3D Washout Analysis of Bridge Superstructures Using the Explicit MPS Method

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***Abstract*—The Great East Japan Earthquake, which occurred on March 11, 2011, inflicted serious damage on civil and architectural structures in northeastern Japan. The tsunami generated by the earthquake washed away the superstructures of road bridges. The damage caused by this disaster had a strong impact on the design of road bridges. Against this background, numerical simulation is expected to become an important tool to evaluate the tsunami force acting on bridge structures. This study aims to apply explicit moving particle simulation (E-MPS), which is one of the particle methods, to the three-dimensional (3D) simulation of fractures of a bridge superstructure. A dam break problem with a floating rigid body is solved by E-MPS, and the results are compared with those obtained by the corresponding experimental approach to verify the developed E-MPS code. The tsunami run-up and simulation of a bridge being washed away are implemented by the E-MPS, and their realistic visualization results are demonstrated by the help of marching cube method.**

***Index Terms*—Explicit MPS (E-MPS), particle method, tsunami simulation, bridge washout analysis**

I. Introduction

The proposed study uses explicit moving particle simulation (E-MPS) to demonstrate three-dimensional (3D) simulation of tsunami run-up in a river and washout of a bridge superstructure.

The Great East Japan Earthquake, which occurred on March 11, 2011, inflicted serious damage on civil and architectural structures [1] in northeastern Japan. In particular, the superstructures of many bridges were washed away when the tsunami shifted the upstream of the river and the water level exceeded the bridge height [2]. Therefore, in recent years, several measures have been taken to make bridges stronger [3] against the possible powerful earthquakes in the future. However, the hydraulic force exerted by the tsunami run-up in the river on the bridge and the fracture mechanism of the bridge resulting from the tsunami are still unclear. Several studies were conducted based on the numerical methods to address these unclear points. For example, Motohashi [4] et al. estimated the hydraulic force acting on the bridge exerted by the tsunami using an open source software program. Wei et al. calculated the hydraulic force of the tsunami using smoothed particle hydrodynamics (SPH), which is one of the particle methods [5].

The particle method is known as a powerful numerical technique, and it approximates the continuum motion by the motion of discrete particles. Therefore, the particle method does not require computational meshes such as the ones used in the finite element method (FEM) and boundary element method (BEM) [6]. Moreover, the particle method can easily handle the large deformation of continuum bodies. The SPH method and moving particle semi-implicit (MPS) are popular particle methods. MPS was first proposed by Koshizuka et al. [7], and the computational method was used for simulating incompressible flows with free surfaces. The MPS algorithm is similar to that of SPH [8]. However, the MPS generally needs more computational time and memory than the SPH for dealing with a large number of particles because the pressure Poisson equation resulting from the Navier–Stokes equation must be solved implicitly. The SPH calculation scheme is explicit and requires less computational cost in comparison with MPS. However, in general, the computational accuracy of the SPH is inferior to that of the MPS for incompressible flow problems.

The explicit MPS (E-MPS) was developed by Shakibaeinia et al. [9] and Oochi et al. [10] to improve the computational intensity of the MPS. In the E-MPS, the implicit computational scheme of the MPS for the pressure Poisson equation is solved explicitly by considering the density as a function of pressure.

Therefore, the proposed study develops a numerical method using the E-MPS to reproduce the destruction of the bridge superstructure caused by the tsunami run-up in a river. The following text explains the problem statement and its analysis model in this research. Next, brief description of the E-MPS is given, and the treatment of the bridge superstructure, which is considered a rigid body, is explained. Then, a dam break problem with a floating rigid body is solved to verify the proposed E-MPS code. Finally, the 3D washout analysis of bridge superstructure is demonstrated using the E-MPS, and the results are visualized by the help of the marching cube method.

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II. E-MPS Formulation

A. Problems

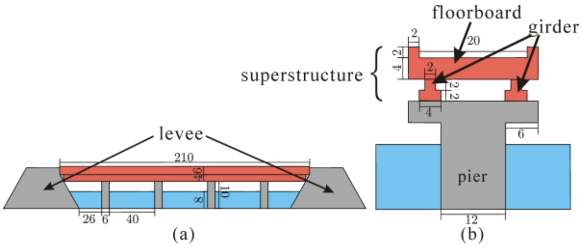


Figure 1. Analysis model (a) bridge structure and (b) its cross section. The unit of length is (m).

In this research, we consider a bridge with 5 spans, its piers, and levees, as shown in Fig.1. A tsunami hits the bridge and washes out the superstructure. For simplicity, the deformation of the piers, levees, and superstructure is neglected because it is much smaller than that of fluid itself. In other words, in this work, the piers, levees, and superstructure are assumed as rigid bodies, and the structures and river bed are treated as the wall particles.

B. Governing Equations and Discretization

The continuity and Navier–Stokes equations at time t are written, respectively, as follows:

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\boldsymbol{u} = 0 \tag{1}$$

$$\frac{D\boldsymbol{u}}{Dt} = -\frac{1}{\rho}\nabla P + \nu\nabla^2\boldsymbol{u} + \boldsymbol{g} \tag{2}$$

where ρ is the density, $\boldsymbol{u}$ is the fluid velocity vector, and P is the fluid pressure. In addition, ν and $\boldsymbol{g}$ are the kinematic viscosity and gravity acceleration, respectively. Both the continuity and Navier–Stokes equations (1) and (2) are discretized by using the well-known particle interaction models for ∇ and ∇^2as follows:

$$\langle\nabla P\rangle_i = \frac{d}{n^0_{grad}}\sum_{j\neq i}\left[\frac{(P_j+P_i)(\boldsymbol{r}_j-\boldsymbol{r}_i)}{|\boldsymbol{r}_j-\boldsymbol{r}_i|^2}\omega_{\mathrm{grad}}(|\boldsymbol{r}_j-\boldsymbol{r}_i|)\right] \tag{3}$$

$$\langle\nabla^2\boldsymbol{u}\rangle_i = \frac{2d}{\lambda^0 n^0}\sum_{j\neq i}\left[(\boldsymbol{u}_j-\boldsymbol{u}_i)\omega(|\boldsymbol{r}_j-\boldsymbol{r}_i|)\right] \tag{4}$$

where the subscript (i or j) shows the parameter of i or j -th particle. Further, $d, \boldsymbol{r}$, and λ^0 are the number of space dimension, position vector of the particle, and correction parameter used in the particle method, respectively. In addition, $\omega(|\boldsymbol{r}|)$ and n^0 are the weight function and initial value of the particle number density, respectively. The subscript grad appeared in (3) represents the calculation term about gradient. The weight functions $\omega_{grad}(r)$ and $\omega(r)$ can be calculated as follows:

$$\omega_{grad}(r) = \begin{cases} \dfrac{r_e}{r} - \dfrac{r}{r_e} & (r < r_e) \\ 0 & (r \geq r_e) \end{cases}, \tag{5}$$

$$\omega(r) = \begin{cases} \dfrac{r_e}{r} + \dfrac{r}{r_e} - 2 & (r < r_e) \\ 0 & (r \geq r_e) \end{cases} \tag{6}$$

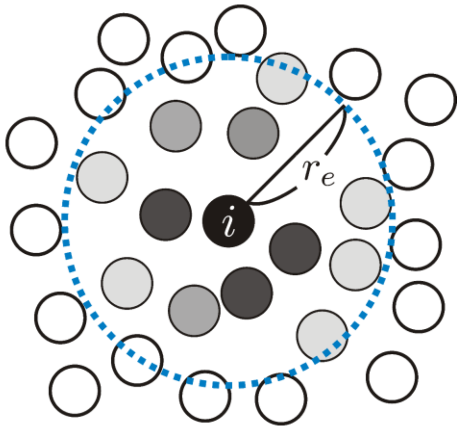


Figure 2. Influence radius r_e of a particle.

where r is the distance between particles and r_e is the influence radius of a particle, as shown in Fig.2. The parameters n^0_{grad} and n^0 in (3) and (4), respectively, can be defined as follows:

$$n^0_{grad} = \sum_{j\neq i}\omega_{grad}(|\boldsymbol{r}_j-\boldsymbol{r}_i|) \tag{7}$$

$$n^0 = \sum_{j\neq i}\omega(|\boldsymbol{r}_j-\boldsymbol{r}_i|). \tag{8}$$

Equations (1) and (2) can be calculated by using (3) and (4). The details of the calculations can be seen in other published papers (for example, see [7]).

C. Numerical Algorythm for E-MPS

In this subsection, the numerical algorithm for E-MPS is described with the flowchart as shown in Fig.3. In the E-MPS algorithm, the left-hand side of Navier–Stokes equation (2) is discretized by using the explicit Euler method. Considering the intermediate velocity $\boldsymbol{u}_i^*$, the left-hand side of equation (2) can be rewritten as follows:

$$\frac{D\boldsymbol{u}}{Dt} = \frac{\boldsymbol{u}_i^{k+1}-\boldsymbol{u}_i^k}{\Delta t} = \frac{\boldsymbol{u}_i^*-\boldsymbol{u}_i^k}{\Delta t} + \frac{\boldsymbol{u}_i^{k+1}-\boldsymbol{u}_i^*}{\Delta t} \tag{9}$$

where Δt is the time increment, and $\boldsymbol{u}_i^k$ is the velocity of particle i at k-th time step. In addition, the superscript * indicates the physical quantity at the intermediate step, and $\boldsymbol{u}_i^*$ can be calculated as follows:

$$\boldsymbol{u}_i^* = \boldsymbol{u}_i^k + \left(\nu\langle\nabla^2\boldsymbol{u}\rangle_i^k + \boldsymbol{g}\right)\Delta t. \tag{10}$$

The particle position $\boldsymbol{r}_i^*$at the intermediate step can be calculated as shown below:

$$\boldsymbol{r}_i^* = \boldsymbol{r}_i^k + \boldsymbol{u}_i^* \Delta t. \quad (11)$$

The velocity $\boldsymbol{u}_i^{k+1}$ and particle position $\boldsymbol{r}_i^{k+1}$ at the $k+1$-th time step can be obtained from the second term on the right-hand side of (9) using the pressure value ∇P^{k+1} at the $k+1$ step as follows:

$$\boldsymbol{u}_i^{k+1} = \boldsymbol{u}_i^* - \frac{\Delta t}{\rho_i^0} \langle \nabla P \rangle_i^{k+1} \quad (12)$$

$$\boldsymbol{r}_i^{k+1} = \boldsymbol{r}_i^* + \left(\boldsymbol{u}_i^{k+1} - \boldsymbol{u}_i^*\right) \Delta t. \quad (13)$$

In the standard MPS, the Poisson equation for the pressure obtained from (12) can be solved implicitly to obtain pressure P^{k+1} at the $k+1$-th step. However, this implicit calculation scheme requires much computational time and memory. Therefore, this implicit scheme is simplified by evaluating the pressure P^{k+1} as a function of density, as follows:

$$P^{k+1} = \begin{cases} c^2(\rho^* - \rho^0) & (\rho^* > \rho^0) \\ 0 & (\rho^* \le \rho^0) \end{cases} \quad (14)$$

where c is the speed of sound. The free surface can be treated by assuming pressure $P = 0$ when the reference pressure P^0 is given by $P^0 = 0$, and the density ρ^* at the intermediate step is smaller than the reference density ρ^0. Assuming density ρ^* at the intermediate step is proportional to the sum of the weight function ω, the density ρ^* is given as follows:

$$\rho^* = \frac{\rho^0}{n^0} \sum_{j \neq i} w\left(\left|\boldsymbol{r}_j^* - \boldsymbol{r}_i^*\right|\right). \quad (15)$$

A large time step size Δt can be set by considering a lower virtual speed of sound than the actual one. In the Navier–Stokes equation, the fluid is assumed incompressible. Consequently, the Mach number $M_a = |u|/c$, which is the dimensionless quantity representing the ratio of flow velocity past a boundary to the local speed of sound, must satisfy $M_a = |u|/c \ll 1.0$. In this research, the Mach number M_a is given by $M_a = 0.2$, and the speed of sound can be calculated as shown below:

$$c = \frac{u_{\max}}{0.2}, \quad (16)$$

where $u_{\max}$ is a predicted value of the maximum fluid velocity, and it is given as $u_{max} = \sqrt{2gH}$ for the dam break problem for height H of the water column.

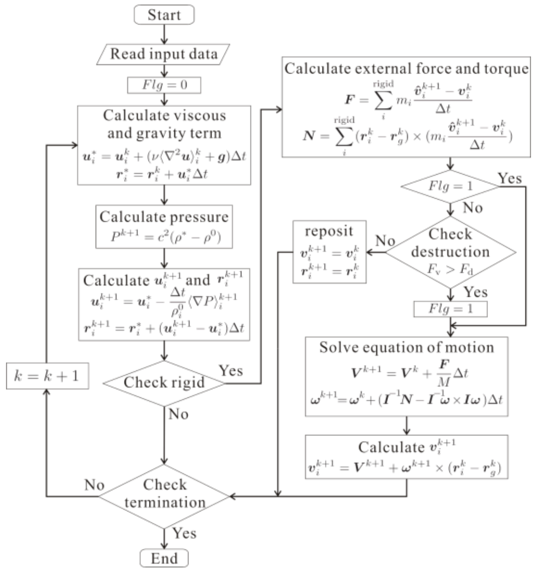


Figure 3. Flowchart of this numerical analysis.

III. Modeling of the Bridge Structure

As mentioned before, the superstructure of the bridge is considered as a rigid body in this research. Therefore, dealing with a rigid body structure using the E-MPS is briefly described in this section.

A. Calculation for a Bridge Superstructure

A rigid body has three translational and rotational degrees of freedom in 3D space. The equations of motion of a rigid body for translation and rotation are written as follows:

$$M \frac{d\boldsymbol{V}}{dt} = \boldsymbol{F}, \quad (17)$$

$$\boldsymbol{I} \frac{d\boldsymbol{\omega}}{dt} + \boldsymbol{\omega} \times \boldsymbol{I\omega} = \boldsymbol{N}, \quad (18)$$

where M and $\boldsymbol{I}$ are the mass and inertia tensor of a rigid body, respectively. In addition, $\boldsymbol{V}$ and $\boldsymbol{\omega}$ are the translational and rotational speed of the gravity center of a rigid body, respectively, and $\boldsymbol{F}$ and $\boldsymbol{N}$ are the external force and torque acting on a rigid body, respectively. For coupling the translation and rotation equations (17), (18) and the Navier–Stokes equation (2), the particles for the rigid body are once considered as those for the fluid, and the calculation is performed. Thereafter, the calculation is performed to preserve the momentum and the angular momentum of each particle, which is regarded as a fluid particle. The external force $\boldsymbol{F}$ and torque $\boldsymbol{N}$ acting on a rigid body can be calculated as follows:

$$\boldsymbol{F} = \sum_{i}^{\text{rigid}} m_i \frac{\widehat{\boldsymbol{v}}_i^{k+1} - \boldsymbol{v}_i^k}{\Delta t}, \quad (19)$$

$$\boldsymbol{N} = \sum_{\text{i}}^{\text{rigid}} (\boldsymbol{r}_i^k - \boldsymbol{r}_g^k) \times (m_i \frac{\widehat{\boldsymbol{v}}_i^{k+1} - \boldsymbol{v}_i^k}{\Delta t}) \quad (20)$$

where m_i is the mass of the i-th particle, $\boldsymbol{r}_g$ is the gravity center position of a rigid body, and the word “rigid” in equations (19) and (20) indicates the total number of the particles for a rigid body. By applying (19) and (20) to the translational and rotational equations of motion (17) and (18), the translational and rotational speed of a rigid body $\boldsymbol{V}^{k+1}$ and $\boldsymbol{\omega}^{k+1}$, respectively. The explicit Euler method is applied to the time discretization of (17) and (18). The particle velocity $\boldsymbol{v}_i^{k+1}$ of the rigid body can be obtained using $\boldsymbol{V}^{k+1}$ and $\boldsymbol{\omega}^{k+1}$ as follows:

$$\boldsymbol{v}_i^{k+1} = \boldsymbol{V}^{k+1} + \boldsymbol{\omega}^{k+1} \times \left(\boldsymbol{r}_i^k - \boldsymbol{r}_g^k\right). \tag{21}$$

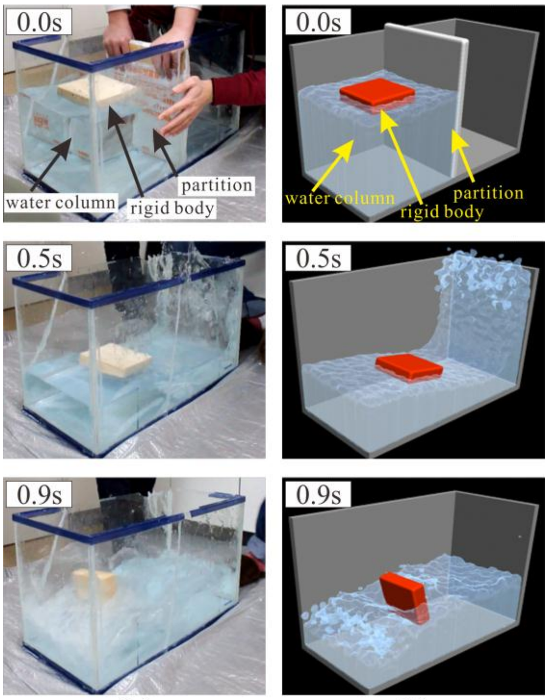
The particle position $\boldsymbol{r}_i^{k+1}$ of the rigid body can be calculated using the rotation with quaternion [11].

B. *Washout Condition of the Bridge Superstructure*

The beam, pier, and substructure of the superstructure are fixed on shoes. The shoes are broken by the upward force acting on the bridge superstructure. In this research, the washout condition of the bridge superstructure is defined as shown below:

$$F_{\mathrm{v}} > F_{\mathrm{d}}, \tag{22}$$

where F_v is the vertical upward external force acting on the rigid body and F_d is the threshold value for judging the destruction of the superstructure. In other words, if (22) is satisfied, the bridge superstructure is washed out. The calculation flow of E-MPS implemented in this research is described in Fig.3. As shown in Fig.3, the computational scheme is implemented explicitly. The parameter Flg in Fig. 3 shows whether the bridge superstructure flows out. The parameter Flg in Fig.3 takes the value $Flg = 1$ after the washout, and since then, the washout calculation for the bridge superstructure is implemented.



(a) (b)

Figure 4. Dam break problem (a) experimental results (b) numerical results.

IV. NUMERICAL EXAMPLE

Some numerical examples obtained by the E-MPS are shown in this section. All the numerical simulations are performed in three dimensions. Therefore, the dimensional parameter d is given as $d = 3$. The influence radius r_e required for the calculation of the weight functions ω_{grad} and ω are set as $r_e = 2.1\Delta s$ and $3.1\Delta s$, respectively, where Δs is the diameter of the particle. The OpenMP parallelization is utilized to save the computational time in this research.

A. *Confirmation of the Validity of the Analysis Code*

First, the validity of the E-MPS code is evaluated by solving a dam break problem, as shown in Fig4. This dam break problem is different from the usual one in which a rigid body with a low density is present as a floating body in the tank of water. The dimensions of the water tank, water column, and floating rigid body are 60 × 45 × 30 cm, 30 × 30 × 30 cm, and 14 × 4 × 14 cm, respectively. The parameters used in this analysis are listed in Table I. The speed of sound c in Table 1 can be obtained from (16), where $u_{\max}$ is calculated by considering the vertical height of the water column H. In addition, the number of particles for the floating rigid body, water tank, fluid, and partition board for the dam break problem are 784, 45616, 25200, and 2700, respectively, and the total number of particles is 74300. The number of total time steps N_{step} is $N_{step} = 7000$.

TABLE I. ANALYSIS PARAMETERS USED IN THE DAM BREAK PROBLEM.

Particle spacing:Δs	0.01m
Time increment: Δt	5.0×10^{-4}s
Density of fluid: ρ_f	1000kg/m^3
Density of rigid: ρ_g	395kg/m^3
Gravity: g	9.8m/s^2
Kinematic viscosity: ν	1.0×10^{-6}m^2/s
Sound speed: c	12.1m/s

TABLE II. ANALYSIS PARAMETERS USED IN THE WASHOUT ANALYSIS OF THE BRIDGE SUPERSTRUCTURE.

Particle spacing:Δs	2.0m
Time increment: Δt	1.2×10^{-2}s
Density of fluid: ρ_f	1000kg/m^3
Density of rigid: ρ_g	3000kg/m^3
Gravity: g	9.8m/s^2
Kinematic viscosity: ν	1.0×10^{-6}m^2/s
Sound speed: c	110.9m/s

Numerical results obtained by E-MPS are shown in Fig.4(b). The three figures in Fig.4(b), from top to bottom, show the initial state of the water column and floating rigid body, the situation when the collapsed water column strikes the wall on the right side, and the time when colliding water returned from the wall. For comparison, the corresponding experimental results are presented for each situation of Fig.4(b) in Fig.4(a). The partition board is rapidly removed at $t = 0.0s$, and the water column collapsed under gravity. Thereafter, the collapsed water column hits the side wall of the water tank at about $t = 0.5$ s, and the hydraulic jump occurs. The floating rigid body is located horizontally just before the water column collapsed at $t = 0.0$ s. However, it can be observed that the floating rigid body rotates at about $t = 0.9$ s when the colliding water returns to the original side. In any case, the numerical results, as shown in Fig. 4(b), obtained by the E-MPS are in good agreement with the experimental results shown in Fig.4(a).

In general, the particle-based method, such as MPS and SPH methods, can solve the dynamic behavior of the fluid. Therefore, to reproduce the dynamic behavior of a fluid, various methods are used to visualize the particles obtained by the numerical analysis and to express them more like fluids. In this research, all the particles analyzed by the E-MPS were polygonized using the marching cube method [12], and the fluid and its surface were reproduced by the superposition of the polygonized objects for realistic visualization.

B. 3D Washout Analysis of the Bridge Superstructure

Next, the numerical results for the 3D washout analysis of the bridge superstructure, as shown in Fig. 1, are presented. Analysis parameters used in this washout analysis, as presented in Table II. In general, the 3D analysis using the particle-based method requires much computational time and memory. Therefore, in this analysis, the inflow and outflow boundary conditions are considered, as shown in Fig. 5, to save the computational load; i.e., the influence of the tsunami that has sufficiently moved upstream of the river on this analysis is not considered in this research. From this viewpoint, the number of particles of fluid required for the modeling is changed in each time step. The number of particle for the bridge superstructure, which is considered as rigid body, for the total of wall, levee and river bed, and fluid are 3380, 97430, and 319690, respectively. Therefore, the total number of particles is 420500 in the final time step. In addition, the total time step, N_{step}, is given by $N_{step} = 1000$. The washout condition F_d for the bridge superstructure in (22) is given by $F_d = 1000\text{MN}$.

Several ways have been proposed to excite a tsunami for particle-based methods. In this research, a pseudo-tsunami was created by setting an inflow boundary on the front side of the bridge. Particles flowing with a constant velocity, u_{in}=10 m/s, from the boundary and a gradually raising the water level, as shown in Fig.5, are considered. The maximum height of the excited tsunami is 20 m, and the particles are arranged in the horizontal direction over a length of 40 m, as shown in Fig. 5. The speed of sound, c, as listed in Table II is calculated by (16). The parameter u_{max}, which is required for (16), is obtained by $u_{max} = \sqrt{u_{in}^2 + 2gH}$.

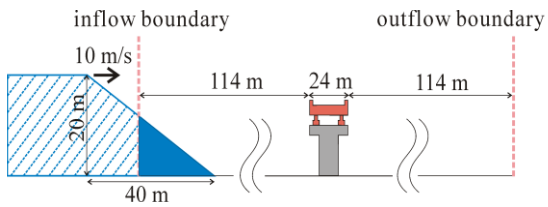


Figure 5. Inflow and outflow boundaries.

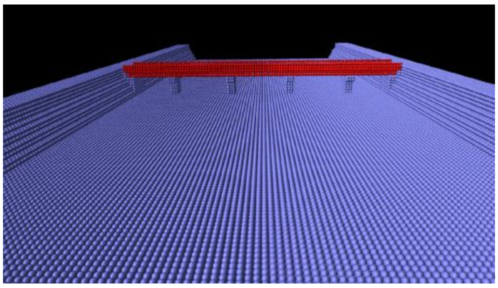


Figure 6. Initial particle locations for the analysis model.

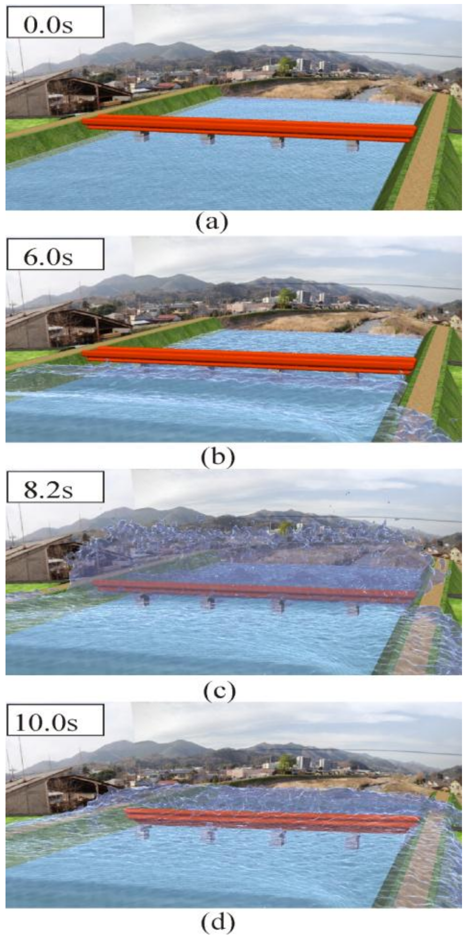


Figure 7. Numerical results for 3-D washout analysis of the bridge superstructure.

The initial location of the particles for the analysis model in Fig. 1 is shown in Fig. 6. In this analysis, the bridge superstructure is considered as the rigid body. Therefore, the particles of the bridge superstructures considered as rigid are shown in red, and the remaining are indicated by blue particles.

Fig. 7 shows the numerical results for the 3D washout analysis using E-MPS. Fig. 7(a)–(d) show the tsunami run-up in the river and the bridge at different points of time, and the run-up is visualized using the marching cube method, as in the previous dam break problem. Note that the tsunami moves upstream along the river from the front of the bridge, and the presence of the tsunami run-up in the river is reproduced by inserting a digital image of a river in the background. The tsunami enters the analysis area from the inflow boundary at $t = 0.0$s, as shown in Fig. 7(a). Subsequently, the tsunami gradually approaches the bridge, as shown in Fig. 7(b). Then, the tsunami swallows the bridge, as shown in Fig. 7(c), and the bridge superstructure is washed away owing to the tsunami attack in Fig. 7(d). Thus, the phenomenon of washing away the bridge superstructure by the tsunami run-up in the river could be qualitatively reproduced by E-MPS simulation.

V. CONCLUSION

In this research, 3D washout analysis of the bridge superstructure was implemented by E-MPS, a particle-based method. The formulation for the E-MPS method and the 3D washout analysis model were explained. The dam break problem with a floating rigid body was solved by the E-MPS to validate the developed E-MPS code, and the results were compared with those obtained by the corresponding experiment. In addition, tsunami run-up and bridge-wash-out simulations were demonstrated. In the future, the hydrodynamic force that the tsunami exerts on the bridge will be estimated using this developed E-MPS code. A realistic visualization of the numerical results obtained by the particle method was accomplished. This research may help the tsunami evacuation training using virtual reality (VR). Therefore, the developed E-MPS code will be integrated into the system of the tsunami simulator developed by us, and it will incorporate a VR system in the future. In addition, the development of the hybrid technique of the E-MPS and the other numerical methods, such as FEM [13] and the innovative time-domain BEM [14], will be tried. The tsunami fluid force acting on bridges will be estimated by the E-MPS.

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