Structural Probabilistic Health Monitoring of a Potentially Damaged Bridge

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***Abstract*—This research consists to determine the security margin reliability with probabilistic analysis using the dynamic computed stresses and the RC reinforced concrete strength of a specific bridge structure. The maximum stresses are obtained from a dynamic finite element damage detection analysis in which each value is computed inside a mesh element under severe loadings [1, 5]. The RC strength stochastic characteristics are calculated from experiments and RC composite materials mixture rules. Reliability of computed stresses from a damage detection analysis [1] became an important structural health monitoring process. The main objective of this research work is the probabilistic analysis of the dynamic operating stresses using their calculated stochastic characteristics. This research work allows quantifying the structural warranty period including unpredictable and stochastic phenomena like natural disasters under severe loadings. An important vital structure with known stochastic characteristics is analyzed by quantifying and increasing its lifetime period. The RC strength stochastic characteristics and structural security margin reliability of a specific designed structure are performed.**

***Index Terms*—reliability, failure, damage detection, security margin, RC strength, stresses, finite element, dynamic, probabilities, statistics, health monitoring**

I. Introduction

Bridges are essential engineering infrastructures. The dynamic response of such structures has often been used as a basis for analysis within the field of structural health monitoring (SHM). However, difficulty often exists when bridges exhibit significant nonlinear behavior due to aging degradation of structural properties and fracture damage as well as uncertainly boundary environmental and dynamic loading conditions.

Structural design takes in consideration probabilistic phenomena like severe winds, tsunamis, earthquakes, hurricanes and vandalisms which are not included in the design analysis and are not included in the RCPR designing code Ref. [5]. Specialized cars manufacturers have used to include the reliability analysis in order to increase their product warranty period which is obtained from the mean time between failures ranging from 100000 to 200000 *km* when quantified in kilometers. This study aims to determine the security margin maximum stresses of a vital structure which has been analyzed with a dynamic damage detection finite element analysis. The operating stresses are computed at each time step iteration inside each mesh element in a transient dynamic analysis. Furthermore, the RC strength is calculated from experiments and RC composite rules Ref. [1, 2]. The RC material is made of reinforcement steel, sands, gravels and cement mixtures.

Reliability is an important analytical decision making method in structural design. Some cars manufacturers have increased their cars life time warranty by a 2 factor with respect to the others. The behavior of a structure under severe or random loadings can generate failures or catastrophic failures such that the material is stressed beyond its strength limit. The types of failures are severe loadings, fatigue or corrosion, use of unpredictable defective materials and unexpected environmental problems. One can introduce some important probabilistic events like natural disasters or vandalisms. Several important natural structural failures and collapses happened on well-known bridges, buildings, aircraft structures, spacecraft vehicles, trade centers Ref. [6]. On the other hand, some works have been done on the reliability estimation studies for different type of bridges and a probabilistic approach has been used Ref. [7] to assess their safety level.

Several research works have been published in international specialized conferences on structural defects and repair and several analysis methods have been suggested. We introduce in this research work a new method for the determination of the stochastic stresses variables characteristics from experiments and computations based on statistical formulas of real experimental samples values and finite element computed ones.

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II. Problem Formulation

In dynamic finite element analysis, stresses are computed inside each element and at each time step iteration ranging from directional stresses to ideal Von Mises stresses of structures with material mixture characteristics.

If one selects to use the equivalent Von Mises stress with RC material strength characteristics, it is then possible to define the security margin as:

$$m = S_{RC} - s_{equiv} \tag{1}$$

where, S_{RC} is the RC strength and S_{equiv} is the equivalent ideal Von Mises maximum stress.

The reliability of a detected potentially damaged element is expressed as follows:

$$R = P(m \geq 0) = P(S_{RC} \geq s_{equiv}) \tag{2}$$

where, m is a stochastic variable.

We know also that $R = P(n \geq 1)$ and $n = 1$ is the optimized boundary security factor such that $n = \frac{S_{RC}}{s_{equiv}}$. For the case where the boundary limit is $m = 0$ which corresponds to the best reliable case and is considered as an acceptable region limit with $S_{RC} = s_{equiv}$.

Tabulated values use the probability of failure using the following relation:

$$F = 1 - R(t) = P(m < 0) = P\left(S_{RC} < s_{equiv}\right) \tag{3}$$

Let $f(m)$ be the probability security margin density function with the normal probability law such that [3, 4]:

$$\int_{-\infty}^{+\infty} f(m)\, dm = 1 \tag{4}$$

$f(t) = \frac{dF(t)}{dt}$ is also called the failure probability density function. And let X be a stochastic variable, then its mean value can be written as follows:

$$\mu_X = E[X] = \int_{-\infty}^{+\infty} x\, f(x)\, dx \tag{5}$$

The corresponding variance is related to the mean value by:

$$V(X) = E[(X - E[X])^2] = \sigma_X^2$$
$$= \int_{-\infty}^{+\infty} (x - \mu_X)^2 f(x)\, dx$$
$$= \int_{-\infty}^{+\infty} x^2 f(x)\, dx - \mu_X^2 \tag{6}$$

We define also for two others stochastic variables the relations:

$$E[X - Y] = E[X] - E[Y] = \mu_X - \mu_Y \tag{7}$$

$$cov(X, Y) = E[XY] - \mu_X \mu_Y \tag{8}$$

If X and Y are independent variables, a zero correlation imply: $cov(X, Y) = 0$ and we obtain:

$$V(X) = E[(X - \mu_X)^2]$$
$$= E[(X - \mu_X)(X - \mu_X)] = cov(X, X) \tag{9}$$

$$V(X - Y) = V(X) - 2\, cov(X, Y) + V(Y)$$
$$= V(X) + V(Y) \tag{10}$$

The two cited operating stresses are independent variables. We have then a zero correlation between them in the difference as in (10). Therefore, the square SD standard deviation of the security margin becomes:

$$\sigma_m^2 = \sigma_{S_{RC}}^2 + \sigma_{s_{equiv}}^2 \tag{11}$$

where, σ_m is the standard deviation of security margin, $\sigma_{S_{RC}}$ is the standard deviation of RC strength and $\sigma_{s_{equiv}}$ is the standard deviation of an ideal equivalent Von Mises maximum computed stress.

The security margin mean value is:

$$\mu_m = \mu_{S_{RC}} - \mu_{s_{equiv}} \tag{12}$$

here, $\mu_{S_{RC}}$ and $\mu_{s_{equiv}}$ are the mean value of RC strength and the mean value of ideal equivalent Von Mises stress, respectively.

The limit value $m = 0$ gives the boundaries $[0\!:6\sigma_m]$ such that we have the following data range repartition:

$$m = {\mu_m}^{\pm 3\sigma_m} \tag{13}$$

The density curve has symmetry of $3\sigma_m$ around the mean value μ_m.

Let be the variable change for the reduced central normal probability law:

$$z = \frac{m - \mu_m}{\sigma_m} \tag{14}$$

In practice, z values are tabulated for the normal probability law and $m = N(\mu_m, \sigma_m)$ where m is a selected security margin value.

It is obvious that for $m = 0$ we have:

$$z = \frac{\mu_{s_{equiv}} - \mu_{S_{RC}}}{\sqrt{\sigma_{S_{RC}}^2 + \sigma_{s_{equiv}}^2}} \tag{15}$$

For $i = 1\ to\ n$ discrete values, we get:

$$\mu_X = \frac{\sum_{i=1}^{i=n} x_i}{n} \tag{16}$$

And:

$$V(X) = \sum_{i=1}^{i=n} (x_i - \mu_X)^2$$
$$= \sum_{i=1}^{i=n} {x_i}^2 + \mu_X \sum_{i=1}^{i=n} 1 - 2\mu_X \sum_{i=1}^{i=n} x_i \tag{17}$$

Then:

$$V(X) = \sum_{i=1}^{i=n} {x_i}^2 - n(\mu_X)^2 \tag{18}$$

RC is a specific material equivalent to a heterogeneous concrete paste with reinforcement steel. If we measure with experiments several times the strength of this material, we obtain several statistical values on these samples.

To get the unknown RC material strength standard deviation of the specific material made from reinforcement steel, sands, gravels and cement, one has to calculate the mean value of these samples and the corresponding standard deviation.

For given n experimental strengths values $x_i = S_{RC_i}$, the following strength characteristics are calculated using:

$$\mu_{S_{RC}} = \frac{1}{n} \sum_{i=1}^{i=n} S_{RC_i} \tag{19}$$

$$V(S_{RC}) = \sum_{i=1}^{i=n} (S_{RC_i})^2 - n\left(\mu_{S_{RC}}\right)^2 \tag{20}$$

Such that:

$$\sigma_{S_{RC}} = \sqrt{V(S_{RC})} \tag{21}$$

The selected computed maximum equivalent stress characteristics are obtained with the following method:
For a selected time step iteration of the dynamic analysis, one gets p neighborhood equivalent stresses values around a detected critical zone among the computed stresses. Consequently, we get:

$$\mu_{s_{equiv}} = \frac{1}{p} \sum_{i=1}^{i=p} s_{equiv_i} \tag{22}$$

$$V\left(s_{equiv}\right) = \sum_{i=1}^{i=p} (s_{equiv_i})^2 - p\left(\mu_{s_{equiv}}\right)^2 \tag{23}$$

Such that:

$$\sigma_{s_{equiv}} = \sqrt{V\left(s_{equiv}\right)} \tag{24}$$

III. Practical Example

Let E be the event of $m \geq 0.01$, the corresponding reliability is obtained as follows:

$$R = P(E) = P(m \geq 0.01)$$

If $S_{RC} = 66.0197\ MPa$ and $s_{equiv} = 32.9000\ MPa$ Ref. [1], then we get:

$$\mu_m = 66.0197 - 32.9000 = 33.1197\ MPa$$

If we assume that $\sigma_m = 11.0399\ MPa$ then we have

$$z = \frac{0.01 - \mu_m}{\sigma_m} = -2.999$$

and $P(E) = P(Z \geq -2.999) = 1 - P(Z < -2.999) = P(Z < +2.999)$

Tabulated values give:

$P(Z < +2.99) = 0.99861$
$P(Z < +3.00) = 0.99865$
$P(Z < +2.999) = 0.99861 + 0.9 * (0.99865 - 0.99861) = 0.998646$

This is obviously a very reliable case for the imposed security margin value. If we set $S = S_{RC}$ and $s = s_{equiv}$ one gets the following repartitions curves as in Fig. 1 of equivalent operating stress and RC material strength stress.

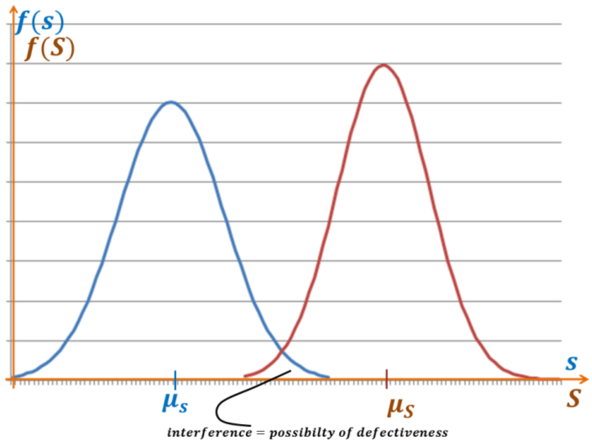

Figure 1. Repartitions curves of equivalent operating stress and RC material strength stress

Where the cited means stresses and their random corresponding values can generate a tolerated interference due to stochastic phenomena as a possibility of defectiveness. The means values must agree for $\mu_S \geq \mu_s$ condition and their intersecting stochastic range values correspond to probabilistic defectiveness.

IV. Conclusion

We have introduced in this research work an efficient method to perform the stochastic stresses characteristics in order to check and improve the structural health using the stochastic security margin variable. Unpredictable and stochastic phenomena like natural disasters under severe loadings are quantified in this research and the structural lifetime period can be handled and increased. Experiments to determine the structural stochastic characteristics for each most important specific structure are a recommended process. The practical RC strength stochastic characteristics and structural security margin reliability of a specific designed structure are also performed.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Moussaoui developed the research and wrote the paper; Chabaat,Kibboua analyzed the paper; all authors had approved the final version.

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