Preliminary Validation of an Automatic Modal Identification Methodology for Structural Health Monitoring of Historical Buildings

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Abstract—Automatic structural modal parameter estimation is a key aspect for a continuous health monitoring process. In the case of existing buildings and historical constructions, the implementation of an automatic process for structural health monitoring (SHM) is important since it provides qualitative information of the structural behavior through time. For parametric system identification, this task is faced with the automatic interpretation of stabilization diagrams and the elimination of the mathematical or spurious modes. The present paper shows a fully automated modal identification methodology based on the application of four processing stages: (i) digital signal pre-processing of the recorded data and the application of the Data-Driven Stochastic Subspace Identification method to obtain modal parameters; (ii) automatic analysis of the stabilization diagram with the application of soft\hard validation criteria and the use of hierarchical clustering approach; (iii) automatic estimation of the modal parameters (frequencies, damping and modal shape) of the structure and (iv) automatic modal tracking of the data with a time-windows technique. This paper presents the adopted methodology to automatically track the variation of the modal parameters of earthen systems and a particular application in a sixteenth century adobe church. The developed methodology shows good results for the task of cleaning of stabilization diagrams which then allows the accurate automatic estimation of the modal parameters.

Index Terms—historical building, earthen structures, automatic modal analysis

I. INTRODUCTION

Long-term structural health monitoring (SHM) systems are spreading widely in many fields [1]-[4], including preservation of historic structures [5]-[7], due to the low invasiveness, accuracy of the results, increase of the level of knowledge of structural system, possibility of remotecheck of the health state of the structure and possibility of predicting and localizing damage [8], [9]. In the case of a vibration-based SHM, an automatic and continuous monitoring system, with the use of ambient-excitation methodology, is required and the evolution in time of the structural conditions is checked to identify possible structural damage [10], [11]. For this purpose, a vibration-based method is used to monitor the modal properties of a building [12]. The main challenge of this technique is the automatic identification of dynamic properties, that, in a parametric system identification, is the automatic interpretation of the stabilization diagram to separate physical from the spurious poles [13].

The goal of this paper is to describe a fully automatic identification algorithm, focused firstly on the automatic cleaning of the stabilization diagram obtained with the Data-Driven Stochastic Subspace Identification method (SSI-Data), secondly with the automatic selection of the modal parameters (natural frequencies, modal damping and mode shapes) and, finally, the use of an innovative modal tracking algorithm. Usually, modal tracking needs a baseline list of pre-selected vibration modes of the investigated structure to clean in time the automatic results of the algorithm [14]. The proposed algorithm with the use of a time-window technique avoids to fix a baseline with the automatic updating during the analysis.

The paper is organized as follows. Section 2 describes the evaluated methodology. Section 3 describes the case study and the results obtained. Section 4 concludes the paper.

II. METHODOLOGY

The proposed methodology considers four stages: the processing of recorded data to obtain the modal parameters, the cleaning of the stabilization diagram, the

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automatic choice of the most representative modal values and the modal tracking for a clear representation of the data for structural health assessment purpose. To evaluate the automatic results of the developed algorithm, a comparison with Artemis software [15] and a frequency domain approach [16] with the averaged auto power spectrum of the time signal were used.

In the stage 1, the dynamic properties (frequency, damping and modal shape) are obtained using an automatic procedure developed in time domain. For this purpose, digital signal pre-processing of recorded data is applied (decimation and filtering according necessities) and subsequently, the Data-Driven Stochastic Subspace Identification (SSI-Data) method [17] is used to process the data. The method is based on the stochastic space model theory from output-only measurements and aims at the identification of the state matrix A and the output matrix C, which contain the information about the resonant frequencies, mode shapes vectors and damping coefficients. For the matrices estimation, the method uses robust mathematical techniques and the main steps are: (i) construction of the Henkel matrix (H) with the recorded data; (ii) LQ factorization of the H matrix; (iii) Singular Value Decomposition of a part of the matrix; (iv) selection of the system order to split the singular values to obtain the extend observability matrix; (v) estimation of the state matrix A and the output matrix C from the observability matrix; (vi) estimation of the modal parameters. In this procedure, the choice of an appropriate system order is an important issue because a large system order increases the number of the identified poles, with an unavoidable identification of spurious or mathematical poles. Whereas, a small system order could exclude physical poles and, therefore, generates an incorrect modal identification. Thus, the modal identification is performed by constructing a stabilization diagram to facilitate the selection of correct model order. The stabilization diagram can be defined as a plot of model order vs eigenfrequencies for a wide range of model orders. The main idea of the stabilization diagram is that the method is run several time with an increasing model order, obtaining different solutions of the matrices (physical and spurious poles). From the empirical observation of the stabilization diagram of numerous modal identification problems, the physical poles appear nearly the same frequency, while the spurious modes tend to spread around the frequency range. In this way, the physical poles are ready visible in the diagram. Several criteria were developed to select the optimal system order, the most used is the selection of the lowest order at which the poles become stable, namely, with a direct userinteraction, there is the choice of the lowest model order at which all the searched poles appear visible.

In the stage 2, to eliminate the user-interaction, a complete automatic algorithm is developed to automatically analyze the stabilization diagram. The diagram is cleaned using hard and soft validation criteria [18] and a hierarchical clustering approach [19]. The procedure consists in the division of the poles of the stabilization diagram in two groups: the first with the

possibly physical modes, the second with spurious modes. The poles assign to the second group are deleted from the analysis, and the poles of the first group goes to the next stage. For this purpose, four hard validation criteria, (1) and (2), and four soft validation criteria, (3), (4), (5) and (6) are used. Hard validation criteria were applied to each pole and frequency (1) and damping (2) limits were fixed to focus the research of the physical poles to a specific range of interest. The poles outside these limits were eliminated. Then, soft validation criteria were applied and, in particular, they are categorized as distance and mode shape criteria. Equations (3), (4) and (5) describe distance criteria determined, respectively, by frequencies, damping ratio and modal shapes (as a function of the MAC, modal assurance criteria). These criteria correspond to a maximum distance between the pole in analysis and the pole associated to the inferior model order. The poles that do not respect these criteria (in terms of distance) are eliminated. Equation (6) is based on the mode shape criteria and determines the minimum value of the modal phase collinearity (MPC). This criterion analyzes the complex part of the modal shape, and in the case of a proportionally damped structure, the modal shape components lie on a straight line in the complex plane. The criterion quantifies this behavior and if there is perfect collinearity, the MPC value is 1, if there is no collinearity, the MPC value is 0. In a real structure, poles with low MPC values are considered spurious poles, and therefore, eliminated. The next step is the grouping of the similar poles in a same cluster with the application of a hierarchical clustering algorithm. In this approach, each pole is evaluated as a single cluster. Then, the two closest clusters are combined into a new aggregate one. This procedure is repeated until all the poles are grouped into several large clusters. In the proposed methodology, a distance frequency criterion is used to group the poles. Generally, the clusters with a high number of poles are "physical" clusters because the physical poles are function of the analyzed structure, therefore, they appear with a higher probability. To separate "physical" clusters with "spurious" ones an automatic threshold (the number that defines the size limit between physical and spurious cluster) is needed. An automatic hierarchical tree distance cut is important because, in every recorded data, the number of detected poles is different, being a function of the structural excitation and ambient noise. It means that, in the case of a low or not sufficient excitation of the structure, a high tree cut may eliminate physical poles, on the contrary in the case of high excitation, a low tree cut may not eliminate spurious poles for the presence of high ambient noise. For this purpose, good results were achieved with a variable clustering limit calculated considering the Root Mean Square (7). In detail, the square root of the mean square with the number of poles present in each cluster is calculated, and the clusters with a lower number of elements are eliminated.

$$f_{min} < f_i < f_{max} \tag{1}$$

$$\xi_{min} < \xi_i < \xi_{max} \tag{2}$$

$$d(f_i^k) < (f_i^k - f_i^{k-1}) / \max(|f_i^k|, |f_i^{k-1}|)$$
(3)

$$d\left(\zeta_{i}^{k}\right) < |\zeta_{i}^{k} - \zeta_{i}^{k-1}| \tag{4}$$

MAC
$$(\phi_i \phi_j) < |\phi_i^k \phi_i^{k-1}|^2 / ||\phi_i^k||_2^2 ||\phi_i^{k-1}||_2^2$$
 (5)

$$MPC_{i} < [2 (\lambda_{1} / (\lambda_{1} - \lambda_{2}) - 0.5)]$$
(6)

$$x_{rms} = \sqrt{(x_1^2 + x_1^2 + \dots + x_m^2)/m}$$
(7)

Here, f_{min} , f_{max} , ξ_{min} and ξ_{max} are the upper and lower frequency and damping limits of the hard validation criteria and f_i , ξ_i , and ϕ_i are, respectively, the frequency, the damping ratio and the mode shape of an identified vibration mode, and the superscript is associated to the model order and the subscript to each mode identified by each model order. Finally, λ_1 and λ_2 are the eigenvalues of the variance-covariance matrix [20] and x_{rms} is the variable clustering limit with m the number of obteined clusters and x the number of elements in each cluster.

The stage 3 is the automatic choice of the modal parameters. Good results were achieved with the choice of the pole with the highest MPC coefficient (modal shape complexity criteria) for each cluster. In the case of poles with the same MPC coefficient, the pole with the lowest model order is chosen. Then, for every recorded dynamic data, a set of poles are obtained in the automatic way.

In the stage 4, an automatic tracking of the evolution in time of the identified modal parameters is proposed with a self-adaptable filter time-window. The previous stages are able to filter the stabilization diagram and to detect the most representative pole for each cluster, however, a further cleaning of the data is needed for a clearer visualization of data in time. The proposed procedure analyses each event, the poles obtained for every recorded dynamic data, with the n previous events (Fig. 1). The poles of the n+1 analyzed event that are present more time in the n previous events are defined as physical poles and the other are eliminated. The main feature of the procedure is that the n previous events are a sliding time-window. Equations (8) and (9) are the distance criteria used to cluster the poles of the n evens, using the poles of the n+1 event as referent vector to determine the physical and spurious poles.



Figure 1. Summary of the proposed automatic procedure for the modal tracking.

$$f_{\text{event } n+1}^{h} - f_l^{h} \mid / f_{\text{event } n+1}^{h} < a$$
(8)

$$1 - \text{MAC} \left(\phi_{\text{event } n+1}^{\quad h} \phi_l^{\quad h} \right) < b \tag{9}$$

Here, f_i , and ϕ_i are the frequency and the mode shape of an identified vibration mode and the superscript h is associated to the pole in each event and the subscript to each event. The coefficients *a* and *b* are user-defined distance criteria to group the similar poles in the modal tracking procedure.

III. VALIDATION EXPERIMENTS

A. Laboratory Validation: SHM in Adobe Walls

Adobe blocks ($320 \times 220 \times 120 \text{ mm}$) were built in laboratory and dried for two months approximately. Then, three full-scale walls (Fig. 2a) were built using these blocks and they were continuously monitored for more than 9 months. This paper reports in detail only the modal results of one of the adobe walls (with dimensions 1660 x 2000 x 220 mm) (Fig. 2b). A concrete plinth was built as walls foundation.

The installed equipment is composed by a multichannel acquisition system cDAQ-9234 (24-bit resolution, 102 dB dynamic range and anti-aliasing filters) (Fig.2c) and by three PCB 393B31 uniaxial accelerometers for each wall (dynamic range of ± 0.5 g, sensitivity of 10 V/g, a frequency range of 0.1-200 Hz and a weight of 210 g) (Fig.2d). These sensors include a thermal jacket for outdoor protection. The three accelerometers were placed, as shown in Fig. 2b, at the top of the wall, two in transversal direction and one parallel to the wall to be able to measure in plane and out of plane behavior. The data acquisition parameters were set as 256 Hz of sampling rate, 600 seconds of sampling time and the recurrence of events of 1 hour (a total of 5100 of events were recorded in 9 months). Table I and Table II show the parameters used for the time-domain analysis and the modal tracking.

TABLE I. PARAMETERS TO FILTER THE STABILIZATION DIAGRAM

Freq. range	1-45 Hz	Min MAC	0.9
Max Freq. Std Dev.	0.05	Max MAC Std Dev.	0.2
Damp. range	5-1E-5	Min MPC	0.9
Max Damp. Std Dev.	2	Model order range	20-150

TABLE II. PARAMETERS TO MODAL TRACKING

No. of events	6	а	0.01
Size of cluster	2	b	0.04

Fig. 3 shows step by step the methodology to filter the stabilization diagram. As an example, the event on 01/12/2016 at 10:00 am was analyzed. Fig. 3a shows the stabilization diagram obtained by the stage 1, the recorded data was pre-processed and processed by the SSI-Data method with a 130 analyzed model order.

Straight lines are identifiable but the identification of the physical poles is not possible. Subsequently, there is the stage 2 of the developed methodology. Fig. 3b, Fig. 3c, Fig. 3d show the filtering of the stabilization diagram by frequency (equations (1) and (3)), damping (equations (2) and (4)) and MAC (equation (5)). A part of the spurious poles is eliminated in each step for a clearer legibility of the diagram: nevertheless, the physical poles are not detectable. Fig. 3e shows the filtering by MPC (equation (6)). Clearly, four groups of poles are present, respectively, close to 5 Hz, 20 Hz, 27 Hz and 35 Hz. The results show a good capacity of the algorithm to eliminate the spurious poles. Then, there is the grouping of the similar poles in a same cluster with the application of the hierarchical clustering algorithm with the variable clustering limit (Fig. 3f). All the clusters lower than the horizontal line (cluster limit) in Fig. 3f are eliminated. The final results of this stage are the cluster with a number of poles larger than the automatic cluster limit. Then, there is the stage 3. The pole with the highest MPC is chosen for each cluster. For the event on 01/12/2016 six frequencies, six damping values and six modal shapes were found. Finally, with the stage 4 and the application of the automatic modal tracking of the data with a timewindow technique, the final values are four poles with 6.09 Hz, 20.91 Hz, 25.68 Hz and 34.62 Hz natural modal frequencies.





Figure 2. (a) The three full-scale adobe walls. (b) Geometric details in mm and accelerometer positions. (c) Used cDAQ-9234 acquisition system and used PCB 393B12 – accelerometer sensor.

To validate the results, a comparison with the Artemis software was done. Fig. 4a and Fig.4b show the spectral density and the stabilization diagram of the analyzed event. In Fig. 4a the four natural frequencies are clear and correspond to the found values. Fig. 4b shows similar results to the Fig. 3e, and the final values obtained by Artemis are 6.11 Hz. 20.76 Hz. 25.77 Hz and 34.91 Hz. The difference between the calculated frequencies and the obtained values by Artemis are 0.33%, 0.72%, 0.35% and 0.84% respectively for the first, second, third and fourth frequency. Furthermore, the MAC was calculated between the obtained modal shape by the developed algorithm and the obtained modal shapes obtained by Artemis. The values are 0.99, 0.92, 0.99 and 0.99 for the four poles. Table III shows a comparison between the obtained results. Finally, for a direct and complete comparison a FMAC analysis was developed (Fig. 5). The FMAC is a graphical representation that provides a general comparison of modal properties by considering simultaneously the modal shape correlation (MAC), the degree of spatial aliasing and the frequency comparison. FMAC graphic shows a high correlation between calculated poles and the obtained one by Artemis.





Figure 4. (a) Spectral Density of the analyzed event. (b) Artemis results.

	Obtained frequency by algorithm [Hz]	Obtained frequency by Artemis [Hz]	Difference Freq. [%]	MAC
Pole 1	6.09	6.11	0.33	0.9997
Pole 2	20.91	20.76	0.72	0.9197
Pole 3	25.68	25.77	0.35	0.9999
Pole 4	34.62	34.91	0.84	0.9998

TABLE III. COMPARISON RESULTS



Figure 5. FMAC comparison between results of the developed algorithm and Artemis.

The results of the entire monitoring period are shown in Fig. 6. The four frequencies are clearly observed and the developed algorithm is able to detect the drying effect in the adobe structural system, demonstrating a high accuracy of the methodology in the case of modal parameters changes. To validate the results, a frequencydomain analysis using the averaged auto power spectrum of the time signal is applied to determine frequency content of the recorded data. The results are plotted in Fig. 7 and confirm the efficiency of the methodology.



Figure 6. Monitoring results of the adobe wall in terms of frequencies' evolution: with the developed algorithm.



Figure 7. Monitoring results of the adobe wall in terms of frequencies' evolution with the frequency domain approach.

B. Field Validation: SHM in the Adobe Church of 'San Juan Bautista de Huaro'

The church of 'San Juan Bautista de Huaro' in Peru is selected as case study (Fig. 8). This adobe church is located about 41 km to the south-east of the city of Cusco, Peru and strategically placed on an important Inca road system. Due to its architectural characteristics, its construction dates back to the 16th century and was built by Spanish Jesuits. The church was declared as a Peruvian historical monument in 1972 [21]. In order to study the dynamic behavior of the church, a dynamic monitoring system composed by four accelerometers (Fig. 9) is operating since March 2017. The accelerometers installed and the data acquisition system have the same characteristics as in the previous case study. The data acquisition parameters were set as 200 Hz of sampling rate, 900 seconds of sampling time and the recurrence of events of 1 hour. In this paper, the dynamic data of two months were analyzed for a total of about 1400 events. Table IV and Table V show the used parameters for the time-domain analysis and the subsequently modal tracking.



Figure 8. General view of the church.



Figure 9. Location of the sensors.

Freq. range	1-4 Hz	Min MAC	0.9
Max Freq. Std Dev.	0.02	Max MAC Std Dev.	0.2
Damp. range	5-1E-5	Min MPC	0.8
Max Damp. Std Dev.	1.5	Model order range	20-150

TABLE IV. PARAMETERS TO FILTER THE STABILIZATION DIAGRAM

TABLE V. PARAMETERS TO MODAL TRACKING

No. of events	8	a	0.02
Size of cluster	2	b	0.2

Fig. 10a shows the natural frequencies of the church in the range 1-4Hz obtained by the developed methodology. Four frequencies are detectable (1.98 Hz, 2.16 Hz, 2.56 Hz, 3.18 Hz and 3.70) and also their evolution and change in time. Fig. 10b shows the frequency-domain analysis applied to the recorded data of the church. The analysis is able to detect three frequencies in the range 2-3 Hz but, as expected, it shows a lower accuracy than the developed methodology.



Figure 10. Results of the Church of 'San Juan Bautista de Huaro': (a) Developed algorithm. (b) Frequency domain approach.

IV. CONCLUSION

A new automatic modal identification methodology was successfully evaluated in laboratory and field experiences. Hard and soft validation criteria, together with automatic clustering and selection procedure showed that the most representative modal properties can be estimated in complex earthen systems. In a comparative analysis between the developed algorithm and the results of the software Artemis and the frequency-domain approach, the methodology showed high accuracy and reliability. In the case of frequencies, the difference between obtained frequencies by the developed algorithm and by the Artemis software are 0.33%, 0.72%, 0.35% and 0.84%. High values of Modal Assurance Criteria coefficients of up to 92% were also found, showing similar modal shapes estimation. The high accuracy of the developed algorithm makes it a useful tool for monitoring complex masonry structures and for reducing uncertainties regarding their structural performance trough time. Future studies should examine the possibility to introduce an automatic alarm in the case of changes of modal parameters due to structural damage. The studies should face the influence of environmental and operation conditions and the creation of a base-line of the studied buildings.

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