Evaluation of the Performance of an Enhanced Damage Plasticity Model for Predicting the Cyclic Response of Plain Concrete under Multiaxial Loading Conditions

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***Abstract*—Reliable design of reinforced concrete (RC) structures against earthquake has received considerable attention for many decades. It is vital that RC members exhibit sufficient strength and ductility under combinations of gravity loads and cyclic lateral excitations caused by earthquakes. To that end, this study presents an Enhanced Concrete Damage Plasticity Model (ECDPM) for predicting the cyclic behavior of plain concrete under multiaxial loading conditions, which combines the theories of classic plasticity and continuum damage mechanics. This model employs two damage variables for describing the influences of tensile and compressive damages on overall behavior. The capability of the model to predict the cyclic response of plain concrete is evaluated using experimental data from a uniaxial tension test, as well as uniaxial, biaxial and triaxial compression tests. Very good agreement is generally observed between the numerical predictions and test data. Various shortcomings of the model are also identified to aid future development efforts.**

***Index Terms*—damage-plasticity model, plain concrete, cyclic loading, multiaxial loading conditions**

I. Introduction

Several predictive response models at the material and structural level exist in the literature. Some serve for evaluating the behavior of structural components under seismic loading [1-7]. Some consider the uniaxial response of plain unconfined or confined concrete [8-10]. In addition to the previously mentioned models, several constitutive models have been proposed for simulating the behavior of concrete under monotonic or cyclic loading. One of the popular models for describing the response of concrete under monotonic loading was proposed by Grassl and Jirásek, which is typically referred to as CDPM1 [11]. The performance of this model for predicting the behavior of plain concrete and RC structures subjected to multiaxial loads has been demonstrated in some studies [12, 13]. However, this model cannot describe the concrete response under cyclic loading since it employs only one single damage variable for both tension and compressive regimes. Among the existing models for concrete, the model proposed by Lee and Fenves, which is called CDPM, is the most practically used one for simulating the cyclic behavior of plain and reinforced concrete under both monotonic and cyclic loading [14]. It contains two damage variables, one for tensile damage caused by cracking and one for compressive damage caused by crushing. This model can describe the stiffness degradation caused by tensile or compressive damage, the effect of crack opening or closing on the strength recovery, as well as the influences of lateral confinement on strength and ductility . Furthermore, Recently, Grassl et al. extended CDPM1 to describe the transition between tension and compression regimes by means of two damage variables and they denoted it as CDPM2 [15]. Despite of the advanced features of CDP and CDPM2, their capabilities to predict the cyclic behavior of plain concrete under biaxial and triaxial cyclic compression were not demonstrated in [14,

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15]. Hence, this study presents an enhanced damage plasticity model (henceforth denoted as ECDPM) as an extension of CDPM1. The performance of ECDPM in simulating the cyclic response of plain concrete under different loading scenarios is evaluated. In this regard, the ECDPM predictions were compared against test data including a uniaxial tension test by Reinhardt et al. [16] and uniaxial, biaxial and triaxial compression tests by Van Mier [17].

II. Brief Description of ECDPM

In this study a damage plasticity model for concrete as an extension of a previous model proposed by Grassl and Jirásek [11] is presented. For sake of brevity, the presentation is restricted on its extension, more information is available in [11]. The nominal stress-strain formulation of ECDPM reads

$$\boldsymbol{\sigma} = (1-\omega)\boldsymbol{E}_0 : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^P). \tag{1}$$

Therein, $\boldsymbol{\sigma}$ denotes the nominal stress tensor, ω is the scalar isotropic damage parameter ranging from 0 (undamaged material) to 1 (fully damaged material), $\boldsymbol{E}_0$ is the fourth order elastic stiffness tensor, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\varepsilon}^P$ are the total and plastic strain tensors, respectively. The model describes the softening response in the post-peak regime by splitting the damage parameter ω into two damage parameters for tension and compression as follows,

$$(1-\omega) = (1-\omega_c)(1-r_w\omega_t), \tag{2}$$

where, ω_c and ω_t denote the scalar damage parameters in compression and tension, respectively, and the split weight factor r_w ranging from 0 (pure 3D compressive strain state) to 1 (pure tension) as follows [18]

$$r_w = \frac{\sum_{i=1}^{3}\langle\varepsilon_i\rangle}{\sum_{i=1}^{3}|\varepsilon_i|}, \tag{3}$$

With $\langle\varepsilon_i\rangle$ denoting the positive parts of the principal strains. The parameters ω_c and ω_t in (2) take the form

$$\omega_c = 1 - e^{-\frac{\alpha_{dc}}{\epsilon_{fc}}} \tag{4a}$$

$$\omega_t = 1 - e^{-\frac{\alpha_{dt}}{\epsilon_{ft}}} \tag{4b}$$

With α_{dc} and α_{dt} denoting the damage driving internal variables for compression and tension, respectively, ϵ_{fc} and ϵ_{ft} are the softening modulus for compression and tension, respectively. The evolution of α_{dc} and α_{dt} is defined as

$$\dot{\alpha}_{dc} = \begin{cases} \dfrac{\alpha_c\dot{\varepsilon}_V^P}{x_s(\bar{\sigma}_m)} & \text{if } \alpha_p > 1 \wedge \dot{\varepsilon}_V^P > 0, \\ 0 & \text{otherwise} \end{cases} \tag{5a}$$

$$\dot{\alpha}_{dt} = \begin{cases} \dfrac{(1-\alpha_c)\dot{\varepsilon}_V^P}{x_s(\bar{\sigma}_m)} & \text{if } \alpha_p > 1 \wedge \dot{\varepsilon}_V^P > 0, \\ 0 & \text{otherwise} \end{cases} \tag{5b}$$

where $\alpha_c = \sum_{I=1}^{3}\frac{\bar{\sigma}_{PCI}(\bar{\sigma}_{PCI}+\bar{\sigma}_{PTI})}{\|\bar{\sigma}_P\|^2}$ is a variable ranging from 0 for uniaxial tension to 1 for uniaxial compression according to [15], $\bar{\sigma}_{PCI}$ and $\bar{\sigma}_{PTI}$ are the negative and positive components of the principal effective stresses, $\dot{\varepsilon}_V^P$ is the rate of volumetric plastic strain, $x_s(\bar{\sigma}_m)$ is the softening ductility measure, and α_p is the strain-like internal hardening variable and its rate takes the form according to [19] as

$$\dot{\alpha}_p = \frac{\|\dot{\varepsilon}^P\|}{x_h(\bar{\sigma}_m)}\left(1 + 3\frac{\bar{\rho}^2}{\bar{\rho}^2 + 10^{-8}f_c'^2}\cos^2\left(\frac{3\theta}{2}\right)\right), \tag{6}$$

where $x_h(\bar{\sigma}_m)$ is a hardening ductility measure, $\bar{\rho}$ and θ denote the effective deviatoric radius and the Lode angle, respectively . The compressive softening modulus is defined as [15]

$$\epsilon_{fc} = \frac{G_{FC}}{f_c' l_{char} A_s}, \tag{7}$$

where the compressive fracture energy can be estimated as $G_{FC} = \left(\frac{f_c'}{f_t}\right)^2 G_{FI}$ [20] with f_c' and f_t denoting the compressive and tensile strength of concrete, respectively, G_{FI} is the Mode-I fracture energy, l_{char} is the characteristic length and A_s is a model parameter.

The tensile softening modulus can be determined as [21]

$$\epsilon_{ft} = \frac{G_{FI}}{f_t l_{char}} - \frac{f_t}{2E} \tag{8}$$

with E denoting the Young's modulus.

III. Validation of ECDPM

In this section, the results, computed by ECDPM, are compared to test results. In this regard, four tests including uniaxial tension and compression, biaxial and triaxial compression tests are used.

A. Cyclic Uniaxial Tension Test Conducted by Reinhardt et al. (1986)

In the test, conducted by Reinhardt et al. [16], the response of concrete in cyclic uniaxial tensile loading is investigated. The material parameters for concrete are characterized as E = 24 GPa, Poisson's ratio ν = 0.2, f_c' = 47.1 MPa, f_t = 3.24 MPa, G_{FI} = 0.06 N/mm, l_{char} = 35 mm, and the model parameter A_s is identified as 15. The comparison between the results computed by ECDPM and the measured ones is displayed in Fig. 1. Good agreement between the numerical result predicted by ECDPM and the test result is concluded.

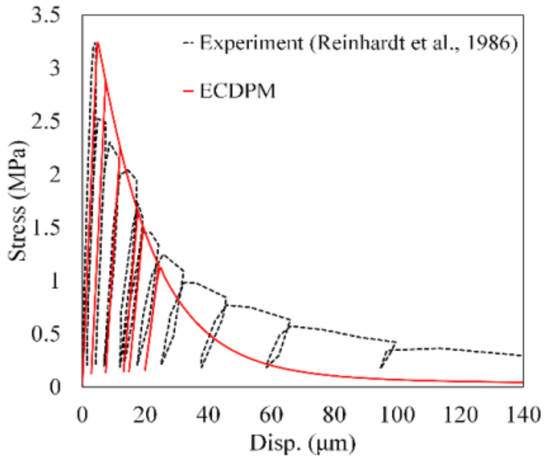
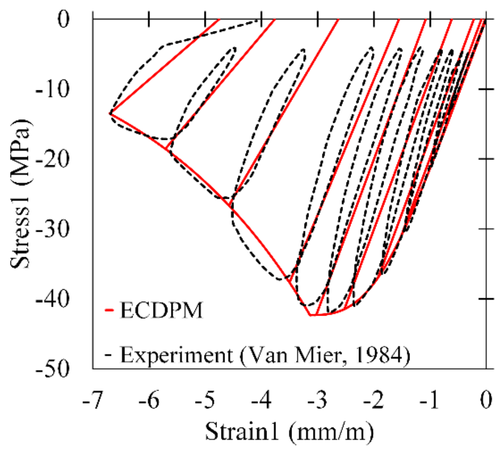


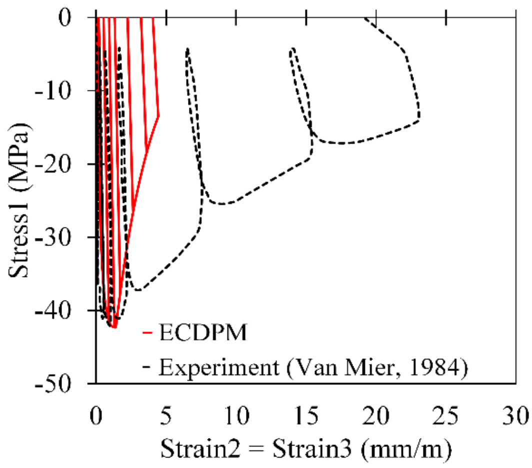
Figure 1. Comparison of the computed and measured stress-displacement diagram for concrete in a cyclic uniaxial tension test conducted by Reinhardt et al. [16].

B. *Cyclic Uniaxial Compression Test Conducted by Van Mier (1984)*

For examining the capability of ECDPM to predict the cyclic response of plain concrete under uniaxial compressive loading regime the test conducted by Van Mier [17] is employed. The material parameters are given as ν = 0.2, f_c' = 42.3 MPa, and f_t = 2.8 MPa. E and G_{FI} were estimated as 21.7 GPa and 0.15 N/mm, respectively, l_{char} is taken as 100 mm, and A_s is identified as 10. In Fig. 2 the predicted and observed results are compared. It can be concluded from Fig. 2(a) that the model captures the cyclic response of concrete in terms of the $\sigma_1 - \varepsilon_1$ diagram very well. However, it can be observed in Fig. 2(b) that the model significantly underestimates the lateral strains measured at the free surfaces. It was stated in the test report that the measurements of the lateral strains were probably distorted by splitting phenomena. Hence, it cannot be clarified if the discrepancy between the computed and measured results is caused by a shortcoming of ECDPM or caused by the measurement setup.



(a)

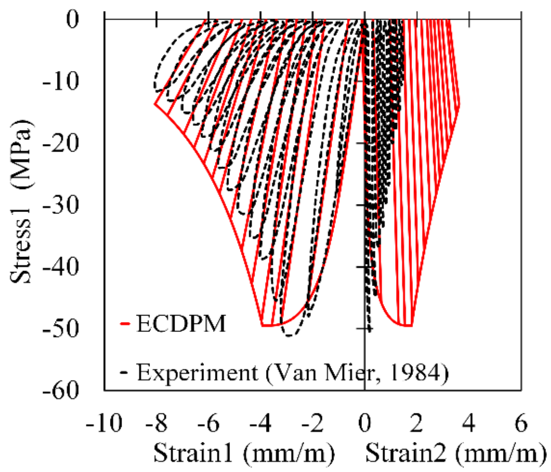


(b)

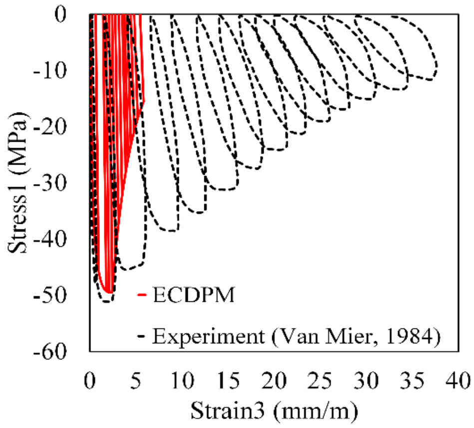
Figure 2. Comparison of the predicted and observed concrete response in a cyclic uniaxial compression test conducted by Van Mier [17].

C. *Cyclic Biaxial Compression Test Conducted by Van Mier (1984)*

In this subsection the results predicted by ECDPM are compared with the observed ones for the biaxial compression test conducted by Van Mier [17]. The concrete parameters are set as E = 29 GPa, ν = 0.2, f_c' = 45.3 MPa, f_t = 2.8 MPa, G_{FI} = 0.15 N/mm, l_{char} = 100 mm, and A_s = 15. A low lateral confining pressure of $\sigma_2 = 0.1f_c'$ was applied to the specimen, whereas $\sigma_3 = 0$. In Fig. 3 the computed results are compared to the measured ones. Fig. 3(a) demonstrates that ECDPM predicts the cyclic behavior of concrete including the softening response and the influence of low confinement on strength and ductility along the loading direction very well. On the other hand, it can be seen that the lateral strain ε_2 is slightly overestimated, whereas, the lateral strain ε_3 at the free surface is strongly underestimated.



(a)

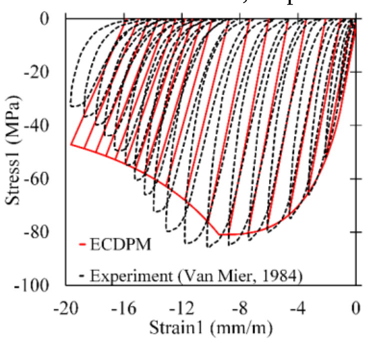


(b)

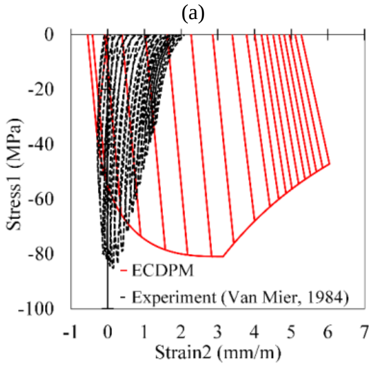
Figure 3. Comparison of the computed and observed concrete response in a cyclic biaxial compression test conducted by Van Mier [17].

D. *Cyclic Triaxial Compression Test Conducted by Van Mier (1984)*

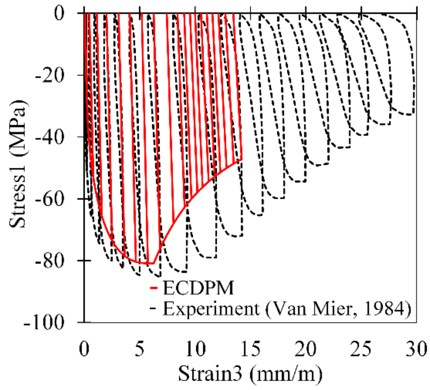
The performance of ECDPM is further evaluated by simulating the concrete response in a cyclic triaxial compression test conducted by Van Mier [17]. The material parameters for this test are E = 29 GPa, ν = 0.2, f_c' = 50 MPa, f_t = 2.8 MPa, G_{FI} = 0.15 N/mm, l_{char} = 100 mm, and A_s = 10. The applied lateral confinement pressures are σ_2 = 0.33 f_c' and σ_3 = 0.05 f_c' . The comparison of results predicted by ECDPM and the measured ones are displayed in Fig. 4. As observed for the previous tests in this study, it can be seen in Fig. 4(a) that the stress-strain response is computed very well by ECDPM along the loading direction. However, Fig. 4(b),(c) show that the strain components ε_2 and ε_3 are underestimated and overestimated, respectively.



(a)



(b)



(c)

Figure 4. Comparison of the predicted and observed concrete response in a cyclic triaxial compression test conducted by Van Mier [17].

IV. Summary and Conclusion

In this study an Enhanced Concrete Damage Plasticity Model (ECDPM) for predicting the cyclic behavior of plain concrete under multiaxial loading conditions was presented. The proposed model can describe the stiffness degradation due to compressive and tensile damage by using two damage variables. The performance of the model was evaluated by experiments conducted for various loading scenarios such as cyclic uniaxial tension and compression as well as biaxial and triaxial compression. Comparisons between the computed results by ECDPM and the test data demonstrated the capability of the model to capture the cyclic response of concrete along the loading direction under multiaxial loading such as the effects of lateral confinement on strength and ductility along the loading direction. However, additional test data from plain concrete specimens subjected to cyclic multiaxial loading is required for further validation and improvement of ECDPM.

Conflict of Interest

The authors declare no conflict of interest

Author Contributions

Mohammad Reza A. Azadi Kakavand wrote the paper; Günter C. Hofstetter conducted the research; Ertugrul B. Taciroglu analyzed the data; all authors had approved the final version.

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