Ground Displacement Analysis of Double-tube Parallel Shield Tunnels

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Abstract—Ground surface settlement is one of the vital factors to assess the structural safety during tunnel construction. According to the well-known Mindlin displacement solution, we fully analyze the mechanical behavior of ground surface settlement caused by the tunnel construction. Modified formula of ground deformation induced by ground loss is proposed in this paper, which is employed to predict the ground surface settlement induced by the double-line tunnel construction with superposition principle. The successive construction process of double-tube parallel shield tunnels in the same direction is simulated with finite element software FLAC3D, and the simulated results is compared with values calculated by the modified theoretical formula. Finally, we find that our numerical results are in an excellent agreement with the theoretical results.

Index Terms—Double-tube parallel shield tunnels; Ground surface settlement; Soil loss; Numerical analysis

I. INTRODUCTION

With the large scale development and utilization of underground space, it’s common that tunnel construction sites are located in city building dense area, therefore, accurate evaluation of soil displacement caused by tunnel construction is important to building a better security protection scheme for surrounding buildings. At present, there are numerous studies on ground surface settlement. For the single-track tunnel, there are several prediction methods to evaluate the construction settlement as follow: Peck experience formula method, stochastic medium theory, laboratory model tests and Numerical Methods. For tunnel construction, based on Mindlin displacement solution, a method had to be applied for ground surface settlement calculation, taking into account of the excess earth pressure, the lateral friction between shield skin and the surrounding soil and ground loss [1-3]. In this paper, we offer a modified formula for the evaluation of the ground deformation induced by ground loss. Based on Mindlin displacement solution, an improved method for calculation of ground surface settlement is proposed. Eventually, we fully analyze the soil mechanical behavior by using FLAC3D to simulate the construction process of double-line tunnel construction.

II. CALCULATION OF GROUND SURFACE SETTLEMENT INDUCED BY DOUBLE-TUBE PARALLEL SHIELD TUNNELS CONSTRUCTION

A. Mindlin Displacement Solution after Coordinate Conversion

As shown in Figure 1, a vertical and horizontal point load are applied at a semi-infinite solid respectively. Through the theory formula deduction, Mindlin [4] found the expressions (1-6) for the displacement in symmetrical cylindrical coordinates. Any point \((x, y, z)\) corresponds to the axis displacements \((u, v, w)\).

\[
\begin{align*}
&u_h = \frac{P}{16\pi G(1-\nu)} \left[ \frac{3-4\nu}{R_1^3} + \frac{1}{R_2^3} + \frac{x^2}{R_1^3} + \left( \frac{3-4\nu}{R_2^3} \right) \frac{x^2}{R_1^3} \right]
\end{align*}
\]

(1)

\[
\begin{align*}
v_h &= \frac{Pxy}{16\pi G(1-\nu)} \left[ \frac{1}{R_1^3} + \frac{3-4\nu}{R_2^3} \frac{6cz}{R_2^3} \frac{4(1-v)(1-2\nu)}{R_2^3} \right]
\end{align*}
\]

(2)

\[
\begin{align*}
w_h &= \frac{Px}{16\pi G(1-\nu)} \left[ \frac{c^2}{R_1^3} + \frac{3-4\nu}{R_2^3} \frac{6cz}{R_2^3} \frac{4(1-v)(1-2\nu)}{R_2^3} \right]
\end{align*}
\]

(3)

Displacement caused by a vertical point load:
In order to obtain the displacement caused by a lateral, longitudinal and vertical point load applied at any location, we transform the coordinate of displacement solution, as depicted in Fig. 2.

![Coordinate Transformation Diagram](image)

Figure 2. Calculation diagram for coordinate transformation.

A lateral force is applied at point \((0, r \cos \theta, H - r \sin \theta)\) and causes vertical displacement \(w_{h1}\) and lateral displacement \(v_{h1}\) at point \((x,y,z)\) in a new coordinate system. The \(w_{h1}\) and \(v_{h1}\) are given by:

\[
v_{h1} = \frac{P_{\text{external}}x}{16\pi G(1-\nu)} \left[ \frac{(3-4\nu)(z-H-r\sin \theta)}{R_1^3} + \frac{6(1-\nu)^2}{R_2} + \frac{6c(z+c)}{R_2^3} \right]
\]

\[
v_{h1} = \frac{P_{\text{external}}y}{16\pi G(1-\nu)} \left[ \frac{(3-4\nu)(z-H-r\sin \theta)}{R_1^3} + \frac{6(1-\nu)^2}{R_2} + \frac{6c(z+c)}{R_2^3} \right]
\]

where \(\nu\) is Poisson's ratio; \(E_s\) is soil compression modulus (MPa); \(K_0\) is coefficient of earth pressure at rest; \(G\) is shear stiffness (MPa). In soil mechanics, the shear stiffness is given by:

\[
G = \frac{(1-2\nu)K_0}{2(1+\nu)}
\]

A longitudinal force and a vertical force are applied at point \((0, r \cos \theta, H - r \sin \theta)\) respectively and cause vertical displacement \(w_{l2}\) and lateral displacement \(v_{l2}\) at point \((x,y,z)\) in new coordinate system. The \(w_{l2}\) and \(v_{l2}\) are given by:

\[
w_{l2} = \frac{P_{\text{external}}(y - r \cos \theta)}{16\pi G(1-\nu)} \left[ \frac{(3-4\nu)(z-H+r\sin \theta)}{R_2^3} + \frac{6(1-\nu)^2}{R_2} + \frac{6c(z+c)}{R_2^3} \right]
\]

\[
v_{l2} = \frac{P_{\text{external}}y}{16\pi G(1-\nu)} \left[ \frac{(3-4\nu)(z-H+r\sin \theta)}{R_2^3} + \frac{6(1-\nu)^2}{R_2} + \frac{6c(z+c)}{R_2^3} \right]
\]

B. Basic Assumptions

In order to use classical mathematics and mechanics to calculate the settlement caused by shield construction, we make the following assumptions to simplify the conditions.
In this paper, the TBM position changes while driving and the “kill” or “alive” of the soils are taking into account. The consolidation of soils is not in consideration;

(2) The model of soils is homogeneous linear elastic semi-infinite solid and the excavation of tunnel construction can’t change the boundary of the model;

(3) TBM continues to move uniformly in a straight line, the change of the driving route caused by the correction or rotation of the machine and the angles of tunnel are not in consideration;

(4) During construction, tunnels can’t interact with others.

C. The Settlement Caused by Construction Factors

During the construction of double-tube parallel shield tunnels, the influencing factors on ground surface settlement are the excess earth pressure, the lateral friction between shield skin and the surrounding soil, the friction force between cutter-head and soil, the grouting pressure at shield tail and the gravity of the shield machine [5-6]. Their calculation diagram is shown in Figure 3.

\[
\begin{align*}
w_{q} &= \int_{0}^{2\pi} \int_{0}^{R} w_{q}(x, y, z, r, \theta, q) r \, dr \, d\theta \\
\end{align*}
\]  

(18)

The ground settlement induced by the lateral friction \( f \) of single-track tunnel between shield skin and the surrounding soil is given by:

\[
\begin{align*}
w_{2f} &= \int_{L} w_{2f}(x-W-l, y, z, r, \theta, f) \, Rd\theta \, dl
\end{align*}
\]  

(19)

In the same way, the ground settlement induced by the friction force between the front of cutter-head and soil is given by:

\[
\begin{align*}
w_{3f-1} &= \sum_{n=1}^{n} \left( \int_{0}^{R} w_{3f-1}(x, y, z, r, \alpha, p_{h}) \times r \, dr \right)
\end{align*}
\]  

(20)

The ground settlement induced by the friction force between the side of cutter-head and soil is given by:

\[
\begin{align*}
w_{3f-2} &= \int_{0}^{w} \left[ w_{3f-2}(x-w, y, z, \phi, p_{h}) \\
&+ w_{3f-2}(x-w, y, z, \phi, p_{h}) \right] \, Rd\theta \, dw
\end{align*}
\]  

(21)

\[
\begin{align*}
p_{1} &= \frac{P_{1} \cdot r \cdot \cos \alpha}{R}, \quad p_{h} = \frac{P_{h} \cdot r \cdot \sin \alpha}{R} \\
p_{2} &= \frac{P_{2} \cdot r \cdot \cos \alpha}{R}, \quad p_{h} = \frac{P_{h} \cdot r \cdot \sin \alpha}{R}
\end{align*}
\]  

(22)

\[
\begin{align*}
\alpha &= \phi + 2k\pi / n, \quad p_{1} = \frac{3(T_{1} + T_{k})}{nR^{2}}, \quad p_{2} = \frac{T_{2}}{2\pi R^{2}w}
\end{align*}
\]  

(23)

where \( p_{1} \) is the max friction force on the far end of cutter-head; \( p_{2} \) is the friction force between the side of cutterhead and soil; \( \phi \) is entrance angle of cutters; \( n \) is the number of lines where cutters are installed, \( k \) means the K-th line; \( T_{1} \) is the torque between the front of cutter-head and soil; \( T_{k} \) is the torque between the side of cutter-head and soil; \( T_{2} \) is the torque of cutter-head for cutting the muck.

The ground settlement induced by grouting pressure is given by:

\[
\begin{align*}
w_{4p} &= \int_{0}^{s} \int_{0}^{2\pi} \left[ w_{4p} + w_{4} \right] \, Rd\theta \, ds
\end{align*}
\]  

(24)

\[
\begin{align*}
p_{v} &= \frac{P \cdot r \cdot \sin \theta}{R}, \quad p_{h} = \frac{P \cdot r \cdot \cos \theta}{R}
\end{align*}
\]  

(25)

where \( s \) is the length of grouting area of shield; \( L \) is the length of shield; \( R \) is the radius of shield.

The ground settlement induced by the weight of the shield machine is given by:

\[
\begin{align*}
w_{5G} &= \int_{0}^{L} \int_{0}^{2\pi} \left[ w_{5G} \right] \, Rd\theta \, dl
\end{align*}
\]  

(26)

\[
\begin{align*}
\int_{0}^{2\pi} P \sin \theta \, R \, d\theta \cdot L = G
\end{align*}
\]  

(27)

where \( G \) is the total weight of shield machine; \( P \) is the weight component in the tunnel axis.
D. The Ground Settlement Caused by Strata Loss

There are several factors influencing the subsidence and the main driving factor is the deformation caused by strata loss. Combining the research of Loganathan [7-8] and Wei [2] obtained the subsidence expressions of strata loss which represented the change of Poisson’s ratio and the non-equivalent, oval and radial movement of strata. The expressions were found to be

\[ w_{sl}(x, y, K) = 2(1-v) \frac{V_{min}}{\pi} \times (1- \frac{x}{\sqrt{x^2+y^2+H^2}}) \exp\left( -\frac{1.38 y^2}{(H+R)^2} \right) \]  

(28)

According to the equivalent parametric method proposed by Attewell [9] for the estimated value of subsidence caused by strata loss \( V_{loss} \), and the value of strata loss, can be represented by Eqs. (33) and assigned to the constants \( \pi, R, \eta \) is strata loss ratio.

\[ V_{loss} = \pi R^2 \eta \]  

(29)

However, revealed by Eqs. (28), the combination of previous research, the values of widths of settlement tank is smaller than the measured values and can’t change with the actual nature of the strata. After a lot of research of strata in London, O’Reilly [10] held that there was a linear relation between the width coefficient \( i \) of settlement tank and the depth \( H \) of tunnel. Based on the above, Han et al. [11] carried the comparison research on the relationship between actual nature of the strata and the width coefficient, and then he provided a set of suggestive values \( K \), varied from area to area, as the width parameter of settlement tank. Combing with the results above, we obtain the semi-theoretical and semi-empirical expression. The relationship between geological conditions and ground settlement is taking into account.

\[ w_{sl}(x, y, K) = 2(1-v) \frac{V_{min}}{\pi} \times (1- \frac{x}{\sqrt{x^2+y^2+H^2}}) \exp\left( -\frac{y^2}{2(KH)^2} \right) \]  

(30)

where \( x = 0 \) is tunnel face; The suggestive value \( K \) of Guangzhou is 0.76, others refer to the research of Han et al. [11].

E. The Total Ground Surface Settlement Caused by Double-tube Parallel Shield Tunnels

In conclusion, combing with solutions of ground settlement caused by different kinds of construction factors, we obtain:

\[ w(x, y) = w_{sl}(x, y, 0, q) + w_{sl}(x, y, 0, f) + w_{sl}(x, y, 0, 0, p_h, p_s, p_v) + w_{sl}(x, y, 0, 0, p_h, p_v, p_s) + w_{sl}(x, y, 0, 0, p_v, p_s) + w_{sl}(x, y, 0, 0, p_s, p_v) \]  

(31)

In order to obtain the expression of settlement of double-tube parallel shield tunnels, we superimpose the expression of settlement of single-tube and the calculation diagram is shown in Figure 4.

\[ W(x, y) = w(x, y - \frac{L}{2}) + w(x, y + \frac{L}{2}) \]  

(32)

III. Numerical Simulation

A. Model Parameter and Grid

Based on the shield construction of the running tunnel of Tianhebei-Wushan project in Guangzhou, the characteristics of transverse and longitudinal surface displacements are studied by numerical simulation with FLAC3D. The outside diameters of segments is 6.0 m while the inside is 5.4 m. The depth of tunnel in the model is 15 m. The material properties and geometric parameters of model are shown in Table I.

<table>
<thead>
<tr>
<th>Layer</th>
<th>t (m)</th>
<th>P (kN/m³)</th>
<th>E (kPa)</th>
<th>v</th>
<th>C (kPa)</th>
<th>ϕ (º )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miscellaneous fill</td>
<td>1</td>
<td>19</td>
<td>8200</td>
<td>0.43</td>
<td>22</td>
<td>12</td>
</tr>
<tr>
<td>Alluvial and diluvial deposit</td>
<td>4.5</td>
<td>19</td>
<td>18000</td>
<td>0.40</td>
<td>23</td>
<td>22</td>
</tr>
<tr>
<td>Hard plastic residual soil</td>
<td>3.5</td>
<td>18.5</td>
<td>23000</td>
<td>0.30</td>
<td>17</td>
<td>21</td>
</tr>
<tr>
<td>Completely-weathered granite</td>
<td>10</td>
<td>19.1</td>
<td>21000</td>
<td>0.38</td>
<td>28</td>
<td>22</td>
</tr>
<tr>
<td>Highly-weathered granite</td>
<td>16</td>
<td>19.8</td>
<td>26000</td>
<td>0.36</td>
<td>30</td>
<td>25</td>
</tr>
<tr>
<td>Over-break</td>
<td>0.07</td>
<td>10</td>
<td>12000</td>
<td>0.38</td>
<td>28</td>
<td>22</td>
</tr>
<tr>
<td>Shield (steel)</td>
<td>0.045</td>
<td>7850</td>
<td>2.06×10⁶</td>
<td>0.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lining segment (C30)</td>
<td>0.4</td>
<td>25</td>
<td></td>
<td>0.15</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The parameters of construction are as follows: the excess earth pressure is -0.3 MPa; the ground settlement induced by the friction force between the side of cutter-head and soil is 46 kPa; the torque between cutter-head and soil is 2000 kN·m; the grouting pressure at shied tail is -0.2 MPa; the gravity of the shield machine is 520 t; the lateral friction between shield skin and the surrounding soil is 8 kPa.

The model established in FLAC3D is shown in Figure 5. Using Mohr-Coulomb elastic-plastic model as constitutive model, the strata and grouting material are created with solid element while the shield and lining
segments made out of isotropic material are created with shell element.

![Figure 5. The classification of soil layer and the grid of surrounding rock of tunnel.](image)

B. Excavation Procedure

When modeling, first, the gravity should be applied and its interior stress reach equilibrium under its own weight [12]; second, the initial geostress field should be reset. All the steps (see Figure 6) are in the following:

Step 1: Excavate 1.5 m along the Y axis within grouting circle, then applied the excess earth pressure and the gravity of the shield machine. The torque in front of cutter-head is simplified as distributed force produced by 4 lines of cutters. The friction in side of cutter-head is applied surround the layer of overbreak. The distributed force applied surround the shield is provided to simulate part of jacking force for overcoming the friction between shield and soil. Running a certain step. Finally, run a certain time step. The running step determined by trial excavation is the time step that soil on the roof of tunnel need to reach the max manoeuvring space.

Step 2-4: Install shield shells outside the grouting circle and drive 3 rings. There is 7 cm thickness of disturbed soil surround the shield.

Step 5: Delete the shield shells and the layer of overbreak and apply grouting pressure 0.2MPa around the strata in 6-7.5 m from the tunnel face. Run a certain time step.

Step 6: Delete the grouting pressure and create grouting circle which is as solid as concrete. Run a certain time step.

Step 7: Erect the segments and run a certain time step.

![Figure 6. Excavation procedure](image)

IV. RESULT COMPARISON BETWEEN EXPRESSIONS AND NUMERICAL SIMULATION

The value of strata loss is determined by calculation as follow. The thickness of shell is 45 mm and the manoeuvring space of shield tail is 10 mm. According to the study, strata loss ratio \( \eta \) is given by:

\[
\eta = \frac{(4Rg + g^2)}{(4R^3)} = 0.92\% \tag{33}
\]

The value of strata loss \( V_{loss} \) is:

\[
g = aGw + w + U_{3D}^* = 28.9mm \tag{34}
\]

Through numerical integration in MATLAB, the solutions introduced above can be shown in Figure 7 and Figure 8.

![Figure 7. The result comparison of longitudinal ground settlement between expressions and numerical simulation when the left tunnel advancing 16.5 m](image)

![Figure 8. The result comparison of lateral ground settlement of section X=6m between expressions and numerical simulation when the left tunnel advancing 22.5 m](image)
As shown in Fig. 7 and Fig. 8, the excess earth pressure, the lateral friction between shield skin and the surrounding soil and the grouting pressure have greater pressure, the lateral friction between shield skin and the deformation induced by shield tunnel is assumed as an S-shaped curve. Surfaces on both sides hunch up at the axis of excavation face. As shown in analytical calculation (see Figure 9), the calculated results of lateral and longitudinal ground settlement is in accordance with the data from numerical simulation.

![Figure 9. The result comparison of lateral ground settlement of section X=6m between expressions and numerical simulation when the double-tube tunnels advancing 22.5m.](image)

Because of ignoring the interaction of two tunnels when calculation, the curves of ground deformation obtained by simple superposition is smaller than those from numerical simulation. The max displacement occur in Y=0 m and the calculated value is 14.9 m while the numerical simulated is 15.6 m. Two curves are similar and the max error is less than 5% that is a proof for the reliability of the two methods.

V. CONCLUSION

On the basis of the Mindlin displacement solution, we fully analyzed the soils mechanical behavior during tunnel construction. In this paper, modified formula of ground deformation induced by ground loss was offered and applied to estimate settlement induced by the double-tube tunnels construction with superposition principle. The successive construction process of double-tube parallel shield tunnels in the same direction was simulated with FLAC3D and we could draw the conclusions as follow:

1. On the basis of the Mindlin displacement solution, through the coordinate transforming and mathematical deduction, we can obtain the expression of the ground settlement caused by the factors of tunnel construction respectively. With the modified settlement formula of soil loss and the superposition theory, we can predict the ground surface settlement curve of double-tube parallel shield tunnels.

2. The process of tunnel construction was simulated with FLAC3D, taking account of the construction parameters, equipment and geological conditions of construction. By comparing the simulated results with analytic solutions, we found that the calculations were consistent with the simulated results.

ACKNOWLEDGMENT

The authors gratefully acknowledge the financial support provided by the National Natural Science Foundation of China (No. 11032005), the Science and Technology Scheme of Guangdong Province (No. 2012A030200003), the Science and Technology Scheme of Guangzhou City (No. 1563000451) and the Fundamental Research Funds for the Central Universities of China (No. 17817004).

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