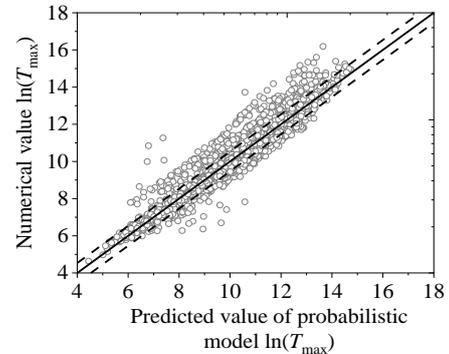
$$\ln(D_r) = \ln(\hat{d}_r) + 3.664 + 0.257 \ln(PGA) - 0.72 \ln(C_s) - 0.862 \ln(F) + 0.62 \ln(r) - 0.071 \ln(R_{max}) + 0.5342\varepsilon \quad (10)$$

#### 2) Comparison of models

Fig. 8 shows the comparison between the predicted value of axial force demand and the numerical solution. The data points in the Fig. 8 (a) are mostly distributed on the right side of the 1:1 solid line, and the deviation degree is large, indicating that the conservatism of axial force calculated is very strong. There is a large-scale dispersion at each point in the figure, which shows that the consistency of the calculation results of the two methods is relatively poor. The solid line with a distance of 1:1 in the Fig. 8 (b) delimits an area within the range of standard deviation. The prediction result of the probabilistic model in the figure clearly corrects the deviation of the deterministic model. Compared with the existing deterministic models, the width of the whole data band is much smaller, indicating that the overall deviation between the predicted value of the probabilistic model and the numerical simulation result is smaller, the prediction is more accurate, and the accuracy also meets the demands of engineering design.



(a) Numerical solution and predicted value of deterministic model



(b) Numerical solution and predicted value of probabilistic model

Figure 8. Comparison of axial force deterministic model and probabilistic model

### D. Application of Probabilistic Models

In order to illustrate the probabilistic model established in this paper, a typical transverse seismic response design

example of a circular tunnel in the paper of Hashash et al. [1] is evaluated for seismic demands. The design example parameters are as follows:

Seismic parameters: earthquake magnitude  $M_W = 7.5$ , fault distance 10 km, peak ground motion acceleration  $PGA = 0.5$  g; Soil parameters: hard soil, density  $\rho_m = 1920$  kg/m<sup>3</sup>, shear wave velocity  $C_S = 250$  m/s, Poisson's ratio  $\nu_m = 0.3$ ; tunnel parameters: tunnel radius  $r = 3$  m, lining thickness  $t = 0.3$  m, buried depth = 15 m, lining elastic modulus  $E_c = 24.8 \times 10^6$  kPa, Poisson's ratio  $\nu_c = 0.2$ , tunnel lining area (per unit width)  $A_c = 0.3$  m<sup>2</sup>/m, the moment of inertia of the tunnel lining (per unit width)  $I = 0.0023$  m<sup>4</sup>/m.

According to the above example, the probabilistic density function of bending moment demand and axial force demand is drawn by using the probabilistic model proposed in this paper, as shown in Fig 9. The figure shows the results obtained by Hashash et al. using the deterministic method to calculate the demand and the mean estimate of the probabilistic model. It can be seen from the figure that for the probabilistic density function of the bending moment demand, the calculation result of the deterministic method is on the right side of the mean value of the probabilistic model, which is relatively conservative. For the axial force, the calculation result of the deterministic method under the assumption of full-slip is on the right side of the mean value of the probabilistic model, indicating that the axial force demand of the tunnel lining is underestimated, while the axial force calculated by the deterministic method under the non-slip assumption is much larger than the mean value of the probabilistic model, indicating that the deterministic method under the non-slip assumption gives an excessively high demand estimate. This result is in line with the view of Hashash et al. [1].

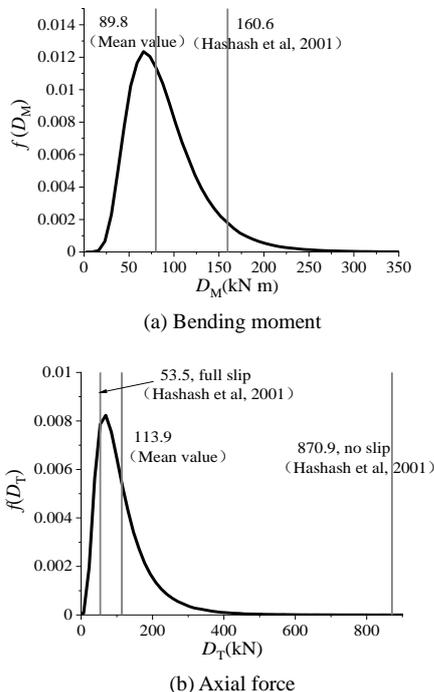


Figure 9. Demand probabilistic density functions

## VI. CONCLUSION

Based on the site seismic response analysis, the soil displacement which is corresponding to the largest shear strain is obtained. And then it is used as the seismic input to the model. The internal force of tunnel lining is obtained through the quasi-static numerical analysis. Finally, probabilistic demand models for bending moment and axial force of tunnel linings subjected to transverse seismic load are established. The research work mainly has the following findings:

Compared with the numerical method, the deterministic method is easier to be applied in practice, but in general, the bending moment and axial force demands are conservative. The probabilistic demand model established in this paper corrects the inherent bias of the deterministic model, and can obtain the probability distribution of the seismic demand, rather than a fixed value. The established model can be used for engineering design and structural vulnerability assessment.

In the process of stepwise regression model optimization, an optimal model can be chosen according to the size of the change of error standard deviation. To reach a compromise between the simplicity of the model and the accuracy, the process is stopped when the standard deviation of the model has slight change after adding another new explanatory function. According to the results of the stepwise regression process the soil shear wave velocity has the greatest effect on the bending moment model, and the deflection ratio has the greatest effect on the axial force model.

## CONFLICT OF INTEREST

The authors declare no conflict of interest.

## AUTHOR CONTRIBUTIONS

Min H. and Renzheng Z. are responsible for the paper, conceiving the idea, conducting the research and finalizing the paper for submission; Dr. Zhao supervised the research and helped to analyze the data; Min H. and Renzheng Z. contributed equally; all authors had approved the final version.

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