# Analytical Study of Internal Phenomena of Inflatable Flexible Membrane Dams in Hightemperature Environments

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Abstract-In this study, a material model for inflatable flexible membrane dam (IFMD) rubber membranes was developed to determine the material properties for finite element method (FEM) analyses by considering the hyperelasticity of rubber composites. In addition to investigating the analytical model at room temperature, realistic models at higher temperatures of 40  $\,$   $^\circ C$  and 60  $\,$   $^\circ C$ were studied to simulate the actual dry, hot daytime field environments under which IFMDs operate. Validation analyses were performed for these developed models, comparing the tensile test results of rubbers and nylon woven fabrics obtained from several temperature environments. Internal stress analyses were conducted with the developed model, and the concentrations of stress and strain within the rubber membrane, which had not vet been studied precisely, were investigated. The internal stresses, strains, and deformations under room temperature and high temperatures were analyzed and compared to confirm the mechanical behaviors of the rubber membrane under tensile loading. In addition, bending analyses of the rubber membrane were conducted to understand the internal phenomena of the bending portions.

*Index Terms*—inflatable dam, rubber dam, rubber-nylon composite, hyperplastic body modeling, inner stress, FEM

## I. INTRODUCTION

Inflatable flexible membrane dams (IFMDs), or inflatable dams and rubber dams, are river weirs that use air or water to inflate and deflate their structures to control water levels. The first IFMD was introduced by the US in the 1960s in Japan; approximately 3900 IFMDs are currently used in Japan. IFMDs operated by the Japanese Ministry of Land, Infrastructure and Transport have various designs, ranging in span from 2 to 50 m and height from 0.5 to 5 m. Some IFMDs are designed to maintain service life for over 30 years. In order to realize both long-term use and safety of the weirs, it is necessary to establish effective maintenance and management techniques for these structures. Fig. 1 shows an example of an IFMD in Japan. Fig. 2 depicts a cross-section of one rubber membrane used for IFMD structural bodies; it uses a four-layer woven nylon fabric to provide strength to the material.



Figure 1. General view of IFMD



Figure 2. Cross-section of rubber membrane with joint portion

Regarding the damage to IFMDs during operation, some dam failure modes relate to deformations of the rubber membranes used for the structural bodies. The fracture of the rubber portion between woven fabric layers within the membrane is one of these failure modes, and can cause serious structural damage. Therefore, it is important to investigate the internal stress and deformation of the membrane around joints and gaps in the woven fabrics under actual operational environments.

Although some studies have investigated the influence of water flow on IFMD dynamic behaviors and overall structural vibrations, little research has been conducted regarding the internal phenomena of the rubber membranes under operational conditions [1]-[3]. In addition, insufficient analytical information is available regarding the material lifetime of rubber membranes; many questions about the operational durability of IFMDs remain, particularly regarding long-term use multiple decades [4].

One study investigated rubber membranes enforced by woven fabrics for another civil engineering structure: the long-term durability of rubber used for submerged tunnel joints was examined via computer simulation [5]. Other research studied fiber-reinforced rubbers as civil engineering materials and mechanical parts via analytical methods, in which material models were provided for rubber membranes that accommodated rubber hyperelasticity and fabric viscosity, as well as dynamic property anisotropies [6]–[9]. These analytical models are applicable to IFMD rubber membranes to study their internal phenomena, which could clarify the complex internal stress fields of rubber composites, permit investigation of the rubber membrane structural failure mechanisms, allow prediction the lifetimes of rubbers, and enable optimization of the material strength and overall design of IFMDs.

In this study, a material model for IFMD rubber membranes was developed to determine the material properties for finite element method (FEM) analyses, with consideration of the hyperelasticity of rubber composites. In addition to investigating the analytical model at room temperature, realistic models in hightemperature conditions at 40 °C and 60 °C were studied to simulate the actual field environments under which IFMDs operate, particularly in dry and hot daytime conditions. Validation analyses were performed for these developed models, comparing the tensile tests results of multiple rubbers and woven nylon fabrics under several temperature environments. Internal stress analyses were conducted with the developed model, and the concentrations of stress and strain within the rubber membrane, which have not yet been studied precisely, were investigated. The internal stresses, strains, and deformations at room temperature and higher temperatures were analyzed and compared in order to confirm the mechanical behaviors of rubber membranes under tensile loading. In addition, bending analyses of the rubber membrane were conducted to understand the internal phenomena of the bending portions.

#### II. MATERIAL MODEL AND VALIDATION

#### A. Material Model

Analytical models were developed to simulate large deformations of IFMD rubber membranes via FEM. As the material components, ethylene propylene diene monomer (EPDM) rubber was used for the rubber portion and woven nylon woven fabric was used for the layered strengthening material.

Rubber is an incompressible material that experiences large-strain nonlinear behavior. For modeling these properties, the nine-parameter Mooney–Rivlin model was employed in this study [10]. The form of the strain energy potential for the nine-parameter Mooney–Rivlin model is:

$$W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) + C_{20}(I_1 - 3) + C_{11}(I_1 - 3)(I_2 - 3) + C_{02}(I_1 - 3)^2 + C_{30}(I_1 - 3)^2 + C_{21}(I_1 - 3)^2(I_2 - 3) + C_{12}(I_1 - 3)(I_2 - 3)^2 + C_{03}(I_2 - 3)^3 + 1/d (J - 1)^2,$$
(1)

where: W = strain energy potential

 $I_1, I_2$  = strain-invariant deviatory

 $C_{ij}$  = material constants characterizing the deviation deformations of the material

d = material compressibility parameter

J = determinant of the elastic deformation gradient F

For modeling the woven fabric portion, a linear elastic model was used.

With these analytical models, the internal phenomena of rubber membranes under tensile loading and bending stresses were investigated with FEM.

## B. Tensile Testing

Uniaxial tensile tests were conducted to determine the material constants for the rubber and woven fabric models. To understand the material characteristics at both ambient and high temperatures, to which IFMD rubber membranes are exposed during use in dry environments on hot days, the experiments were implemented at room temperature (~23 °C), 40 °C, and 60 °C for each material.

For the rubber tests, the specimens were rectangular strips measuring  $150 \times 10 \times 10$  mm. The initial chuck distance of the specimen was set to 40 mm and the test speed was 50 mm/min. Each test was conducted until sample breakage to obtain the nominal stresses and strains, which were determined by the chuck distances.

Fig. 3 shows the tensile test results of the rubber specimens. The stiffness of EPDM rubber is decreased with increasing test temperatures, and the tensile strength and elongation are drastically decreased at 40 °C and 60 °C. While the surfaces of IFMD membranes in operation can reach temperatures exceeding 60 °C in some cases, the test results show that the tensile stress at 60 °C is decreased by over 60% relative to that at room temperature. This indicates that even small variations in temperature, which often occur in the operation of IFMDs, can cause drastic changes in the strength of the rubber. Regarding the elongation, the elongation at 60 °C is decreased by approximately 40% relative to that at room temperature.

For the woven fabric tests, the specimens were dumbbell-shaped and 300 mm in length with a reference line width of 20 mm. The initial chuck distance was set to 185 mm and the test speed was 50 mm/min. Each test was conducted until sample breakage to obtain the nominal stresses, considering the measured fabric thickness of approximately 2.5 mm, and the nominal strains determined by the chuck distances. Fig. 4 shows the tensile test results for the fabric. Unlike the rubber, the temperature dependency of the woven fabric is relatively small within the measured temperature range. The tensile stresses are approximately 300 N/mm (or 120 MPa) and the elongations are approximately 50-55% at each temperature. Slight decreases in the tensile strength and increases in elongation are confirmed at increased temperatures.



Figure 3. Rubber tensile test results at various temperatures



Figure 4. Fabric tensile test results at various temperatures

## C. Material Properties

The material constants of the rubber and woven fabric were determined from the tensile test results. TABLE I and TABLE II show the constants for rubber and fabric, respectively. The rubber constants were obtained by curve fitting of the tensile test results at each temperature. The woven fabric constants were calculated using the least-squares method for the tensile test results, assuming material isotropy with the Poisson ratio of 0.4.

#### D. Validation

 $C_{02}$ 

 $C_{30}$ 

 $C_{21}$ 

C<sub>12</sub>

 $C_{03}$ 

(

To confirm the validity of the analytical models, verification analyses were conducted. The same sample geometries and tests conditions as in the abovementioned tensile test experiments were defined in FEM programs to conduct tensile test analyses. The experimental and analytical results were then compared. The displacement of the chuck portion under each test condition was compared for verification.

	23 °C [MPa]	40 ℃ [MPa]	60 ℃ [MPa]
C <sub>10</sub>	5.88×10 <sup>-1</sup>	3.46×10 <sup>-1</sup>	2.60×10 <sup>-1</sup>
C <sub>01</sub>	4.71×10 <sup>-2</sup>	1.85×10 <sup>-2</sup>	4.03×10 <sup>-2</sup>
C <sub>20</sub>	1.36×10 <sup>-2</sup>	1.61×10 <sup>-2</sup>	2.30×10 <sup>-2</sup>
C <sub>11</sub>	1.19×10-3	3.42×10-3	-1.29×10 <sup>-3</sup>

-5.46×10-4

-2.83×10<sup>-4</sup>

 $1.06 \times 10^{-5}$ 

9.07×10<sup>-6</sup>

-2.15×10-7

-4.12×10-4

-6.30×10<sup>-4</sup>

 $2.70 \times 10^{-4}$ 

-1.50×10-5

5.20×10-7

-1.78×10<sup>-4</sup>

 $-1.29 \times 10^{-4}$ 

-1.45×10<sup>-5</sup>

3.58×10<sup>-6</sup>

-6.79×10-8

TABLE I. MATERIAL PROPERTY OF RUBBER

TABLE II	. MATERIAL	PROPERTY OF FA	BRIC
	<b>a</b> a <i>m</i>		

	23 °C [MPa]	40 ℃ [MPa]	60 ℃ [MPa]
Young's modulus [MPa]	225.7	223.2	198.5
Poisson's ratio		0.4	

TABLE III . VERIFICATION OF ANALYSIS RESULTS

	Error from experiment results [%]		
	23 °C	40 °C	60 °C
Rubber	-7.6	+4.4	-3.3
Fabric	-21.4	-29.1	-21.4

Table III shows the comparison between the experimental and analytical results, thereby determining the deviation from the experimental results For rubber, behavioral verification tests at 23 °C, 40 °C, and 60 °C were conducted under the tensile loads of 200 N, 141 N, and 88 N. For the fabric, each condition was tested under the tensile load of 6300 N. The analytical results with the nine-parameter Mooney–Rivlin model, considering material hyperelasticity, showed good agreement with the experiment results. However, the woven fabric model results are approximately 20–30% lower than the experiment results, indicating that the analytical model is somewhat stiffer than the actual material. In future studies, the modeling of fabric isotropy and viscosity as nonlinear material characteristics will be considered.

# III. ANALYSIS CONDITIONS

## A. Geometry

Fig. 5 shows the rubber membrane geometric model for FEM analysis. This geometry represents the joint portion of the IFMD rubber membrane, which includes discontinuous woven fabric and is the weakest portion within the IFMD membrane. Four gaps were introduced in the model, and the stress concentrations and large-scale deformations around them were investigated in detail. The composite geometry was defined as having a longitudinal length of 500 mm, width of 10 mm, and layer-direction thickness of 16 mm. The membrane has four nylon layers with gap distances of 8 mm each.

The model was meshed with 38410 elements. In terms of constraint conditions, a weak spring model was set on the longitudinal cross-section to prevent the entire model from moving other than in the longitudinal direction.

Loads were applied in the longitudinal direction on both edges to simulate uniaxial tensile stress, and the stress distribution and deformation within the material were confirmed. In addition to deformation by tensile stresses, IFMD membranes experience large deformations around bent portions under bending while deflated or at the edge of the full structure while inflated. Therefore, a bending analysis as illustrated in Fig. 6 was performed.



Figure 5. Geometry of IFMD rubber membrane FE model



Figure 6. Bending of rubber membrane to 300-mm radius

# B. Analysis Sets

The inner stresses and deformations were confirmed under the tensile load applied to the model representing the inflated IFMD, and under bending displacements applied to the bent portions.

To confirm the interior conditions created by tensile loading, five load sets of 100 N, 250 N, 500 N, 2500 N, and 5000 N were applied to the model in the longitudinal direction. Each load set could be converted to nominal stresses on the fabrics of 10 N/mm, 25 N/mm, 50 N/mm, 250 N/mm, and 500 N/mm, respectively. Here, the reference strength of the rubber membrane (i.e., the fabric strength) is 940 N/mm.

To confirm the interior condition induced by bending, the model was bent to the radius of 300 mm (R300). The deformation was simulated by bending the material to make contact with a friction-free rigid body.

# IV. RESULTS

Fig. 7 shows the analysis results for the tensile loading of 2500 N at the temperature of 23 °C. Stress concentrations develop in the rubber around the gaps, and larger stresses are generated there than around the continuous fabric portions. The same is confirmed in the results for the fabric. In the results of both tensile loading and bending analysis, the portions around the gaps between woven fabrics within the membrane joint show stress concentrations and large deformations under each analysis condition. The following shows detailed analysis results around Gap 1, shown in Fig. 5.

Fig. 8 and Fig. 9 show the analytical results highlighting the major principal stress and principal strain of the rubber. Each figure indicates results at both room and high temperature (23 and 60 °C). Regarding the temperature and high-temperature results are stresses around the stress-concentrated part, the room-temperature and high-temperature results are nearly equivalent. The maximum strain in the high-temperature rubber is approximately 1.6 times larger than that at room temperature. This shows that the decreased stiffness of rubber at higher temperatures causes larger deformation than that observed at the room temperature.



Figure 7. Example of analytical result showing stress concentrations in rubber parts (tensile load of 2500 N at 23 °C)



Figure 9. Analytical result of rubber strain distribution (tensile load of 2500 N)

Fig. 10 and Fig. 11 show the analysis results for the fabric, highlighting the major principal stress and principal strain. The stresses of the fabric around the stress concentrations are nearly equivalent at 23 and 60 °C. The maximum strain at 60 °C is approximately 1.2 times larger than that at 23 °C; however, this difference is relatively small compared to that of rubber mentioned above.

Fig. 12–15 show all analyzed data for the analysis sets, indicating the maximum concentrations of stresses and strains around the gaps. The horizontal lines of the graphs represent the nominal stress applied to the rubber membranes under the loading mentioned in section III; these are normalized by dividing by the membrane reference strength of 940 N/mm.

Fig. 12 shows the relation between the nominal stress and maximum stress concentration within the rubber. The result indicates that loading at less than the nominal stress of 0.27 does not affect the stress difference between temperatures. However, when the nominal stress is increased to 0.53, the stress concentrations at 40  $\,$  °C and 60 °C become remarkably higher than that at 23 °C. In order to investigate the reason for this, the deformation status of the rubber was reviewed. Increasing the temperature caused severe compressive deformation of certain elements in the rubber model around the gap. Therefore, the maximum stress observed at 60  ${\rm C}$  reached approximately twice the value for that at 23 °C, for example. This analysis result suggests the importance of strength design for rubber membranes considering hightemperature conditions.



Figure 11. Analytical result of fabric strain distribution (tensile load of 2500 N)



Figure 12. Comparison between nominal stress of membrane and maximum concentration of stress in rubber



Figure 13. Comparison between nominal stress of membrane and maximum concentration of strain in rubber

Fig. 13 shows the relationship between the nominal stress and the maximum concentration of strain within the rubber. The results indicate that the strains depend on the temperatures; increasing temperatures correspond to increased strains. In this study, the rubber and woven fabric viscosities are not included in the models. However, the membranes practically experience creep; therefore, the internal strains are time-dependent. Thus, larger strains may form in structures during operation, particularly over long-term use, than the strains analyzed here. These results suggest that slight temperature increases affect the maximum strain variations in the rubber. Strains and elongations can be criteria of material failures for polymeric materials. Thus, in addition to the analysis results from this study, considerations of the viscous behaviors of rubbers such as creep phenomena, especially under high-temperature conditions, are necessary for the material design and investigation of IFMD durability.

Fig. 14 shows the relation between the nominal stress and the maximum concentration of stress in the fabric. It indicates a small temperature dependency of the maximum stress of the fabric. Fig. 15 shows the relationship between the nominal stress and maximum strain concentration in the fabric. Here, the differences in the fabric maximum strains at varied temperatures are relatively small compared to those in the rubber. For a deeper understanding of the fabric portions of IFMD membranes, it is necessary to integrate nonlinear material properties, such as anisotropy and viscosity under various temperatures, into the fabric model.

Table IV shows the results of the bending analyses. It indicates the values of the maximum stresses and strains generated by bending around the gaps at each temperature. The analytical results show that concentrations of stress and strain around the gaps are generated even when bending the material to R300 mm. However, these maximum stress values remain small at R300 mm bending, and the stresses developed at each temperature are almost the same. Regarding the strains, they are increased with increasing temperatures, showing the same tendencies as those seen in the tensile analysis results. IFMDs have large bending portions, and largescale deformation-analyzing bending models for smaller bending radiuses are necessary in future study.



Figure 14. Comparison between nominal stress of membrane and maximum concentration of stress in fabric



Figure 15. Comparison between nominal stress of membrane and maximum concentration of strain in fabric

TABLE IV . RESULT OF BENDING ANALYSES

	Rubber stress concentrated portion maximum values		
	23 °C	40 °C	60 °C
Major principal stress [MPa]	0.55	0.39	0.34
Principal shear stress [MPa]	0.19	0.17	0.16
Maximum principal strain	0.12	0.15	0.17

## V. CONCLUSION

In this study, the material properties of IFMD rubber membranes were investigated via tensile testing, and analysis models were developed to consider rubber hyperelasticity. Modeling was conducted for various temperatures corresponding to the actual environments of IFMD application. Using the developed model, the internal phenomena of the rubber–fabric composite were analyzed. The following conclusions could be drawn.

(1) The tensile test results showed that the rubber tensile stresses and elongations at higher temperatures were much lower than those at room temperature. For example, the tensile stress at 60 % decreased by over 60% from that at room temperature; the elongation at 60 % also decreased by approximately 40%. Even slight temperature differences changed the rubber membrane material properties.

(2) In addition to modeling the rubber and woven fabric membrane at room temperature, analytical models for high-temperature conditions were developed. Verification analysis results of the rubber model considering hyper-elasticity showed good agreement with the experimental results. Regarding the model for woven fabric, it is necessary to integrate nonlinear material properties, such as anisotropy and viscosity, to obtain a more precise analytical model.

(3) The joint portions of IFMD rubber membranes developed stress concentrations and large deformations around the gaps between woven fabric pieces in certain conditions. The rubber deformations around stress concentrations were particularly noticeable under hightemperature conditions.

(4) Regarding the rubber membranes with the initial design material properties, which are used under relatively low tensions, the stress concentrations and large deformations of joint portions may not immediately present problems. However, to understand the durability of IFMDs, further study on material degradation, especially under high-temperature conditions, is necessary.

(5) The bending parts of the membrane also developed stress concentrations and large deformations around gaps within the joint portions.

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