An Investigation on Ignoring P-Delta Effects Based on Stability Index

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***Abstract*— Stability coefficient is defined as the ratio of the moment induced by the P-Delta effects to the moment due to the lateral forces. For columns with small lateral displacement, it is perfectly acceptable to neglect the P-Delta effects, but for columns which experience high levels of nonlinearity it is crucial to accurately capture the P-Delta effects. An important issue regarding the P-Delta effect is the threshold of safely ignoring it, without compromising on well-being of the structure or being too conservative. Whenever P-Delta effects can be ignored Caltrans SDC provides predefined displacement ductility levels for designing different structural components. Stability coefficient has been the basis for many scholar works regarding design for P-Delta effects. Conservative limits on elastic and plastic stability coefficient have been introduced in order to prevent P-Delta effects from becoming dominant in the structural response. Three different safe thresholds for ignoring the P-Delta effects which are based on stability coefficient have been subjected to study in this research. The displacement ductility corresponding to ignoring the P-Delta effects obtained from these methods are being compared with the Caltrans SDC design target ductility. Using nonlinear time-history analysis ductility levels corresponding to ignoring the P-Delta effects have been evaluated. This research intends to evaluate Caltrans SDC design target ductility for single column bents supported on fixed foundation and compare it with displacement ductility levels corresponding to ignoring the P-Delta effects obtained from methods which are based on stability coefficient.**

***Index Terms*—stability coefficient, P-Delta effects, nonlinear time history analysis, RC bridge column design, caltrans SDC, ignoring the P-Delta effects**

I. Introduction and Background

P-Delta effects can have a detrimental impact on the seismic response of bridges because of a reduction in both the shear capacity and initial stiffness of RC bridge columns [1]-[2]. The reduction in the initial stiffness imposes an increase in the natural period of the system, and a likely surge in the design displacement demand. However, studies by Jennings and Husid [3] have shown that depending on the profile of an earthquake response spectrum the reverse may actually occur when analyzing or testing slender RC bridge column under the effect of ground motions. They also have concluded that for most cases of interest the time to collapse depends mainly on the ratio of the earthquake strength to the yield strength of the structure, and not upon their individual values. These conclusions highlight some of the needs in predicting within a reasonable degree of accuracy the seismic response of RC bridge columns under P-Delta effects. As design codes are progressing towards performance-based metrics, there is an additional need to quantify the destabilizing effect of gravity loads and its effect on the seismic response of bridge columns.

Regarding the P-Delta effects, two issues are typically of special concern. First concern is the threshold of safely ignoring P-Delta effects and more importantly upper design limit for P-Delta effects[4]. The earlier limit-state is determined by limiting the amplification requirements, whereas the latter one is governed by collapse-prevention criteria [4]. Codes tend to control the P-Delta effects through simplistic procedures involving first order structural linear analysis [5] or by imposing a conservative limit for lateral displacement which prevents the P-Delta effects from becoming dominant in the structures response (drift limits) [6]. Another commonly used method in order to make P-Delta effects negligible is limiting the ratio of the P-Delta induced moment to the moments induced by lateral forces (Stability coefficient limits).

Caltrans SDC [7] provides a procedure that can be used to evaluate whether P-Delta effects can be ignored in design. In design circumstances, not requiring considering P-Delta effects, structural components can be designed based on predefined ductility demands. In cases which (1) is not satisfied, increasing the section size or reinforcement ratio can be used to increase the yielding moment capacity of the colum, Howevre, Caltrans SDC reccommends to perform nonlinear time-history analysis to verify whether the column is capable to resist the P-Delta effects.

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$$P \times D_r \leq 0.2\, M_p^{col} \tag{1}$$

where, D_r is the lateral offset between the point of contra-flexure and the base of the plastic hinge, and M_p^{col} is the idealized plastic moment capacity of a column calculated by M-φ analysis. If (1) is satisfied, predefined ductility demands limits the design of structural components. According to Caltrans SDC target displacement ductility of four is suggested for single column bents.

Regarding the P-Delta effects, two issues are typically of special concern. First concern is the threshold of safely ignoring P-Delta effects, and more importantly upper design limit for P-Delta effects [4]. The earlier limit-state is determined by limiting the amplification requirements, whereas the latter one is governed by collapse-prevention criteria. Codes intend to control the P-Delta effects through simplistic procedures involving first order structural linear analysis [5] or by imposing a conservative limit for lateral displacement which prevents the P-Delta effects from becoming dominant in the structures response (drift limits). Another commonly used method in order to control the P-Delta effects is by limiting the ratio of the P-Delta induced moment to the moments induced by lateral forces (Stability coefficient limits). Finding the threshold for ignoring the P-Delta effect has been the subject of many studies. Variables introduced here have been used by many researchers. Elastic stability index is defined as following.

$$\theta_e = \frac{P}{F_{y0}.H/\Delta_y} \tag{2}$$

Nonlinear stability index is proposed by Paulay [8], and is defined as

$$\theta_\Delta = \theta_e\, \mu \,, \mu = \frac{\Delta_u}{\Delta_y} \tag{3}$$

For SDOF systems, Pauley proposed that P-Delta effects could be neglected if $\theta_\Delta < 0.15$. Bernal [9] and Mahin and Boroschek [1] suggested that if the required strength amplification to achieve a specific ductility was less than 10%, then P- Delta effects could be ignored. Using this criterion, Mahin and Boroschek suggested $\theta_\Delta <$ 0.20 as the threshold for ignoring the P-Delta effects. FEMA 450 [10] identifies $\theta e \leqslant 0.10$ as the design tolerance for P-Delta effects. Priestley et al. [11]contended that to obtain stable structural response without producing significant P-Delta displacement, the stability index θ_Δ should be less than 0.30.

II. Method

In this research the displacement ductility at which P-Delta effects can be ignored based on each method is calculated. The corresponding ductility level at which P-Delta effects can be ignored can be calculated using (4).

$$\mu \frac{P\Delta_y}{F_{y0}.H} = \beta, \begin{cases} Paulay: \beta = 0.15 \\ Boroschek: \beta = 0.2 \\ Priestly: \beta = 0.3 \end{cases} \tag{4}$$

This equation can be further simplified into (5).

$$\mu = \frac{\beta}{\theta_e} \tag{5}$$

In order to obtain the elastic stability index it is required to perform nonlinear static analysis (pushover analysis).

A. Pushover Analysis

In this research using OpenSees [12], nonlinear static analysis (Pushover) is performed to obtain the moment-curvature and subsequently the load-deformation for the columns (Fig. 1). Table I illustrates the column properties.

TABLE I. Column Properties

Concrete Strength, f'c (MPa, ksi)	27.5 (4)
yield Strength, fy (MPa, ksi)	413 (60.0)
Reinforcement ratio	1 to 4%
Modulus of elasticity, Es (MPa, ksi)	2×10^5 (29,000)
Column diameter, L (m, ft)	1.21 (4)
Column height ratio, CHR	4 to 12
Cover concrete (cm, in)	5 (2)
Axial load (kips)	389,584,778,973,1168

The main objective of performing pushover analysis is to obtain the value for the load and displacement at yielding and ultimate capacity of the column.

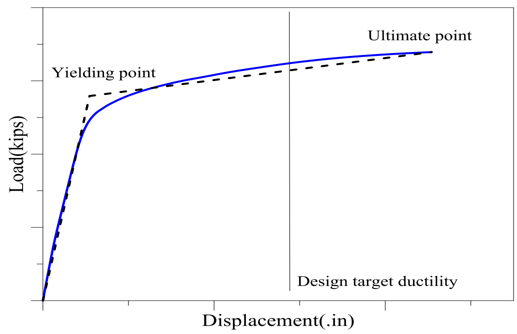

Figure 1. Load-deformation obtained from pushover analysis.

B. Results Evaluation

Nonlinear time-history analysis is used to evaluate the P-Delta effects by comparing the structural response with and without P-Delta effects.

III. Results

Throughout this research nonlinear pushover and time history analyses were performed using the open source object-oriented nonlinear structural analysis program, Open System for Earthquake Engineering Simulation (OpenSees) [12].

In order to obtain the elastic stability index push over analysis has been performed on RC bridge columns with 5 different axial load ratios and 5 different column height ratios. Axial load and column height are being categorized as low, medium and high (Table II).

TABLE II. AXIAL LOAD AND COLUMN HEIGHT CATEGORIES

	Low	Medium	High
Axial Load (kips)	389	584,778	973,1168
Axial load ratio	4	6,8	10,12
Column height (ft)	16, 20, 24	28, 32, 36	40, 44, 48
Column height ratio	4,5,6	7,8,9	10,11,12

Fig. 2 shows a sample result obtained from pushover analysis. Using the load and displacement at yielding point the elastic stability index can be obtained (2).

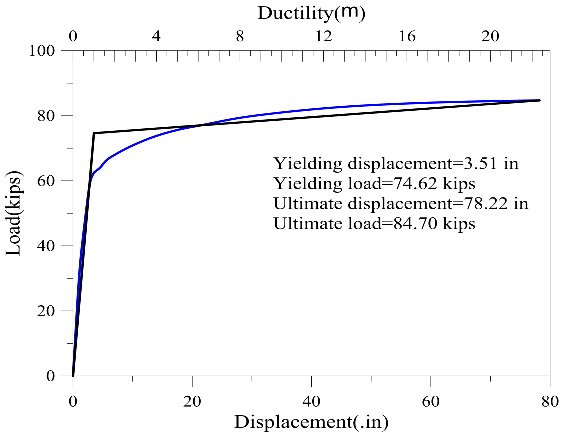

Figure 2. Pushover analysis sample (Col spec: Col Height=32 ft., Axial load=389kips, ρ_L=1%).

Table III shows the results obtained from pushover analysis for columns with 584 and 778 kips axial load. The displacement ductility corresponding to ignoring the P-Delta effects are presented in Fig. 3.

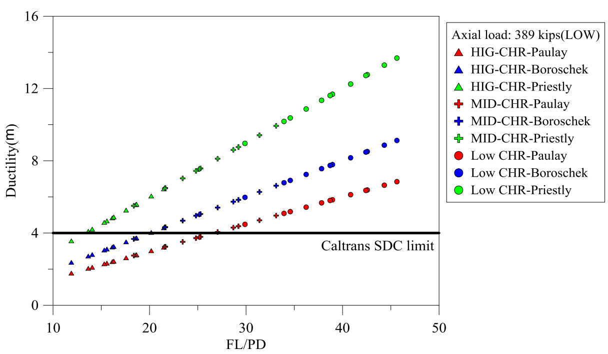

Figure 3. Ductility versus F_yL/PD (389kips).

Fig. 4 and Fig. 5 show the displacement ductility levels corresponding to ignoring the P-Delta effects for columns with 778kips (Medium axial load) and 1168kips (High axial load). Caltrans SDC design target ductility for single column bents supported on fixed foundation is shown by a straight line in the figures.

TABLE III. IGNORING P-DELTA EFFECTS DUCTILITY

	ρ	CHR	FL/PD	Paulay	Boroschek	Priestly		ρ	CHR	FL/PD	Paulay	Boroschek	Priestly
AXL=584 kips	1%	4	26.36	3.95	5.27	7.91	AXL=778 kips	1%	4	20.10	3.02	4.02	6.03
		5	23.51	3.53	4.70	7.05			5	17.96	2.69	3.59	5.39
		6	20.05	3.01	4.01	6.02			6	15.54	2.33	3.11	4.66
	2%	4	28.51	4.28	5.70	8.55		2%	4	21.57	3.24	4.31	6.47
		5	26.08	3.91	5.22	7.83			5	19.65	2.95	3.93	5.89
		6	22.80	3.42	4.56	6.84			6	17.35	2.60	3.47	5.20
	3%	4	29.66	4.45	5.93	8.90		3%	4	22.32	3.35	4.46	6.70
		5	27.36	4.10	5.47	8.21			5	20.60	3.09	4.12	6.18
		6	24.30	3.64	4.86	7.29			6	18.32	2.75	3.66	5.50
	4%	4	30.47	4.57	6.09	9.14		4%	4	22.90	3.43	4.58	6.87
		5	28.36	4.25	5.67	8.51			5	21.31	3.20	4.26	6.39
		6	25.32	3.80	5.06	7.60			6	19.03	2.85	3.81	5.71
AXL=584 kips	1%	7	17.50	2.62	3.50	5.25	AXL=778 kips	1%	7	13.38	2.01	2.68	4.01
		8	14.78	2.22	2.96	4.44			8	11.43	1.71	2.29	3.43
		9	12.54	1.88	2.51	3.76			9	9.68	1.45	1.94	2.90
	2%	7	19.78	2.97	3.96	5.93		2%	7	15.01	2.25	3.00	4.50
		8	16.91	2.54	3.38	5.07			8	12.81	1.92	2.56	3.84
		9	14.59	2.19	2.92	4.38			9	11.04	1.66	2.21	3.31
	3%	7	21.11	3.17	4.22	6.33		3%	7	15.91	2.39	3.18	4.77
		8	18.18	2.73	3.64	5.45			8	13.75	2.06	2.75	4.13
		9	15.72	2.36	3.14	4.72			9	11.89	1.78	2.38	3.57
	4%	7	22.16	3.32	4.43	6.65		4%	7	16.68	2.50	3.34	5.00
		8	19.18	2.88	3.84	5.75			8	14.43	2.16	2.89	4.33
		9	16.58	2.49	3.32	4.97			9	12.51	1.88	2.50	3.75
AXL=584 kips	1%	10	10.82	1.62	2.16	3.25	AXL=778 kips	1%	10	8.34	1.25	1.67	2.50
		11	9.31	1.40	1.86	2.79			11	7.17	1.08	1.43	2.15
		12	8.11	1.22	1.62	2.43			12	6.24	0.94	1.25	1.87
	2%	10	12.55	1.88	2.51	3.76		2%	10	9.57	1.44	1.91	2.87
		11	10.90	1.64	2.18	3.27			11	8.26	1.24	1.65	2.48
		12	9.49	1.42	1.90	2.85			12	7.19	1.08	1.44	2.16
	3%	10	13.61	2.04	2.72	4.08		3%	10	10.22	1.53	2.04	3.07
		11	11.82	1.77	2.36	3.55			11	8.91	1.34	1.78	2.67
		12	10.30	1.55	2.06	3.09			12	7.76	1.16	1.55	2.33
	4%	10	14.36	2.15	2.87	4.31		4%	10	10.83	1.62	2.17	3.25
		11	12.50	1.88	2.50	3.75			11	9.43	1.41	1.89	2.83
		12	10.92	1.64	2.18	3.28			12	8.23	1.23	1.65	2.47

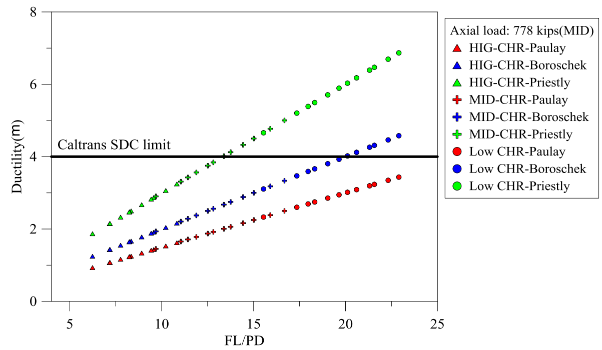

Figure 4. Ductility versus F_yL/PD (778 kips).

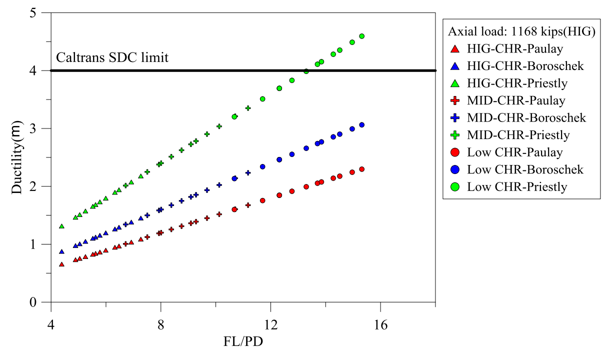

Figure 5. Ductility versus F_yL/PD (1168 kips).

where F_y is the yielding load, and L is column height, and P is the axial load, and D is yielding displacement. Table IV shows the mean and the standard deviation for columns subjected to this study. Columns based on their axial load and column height to diameter ratio were categorized into low, medium, and high. Caltrans SDC target ductility was considered as the benchmark which other method's results were compared to.

TABLE IV. TYPE SIZES FOR CAMERA-READY PAPERS

		Column height ratio								
		LOW			MID			HIG		
Axial load	LOW	Pauly	AVE	5.82	Pauly	AVE	3.87	Pauly	AVE	2.49
			STD	0.70		STD	0.65		STD	0.42
		Boroschek	AVE	7.76	Boroschek	AVE	5.16	Boroschek	AVE	3.32
			STD	0.93		STD	0.86		STD	0.56
		Priestly	AVE	11.65	Priestly	AVE	7.74	Priestly	AVE	4.98
			STD	1.39		STD	1.29		STD	0.84
	MID	Pauly	AVE	3.43	Pauly	AVE	2.30	Pauly	AVE	1.48
			STD	0.62		STD	0.49		STD	0.31
		Boroschek	AVE	4.58	Boroschek	AVE	3.06	Boroschek	AVE	1.97
			STD	0.83		STD	0.65		STD	0.42
		Priestly	AVE	6.87	Priestly	AVE	4.59	Priestly	AVE	2.96
			STD	0.83		STD	0.97		STD	0.63
	HIG	Pauly	AVE	2.19	Pauly	AVE	1.47	Pauly	AVE	0.95
			STD	0.30		STD	0.25		STD	0.16
		Boroschek	AVE	2.92	Boroschek	AVE	1.96	Boroschek	AVE	1.27
			STD	0.40		STD	0.34		STD	0.22
		Priestly	AVE	4.38	Priestly	AVE	2.94	Priestly	AVE	1.90
			STD	0.60		STD	0.51		STD	0.33

Cells which are colord green are proposing a design ductility less than Caltrans SDC, and red cells are suggesting higher ductilities than Caltrans SDC.

IV. CASE STUDY I

In this section a sample column with column height ratio of 8 (32ft tall) and axial load of 389 kips has been studied under multiple earthquake records. The column will fall under category of low axial load and medium column height ratio. Caltrans SDC suggests target ductility of 4 for designing such column. According to the Table III of this paper the average ductility level for ignoring the P-Delta effects for Paulay, Boroschek, and Priestly methods for columns with medium height and low axial load are 3.83, 5.16, and 7.74.

A. Evaluation of the Results

Nonlinear time-history analysis is performed on the column to evaluate whether the P-Delta effects can be ignored at the ductility level suggested by each method.

B. Ground Motion Set

The ground motion set is collected from Pacific Earthquake Engineering Research Center (PEER-NGA) database. Table V and Table VI tabulates the characteristics of the ground motions.

TABLE V. GROUND MOTION PROPERTIES

Distance R	R > 10 km
Large Magnitude Events	M > 6.5
Equal Weighting of Events	≤ 2 records per event
Strong Ground Shaking	PGA > 0.2g /PGV > 15 cm/sec
Source Type	Both Strike-Slip and Thrust Fault Sources
Site Conditions	Rock or Stiff Soil Sites Vs > 180 m/s
Record Quality	Lowest Useable Frequency < 0.25 Hz

TABLE VI. GROUND MOTION PROPERTIES

EQ ID	Earthquake			PGA_{max}	PGV_{max}
	M	Year	Name	(g)	(cm/s.)
120111	6.7	1994	Northridge	0.52	63
120121	6.7	1994	Northridge	0.48	45
120411	7.1	1999	Duzce, Turkey	0.82	62
120521	7.1	1999	Hector Mine	0.34	42

Fig. 6 shows ductility versus PGA for the column with and without P-Delta effects.

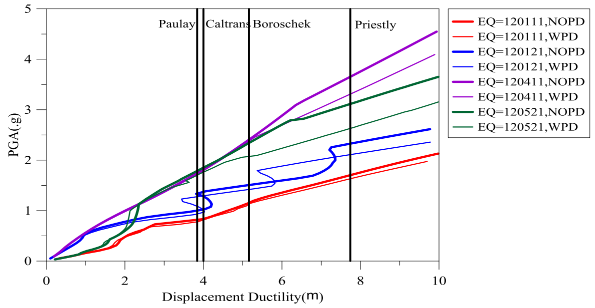

Figure 6. Ductility versus PGA for different ground motions(Case study I).

Fig. 7 and Fig. 8 show the time-history analysis for the column under earth quake records 120111 and 120521.

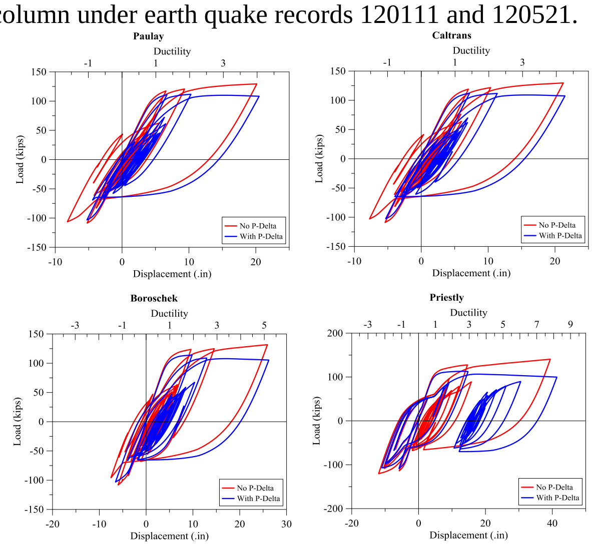

Figure 7. EQID=120111.

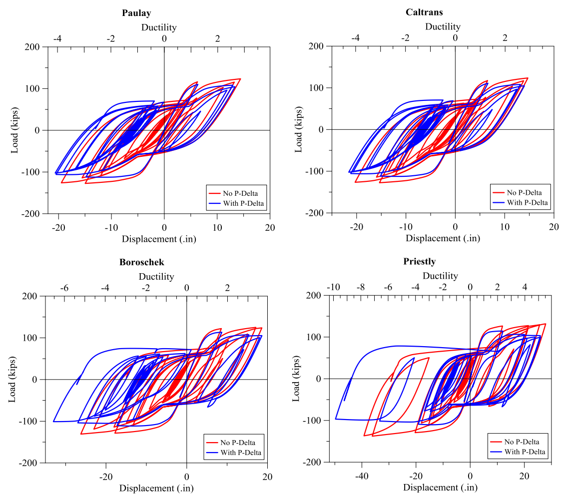

Figure 8. EQID=120521.

The earthquake records have been amplified to the ductility levels at which each method anticipated P-Delta effects can be ignored. Displacement ductility obtained after inclusion of the P-Delta effects from Boroschek, and Priestly methods are 20% or more under EQID120521 which is significant, and cannot be ignored. In all cases at target ductility of four which is suggested by Caltrans SDC P-Delta effects can be ignored.

V. Case Study II

In this section a sample column with column height ratio of 10 (40ft tall) and axial load of 584 kips has been studied under multiple earthquake records. The column will fall under category of medium axial load and high column height ratio. Caltrans SDC suggests target ductility of 4 for designing such column. According to the Table III of this paper the average ductility level for ignoring the P-Delta effects for Paulay, Boroschek, and Priestly methods for columns with medium height and low axial load are 1.48, 1.97, and 2.96.

Same earthquake records as case study I is used.

Displacement ductility obtained with and without the P-Delta effects is shown in Fig. 9 The earthquake records have been amplified to the ductility levels at which each method anticipated P-Delta effects can be ignored (Fig. 10 and Fig. 11). In this case study the difference between the obtained ductility levels with and without P-Delta effects for the first three methods is negligible, but for the Caltrans SDC method under EQID120411 the difference is significant as shown in Fig. 11.

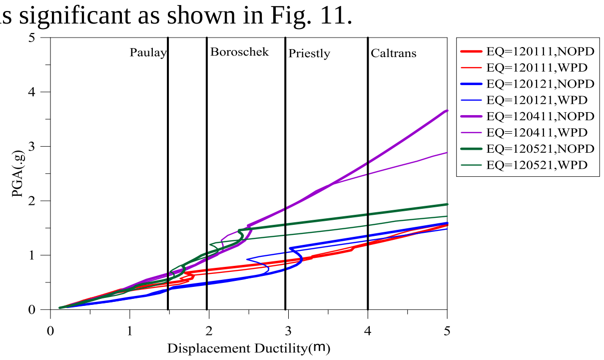

Figure 9. Ductility versus PGA for different ground motions(Case study II).

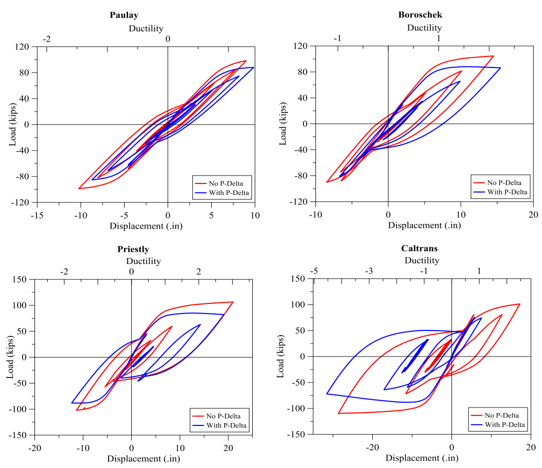

Figure 10. EQID=120121.

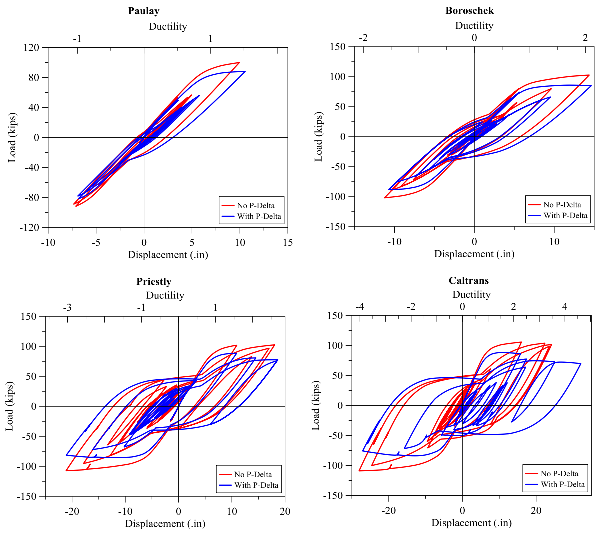

Figure 11. EQID=120411.

VI. Conclusions

The main objective of this research was to study the design target ductility proposed by Caltrans for RC bridge Column. The case studies in this research showed some of the flaws of this method and highlighted the fact that this design target ductility can be improved. Caltrans design target ductility is only sensitive to the ratio of the P-Delta induced moment to the idealized plastic moment capacity, and is insensitive to important parameters such as axial load or the column slenderness as independent variables[13]. First case study showed that the column was capable of achieving high levels of displacement ductility before P-Delta effects significantly deviates the structural response, while the second case study showed that design target ductility of four is unsafe.

Due to complex nature of studying P-Delta effects under earthquake loading and uncertainties associated with accurately capturing the structural model and the random nature of ground motions, obtaining a reliable limit state for ignoring the P-Delta effects for RC bridge columns requires an intensive amount of nonlinear time history analysis for properly capturing all sources of uncertainty.

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