A New Method on Characterizing Particle Crushing State and Strength Revolution of Geotechnical Materials Based on Linear Fitting

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***Abstract*—It's intrinsic properties for geotechnical material to crush after loading or other mechanical impact. Therefore, one of the primary questions need to be solved is to define the degree to which the particles of an element of soil are crushed or broken during loading. Considering the fact that with continued particle crushing the proportion of smaller diameter particles will increase and there is a tendency for the grading to become increasingly "self-similar" or "fractal". Thus, based on the lope and correlation coefficient of fitting line, the article proposes a concise method to describe the revolution characters of particle crushing state. Combining with two sets direct shear tests of specific grading, the relationship between the parameters of particle crushing state and strength index φ is discussed. It's confirmed that the parameters of particle crushing state have clear physical meaning and wide applicability.**

***Index Terms*—particle crushing; particle size distribution (PSD); strength; fitting line**

I. Introduction

Resemble to the minerals, the features of composing the particles are also one of the major indexes of soil classification, determining the engineering properties and basic physical mechanics properties. Based on which and targeted at the diversities of soil, some constitutive models are established. For example, Cam-Clay Model [1]is only applied to fine particles of clay, some other models came up with by Lade and Duncan [2] and by Wang[3] are suitable for more coarse sand particles, while some scholars put forward the constitutive model of the rockfill[4-7]. However, large quantities of experiments and engineering practices prove that under different stress and hydraulic power, the soil is loose granule gathered by particles of various size and shape, and its intrinsic feature is that the particle constitutive feature continuous to evolve as the crushing of particles.(Chen Shuisheng 2008[8]， Hamid etc [9]). This feature shook the foundation of traditional soil mechanics greatly, which means that the same as other parameters, such as space ratio and soil structure, also the particle constitutive feature is changed with the changes of stress and hydraulic power, and turned into model variable from material physical parameter. Huge challenge is brought to the study on the constitutive characteristics of soil strength and deformation.

Aimed at this problem, the researchers from home and abroad have launched a set of study mainly from aspects of the degree of particles crushing of soil [10-14], the effects factors of particle crushing[15-19], the influences of particle crushing on mechanical behavior of soil [20-24], numerical simulation [25-29] and other sides. Among them, being one of knotty, how to define particle crushing lays foundation to the whole study. The key lies on that particle components can be held together casually, correspondingly, the initial PSD is considerably complex, which led to a more complex process of crushing, thus, it is hard to have a quantitative description on crushing in a comprehensive way. While in the other hand, the research of Turcotte [30]、Coop[31]、McDowell、Bolton[32] etc showed that as the development of crushing, the fractal features of PSD curve are much more obvious and there exists a certain crushing limitation; Namely, when the particles continuous to crush from the coarse to the fine, the PSD curve will keep rising and the linear relationships between the particle size and the pass rate are much more visualized easily. According to this phenomenon , this paper will determine the particle crushing and its process by making use of fitting line and put forward a new index to define the crushing；On this

Manuscript received April 13, 2017; revised February 13, 2018.

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doi: 10.18178/ijscer.7.1.51-59

foundation, this paper will also explore the relationships between the index of particle crushing and their strength to study the evolve law of the strength during the crushing process.

II. The Description of Particle Crushing Process Based on Linear Fitting

The characterization of particle crushing is based on the changes of quantifiable indicators during crushing and the researchers of nowadays mainly adopted two approaches to start the research from the perspective of single particle characteristic or from mixed particle statistics.

Starting from the perspective of single particle characteristic can directly come to point. Federico[33] illuminated the features of single particle from aspects of morphology, geometry and energetic and many other aspects, the indexes he adopted including particle size, length-diameter, minor-diameter , abundance etc. By researching crushing state of the sand and coarse material by triaxial test , Miara and O-Hara[34] put forward that crushing volume is presented by added value of the particle surface area. Computed scanning techniques are proposed to measure the particle crushing by Otani[35] etc. Though this sort of method directly comes to point, but for those particles with irregular shape and size, it is still hard to have an overall description on particle crushing with limited indexes and usually in need of much more complex equipment; Meanwhile, geotechnical engineering takes the assemblage of isolated particles as study object and explored their overall physical mechanics characteristics, thus the whole change of material cannot be described by the single particle features. Therefore, starting from statistical point of particle group is a more common method, because PSD curve is the most directly expression of process of the particle crushing. The evolve of crushing is described through the analysis changing features of the PSD curve in today`s research which is defined mainly adopted two specific method- single feature index of particle diameter and overall PSD curve change index. Based on the single feature index of particle size group, the experiments about the isotropic load and proportional load of the sand done by Lee [11]and other scientists proposed that the degree of the crushing of the course sand can be defined by ratio B15 representing the change of constrained diameter D15. Lade[36] studied the permeability effects of the PSD on dam, and proposed to adopt the before-and-after value changes B10 to define the degree of particle crushing of the coarse soil; Bo Shu-tian[37] mentioned using before-and-after constrained diameter differentials B60 to define the degree in the cause of studying the mechanical properties of cypress fields stone body. The particle crushing degree index showed by all those approaches corresponds to the changes of some other fixed point on the PSD curve before and after the text, which is much more simple and visualized, but the characteristic particle size can only reflect well in partial breakage but poorly on overall particle crushing.

While other scholars chose the before-and-after changes of crushing on PSD curve to measure the crushing. For instance, Marsal[12] defined the particle crushing rate Bg through the aggregation of the absolute value of each grain group before crushing. This is a simple but defective method, but it hardly reflects the real volume changes.

Hardin[10] believes that under the same circumstance, large particles are more likely to be crushed and provided that the process will not cease until all particles are crushed into powder with the diameter of 0.074mm.From that, relative crushing rate Br is defined and widely applied, for it is easy to calculate and is able to reflect the crushing situation of particles in all respect. But, Turcotte[30] and Coop[31] and pointed out that after the crushing, instead of being unrestricted broken to powder, PSD curve would present a state of fractal dimension. According to this, based on the fractal theory, Einav[13] proposes to amend the Hardin fragmentation index, and gives a development limits on particle crushing, which perfects the method of crushing description above the whole changes of PSD curve. Likewise, many methods such as volume crushing rate BV put forward by Kong Dezhi[40], IG, state index of PSD defined by Muir Wood[41] all have something in common. Compared with other versions of description to single featured diameter of particles, there still existed some defects though based on the changes of PSD curve can reflect an overall degree of particle crushing.

(1)The PSD situation of soil is excessively complicated, the same parameters like Br 、IG have different PSD, while different initial PSD would have a significant influence on evolve of crushing.

(2) PSD is the major index of soil classification, while particle crushing is only one of factors to change it. Hydraulic permeability and other factors would make the fine particles drain away continuously, and PSD also changed. Obviously, the degree of the crushing cannot be defined by the methods above.

(3)The indexes of particles crushing states are the foundations to study the crushing evolve of particles. It is hard to define all the possibilities of the specific PSD crushing happened by methods above, therefore, which also is difficult to be applied to define constitutive features` study of the particle crushing.

In fact, before the crushing experiment started, all PSD soil samples have experienced the complex stress - hydraulic forces, and all of them are in some state of crushing, which can be relatively regarded as a started point of crushing. As the development of the crushing process, the fractal features of the PSD are much more obvious and existed in some specific breaking boundaries [30-32], which can be regarded as the end of crushing. For the initial PSD cannot certain first, that means the initial crushing state is too complicated to be studied directly. While there still exists the crushing boundary, any specific PSD soil sample has the same end crushing state, as the relationships between initial state and critical state in the framework of critical state soil mechanics. In other hand, evolving from initial crushing state to the

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final crushing state, a large number of experiments proved that as crushing of the particles, the big grain kept crushing into smaller ones, which led to the increasing volume of particles with raising the PSD curve and much more obvious linear relationship (shown as Fig. 1). Therefore, linear fitting can be used to describe the evolution of the particle crushing, and next chapter will put much emphasis on it.

A. State Index to Define Particle Crushing

In order to erasing the influences of different particle sizes, the particle diameters was unified and coordinate system $\ln P(D)$-$\ln(D/D_M)$ was chosen. At meanwhile, the minimum square method is chosen to conduct linear fitting. Among them, D means particle size, D_M is maximum particle size, $P(D)$ is the particle passing ratio and provided that:

(1)Whatever the size of particle is, the density remains same.

(2) The diameter size ranges from the minimum particle size to the maximum within interval [D_m, D_M] and they all are approximated to each other and continuous distributed.

(3)During crushing, the biggest group particles will never be crushed completely.

As showed in Fig. 1, the fitting line of PSD curve includes three parameters: intercept c, slope b and linear correlation coefficient ρ^2. Its expression and meaning are discussed respectively as follows.

Intercept c stands for the corresponding passing ratio of the maximum particle size. Because of the hypothesis that the biggest particle will never be crushed completely[30-31,41], the fitting line and y axis have one fixed intersect point, and intercept $c \equiv \ln P(D) = 4.605$. PSD curve equation is $P = \ln P(D)$; supposed that y is $\ln P(D)$, x is $\ln(D/D_M)$, and fitting line is:

$$y = bx + c = bx + 4.605 \tag{1}$$

AND $y \leqslant 4.605$, $x \leqslant 0$; supposed $a = \ln(D_m/D_M)$ Slope b can be acquired by minimum square law.

$$b = \frac{(1/n)\sum x \sum y - \sum(1/n)x \sum(1/n)y}{(1/n)\sum x^2 - (\sum(1/n)x)^2} \tag{2}$$

Provided that particle size is close to succession, that is $n \to +\infty$.Then,

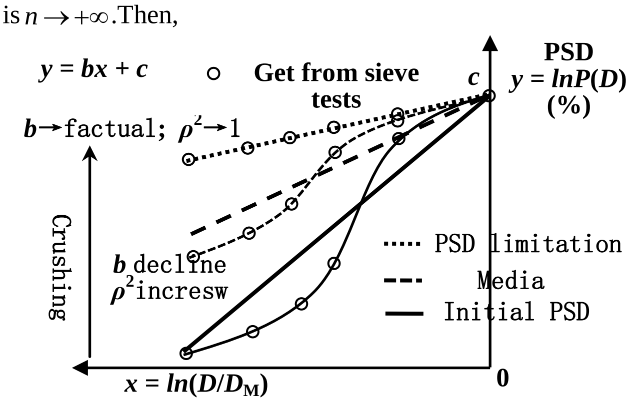

Figure 1. The revolution of fitting line of particle grading contributed by particle crushing

$$b = \frac{\int_a^0 y dx \int_a^0 x dx - \int_a^0 xy dx \int_a^0 1 dx}{\left(\int_a^0 x dx\right)^2 - \int_a^0 x^2 dx \int_a^0 1 dx} = \frac{6a\int_a^0 y dx - 12\int_a^0 xy dx}{a^3} \tag{3}$$

And $dx = dD/D$, the equation can be converted into:

$$b = \frac{6a\int_{D_m}^{D_M} \frac{1}{D} \ln P(D) dD - 12\int_{D_m}^{D_M} \frac{1}{D} \ln\frac{D}{D_M} \ln P(D) dD}{a^3} \tag{4}$$

Slope b of the fitting line reflected the PSD features of material, and its changing regulation also reflected the law of evolve of crushing: with increase of b, the volume of coarse particles is bigger, while the degree of particles crushing is lowered. As the development of the crushing, fine particles gradually increased and b gradually reduced to final crushing state.

Correlation coefficient ρ^2 can be acquired by minimum square law.

$$\rho^2 = \frac{[\sum(x-\bar{x})(y-\bar{y})]^2}{\sum(x-\bar{x})^2 \sum(y-\bar{y})^2} \tag{5}$$

Then

$$\rho^2 = \frac{\{\int_a^0 (x-\bar{x})(y-\bar{y}) dx\}^2}{\int_a^0 (x-\bar{x})^2 dx \int_a^0 (y-\bar{y}) dx} \tag{6}$$

Among them,

$$\bar{x} = \frac{1}{n}\sum x = -\frac{1}{a}\int_a^0 x dx = \frac{1}{2}\ln\frac{D_m}{D_M} \tag{7}$$

$$\bar{y} = \frac{1}{n}\sum y = -\frac{1}{a}\int_{D_m}^{D_M} \frac{1}{D} \ln P(D) dD \tag{8}$$

Bring equation(7)、(8) into(6),then

$$\rho^2 = \frac{a^4 b^2}{12\{[\int_{D_m}^{D_M} \frac{1}{D} \ln P(D) dD]^2 + a\int_{D_m}^{D_M} \frac{1}{D} \ln^2 P(D) dD]\}} \tag{9}$$

The change regulation of linear correlation coefficient ρ^2 also reflected the crushing evolve law: as the development of crushing, ρ^2 is gradually increasing from an arbitrary initial value and get close to 1 which is the final crushing state.

In conclusion, fixed value intercept c shows the biggest particle would not be totally crushed; slope b and correlation coefficient ρ^2 are determined by initial crushing state, as crushing evolved, b gradually reduced while ρ^2 is gradually increasing to fixed value and decided by crushing state. It also can be thought that the changes of b and ρ^2 are corresponding to the crushing evolve. Therefore, these two parameters can be used to define the crushing state. In next chapter, further explanation about b and ρ^2 would be made about the evolve law.

B. The Evolve Law of State Index to Define Crushing

This chapter will give a theoretical testament of b and ρ^2 with development of crushing, as follows:

For b, provided that $P_0(D)$ and $P_1(D)$ stand for passing rate before and after crushing.

But the slop changes of fitting line before and after crushing is

$$\begin{aligned}\Delta b=&\frac{6a\int_{D_m}^{D_M}\frac{1}{D}\ln P_1(D)dD-12\int_{D_m}^{D_M}\frac{1}{D}\ln\frac{D}{D_M}\ln P_1(D)dD}{a^3}\\&-\frac{6a\int_{D_m}^{D_M}\frac{1}{D}\ln P_0(D)dD-12\int_{D_m}^{D_M}\frac{1}{D}\ln\frac{D}{D_M}\ln P_0(D)dD}{a^3}\\=&\frac{6}{a^3}\int_{D_m}^{D_M}\frac{\ln[P_1(D)/P_0(D)]}{D}\ln\frac{D_mD_M}{D^2}dD\end{aligned}\qquad(10)$$

Because,

$$\int_{D_m}^{D_M}\frac{1}{D}\ln\frac{D_mD_M}{D^2}dD=-\frac{1}{2}\ln\frac{D_m}{D_M}>0\qquad(11)$$

Then we can get,

$$\int_{D_m}^{\sqrt{D_mD_M}}\frac{1}{D}\ln\frac{D_mD_M}{D^2}dD>\left|\int_{\sqrt{D_mD_M}}^{D_M}\frac{1}{D}\ln\frac{D_mD_M}{D^2}dD\right|>0\qquad(12)$$

And the particle crushing would raise PSD curve. In particle size range [D_m, D_M], there are $P_0(D)\leqslant P_1(D)$, $\ln[P_1(D)/P_0(D)]\geqslant 0$; Due to percentage composition adopted by PSD curve, in range[D_m, D_M], PSD curve is much closer to small particle size end rises much higher, instead, the gap would be narrowed when PSD curve is closer to large particle size end, just as Fig. 2 shows. So, $\ln[P_1(D)/P_0(D)]$, there is a tendency of monotone decreasing, then,

$$\begin{aligned}&\int_{D_m}^{\sqrt{D_mD_M}}\frac{\ln[P_1(D)/P_0(D)]}{D}\ln\frac{D_mD_M}{D^2}dD>\\&\left|\int_{\sqrt{D_mD_M}}^{D_M}\frac{\ln[P_1(D)/P_0(D)]}{D}\ln\frac{D_mD_M}{D^2}dD\right|>0\end{aligned}\qquad(13)$$

$$\int_{D_m}^{D_M}\frac{\ln[P_1(D)/P_0(D)]}{D}\ln\frac{D_mD_M}{D^2}dD>0\qquad(14)$$

And because $a<0$, and $\Delta b<0$.

In other words, the slope of fitting line will be gradually reduced as the development of the particle crushing.

As for ρ^2, by making hypotheses that different particle size is under the same circumstances as density etc. and substituting particle weight with its volume, S.W.Tyler [42-43] and other researchers deduced the relationships between the weight and fractal dimension of the particle diameter as follows.

$$\ln[M(D<D_s)/M_T]=(3-F)\ln(D_s/D_M)\qquad(15)$$

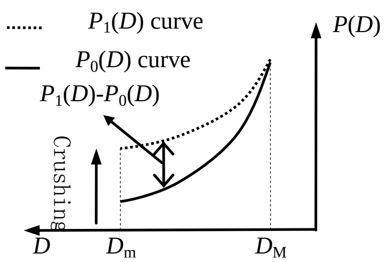

Figure 2. The sketch map of the revolution of grading with particle crushing

It can be converted into：

$$\ln M(D<D_s)=(3-F)\ln(D_s/D_M)+\ln M_T\qquad(16)$$

In equation, $M(D<D_s)$ means that the particle weight is smaller than that of some other specific characterized particle size D_s. M_T stands for the overall weight of particles and F means fractal dimension. It can be known that with fractal characteristics, PSD represents linear relation in double logarithm. Compared with equation(1), the percentage of passing mesh is showed in the left, which can be made certain by sieving test, that is $\ln P(D)$; $(3-F)$ is the slope b of the fitting line; $\ln M_T$ represents corresponding passing rate of biggest particles, namely linear relationship and parameters of c. Therefore, as the development of particle crushing, PSD curve is more likely to go through fractal [30-32].

III. Experimental Verification

This paper summarized some achievements of the typical particle crushing experiments and tested the methods that have established in this thesis.

To explore the relations between the resource PSD and permeability of the dam body materials, High-pressure triaxial tests on dense Cambria sand have been performed by Lade[36]. As the development of the crushing, the changes of PSD curve are showed in Fig. 3 under the circumstances that the confining pressure is 2.22MPa、8.0MPa、26.0MPa and 60MPa respectively. Parameters are showed in table one after conducting linear fitting on PSD curve according to the method put forward in this thesis.

From Fig. 3 and table Ⅰ, the conclusion can be drawn that sand sample is also obliviously crushed under complex stress state. As exacerbating of particle crushing, the curve keeps rising, while the slope of PSD fitting line is correspondingly reduced and the linear relations between filtering rate and diameter of the particles get more obvious.

The evolution of particle crushing was discussed by Mayoraz[44] and other scholars under the condition of confined compression from perspective of morphology and they also compared the influences of two materials of different strength and PSD on particle crushing and 3MPa particles. The crushing situations are showed in Fig. 4 when sandstone is under 0.5MPa and 1MPa of axial compression respectively, and the corresponding fitting line parameters are showed in table Ⅱ.

It is easy to see that under the condition of monotonic loading, particle crushing also happened on sandstone, and with the continuous development of the crushing, the total volume of fine granule is increased, the PSD curve keeps rising. In addition, the linear fitting line revolved round the fixed point (0, 1) in an anticlockwise direction, and the slope is reducing so that the linear related coefficient is close to 1.

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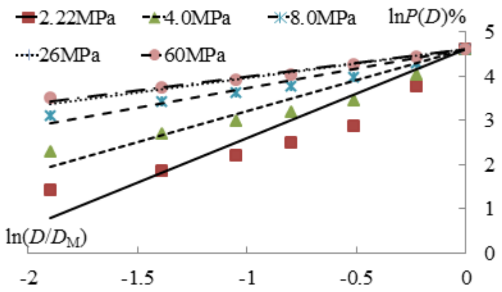

Figure 3. Evolution of grain-size distribution curves for drained triaxial compression tests of Lade

TABLE I. THE VALUE OF LINEAR FIT PARAMETERS-B、P^2 OF TRIAXIAL COMPRESSION TESTS OF LADE

indexes \ σ3/MPa	2.22	4.0	8.0	26.0	60
b	2.0	1.40	0.87	0.64	0.62
ρ^2	0.81	0.86	0.94	0.98	0.98

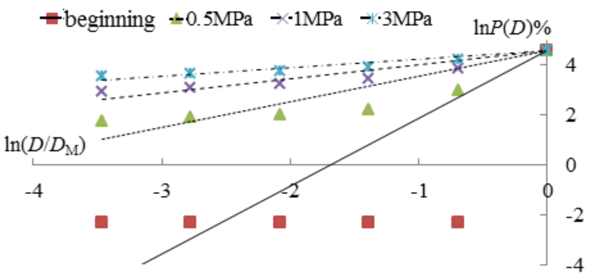

Figure 4. Evolution of grain-size distribution after monotonic loading of Mayoraz

TABLE II. THE VALUE OF LINEAR FIT PARAMETERS-B、P^2 AFTER MONOTONIC LOADING OF MAYORAZ

indexes \ σ1/MPa	Beginning	0.5	1.0	3.0
b	2.72	1.03	0.56	0.35
ρ^2	0.09	0.57	0.78	0.88

Using axial compressive force provided by large direct shear apparatus, this paper did a repeated confined compression tests on basalt of initial specific PSD under the same axial load. The stress controlled each time is 2.4MPa and the terminate conditions of the test is that the sample should not produce any axial strain. After being the compressed and then filtered, the particles are sampled again and sent to the next test; The particles crushing situations are displayed in Fig. 5, the corresponding state parameters can be seen in table 3. Obviously, PSD curve shares a same tendency of development with traixial and confined compressive stress situation.

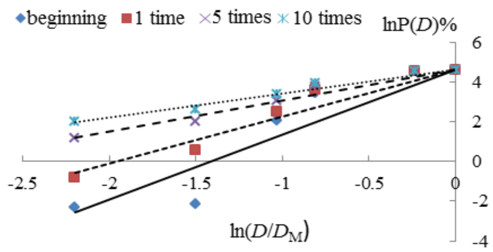

Figure 5. The degree of basalt particles breakage under cycle loading

TABLE III. THE VALUE OF LINEAR FIT PARAMETERS- B、P^2 AFTER DIFFERENT LOADING NUMBERS

indexes \ number	Beginning	1	5	10
b	3.25	2.36	1.55	1.19
ρ^2	0.86	0.94	0.96	0.97

From what have been discussed above (as shown in Fig. 6), corresponding to the evolve of the crushing, the PSD curve has a similar tendency of development however the particles are crushed during load: The slope b of the PSD linear fitting line is also reduced with increasing development of the particles crushing. With more obvious line relations between the ordinate and abscissa in coordinate $\ln P(D)$-$\ln(D/D_M)$, the linear fitting line revolved round the fixed point(0, 4.605)in an anticlockwise direction, which is identified with the method established by this paper. Therefore, the crushing state can be reflected by the changes of the fitting line and parameters related in PSD curve in the cause of crushing development.

IV. THE RELATIONS BETWEEN CRUSHING STATE INDEX AND FRICTION ANGLE

From above, macroscopic mechanical behavior of the particles are changed due to the particle constitutive evolve caused by particles crushing. Therefore studying the quantitative relation between state parameters b, ρ2 between material mechanical indexes mentioned in this paper can further testify rationality and applicability of the new crushing state parameters.

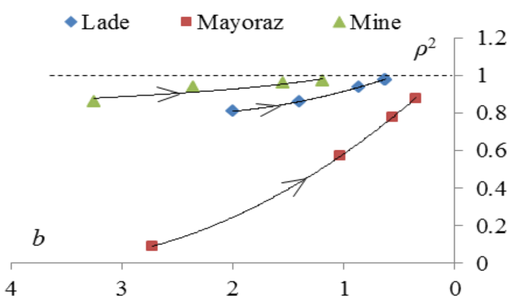

Figure 6. The evolution of the particles crushing state indexes development with crushing

The soil strength theories, the earliest theory being studied and put forward, laid a solid theory foundation for limit equilibrium analysis of soil which is closely bound to many engineering problems. Among them, with simple formation, clear physical meaning and easy accessed, the Mohr-Coulomb strength theory is widely applied to all fields of the geotechnical engineering project. While a large number of practices prove that strength envelope is not the straight line in a strict sense, especially for big particles under the high stress. For example, Leon believes that strength envelope should be a parabolic curve, while De Mello[45] put forward that it is power function relationship that is more suitable and Duncan[46] holds it is logarithmic relationship between confining pressure σ^3 and inner frictional angel φ. All those outlooks mainly explained from the macroscopic phenomenological perspective, while the studies of recent years try to explain this phenomenon from perspective of micro-mechanism. Some other researchers believe that the particle crushing is the main cause to the changes of the internal friction angle, but they do not reach an agreement, for some results of tests are even opposite to it. Take Hassanlourad[47] for example, by comparing two experiments on different materials, he points out that particle crushing has an significant effects on frictional angle of material and further established the relations between crushing and the strength of the soil from the aspect of energetic. And the study of Miura[48] etc. point out that as the particle crushing intensified, the soil strength is lowered obviously. While after a set of ring shear tests, Coop[31] and etc. point out the frictional angel did not change greatly with the development of the particle crushing .

To resolve it, this paper chose the quartz sand (The silica purity reaches more than 99%, and the hardness reaches the 7~8 model; After the experimental verification, the sand was not affected by direct shear test process under 300kPa axial compression, and there is no obvious particle crushing, PSD remains same.) which is characterized difficult to be crushed as experiment material to explore the quantitative relationships between crushing state parameters b and ρ^2 mentioned and internal frictional angel by simulating samples of different PSD artificially and simulating different states of particle crushing.

A. Experimental Steps

The experiments are done under two circumstances. Under the first circumstance, the quartz sand is sampled with crushing state parameters b as1.5 and 0.5 respectively, and ρ^2 as 0.9, 0.8 and 0.6. In the second condition, crushing state parameters ρ^2 remains unchanged as 1, while b is 0.5, 5, 10, 15, 20 and 25 respectively to make different quartz sand samples with different PSD.

We can get samples with same volume by shocking the shear box of different degree, so that every sample has same void ratio e: In order to get strength curve, 50kPa, 100kPa, 200kPa, 300kPa, the axial load of every PSD sample is 50kPa, 100kPa, 200kPa and 300kPa respectively. The velocity of direct shear test remains 0.8mm/min until the sample is destroyed and the peak shear strength is acquired. In addition, all the tests should be done in arid environment and basic physical parameters are showed in table Ⅳ.

B. Data Analysis of Experiment

Under the first condition, relations between crushing state parameters ρ^2 and φ are displayed in Fig. 7. We can tell that there is no correlation between fixed strength parameter φ and parameter ρ^2.

In Fig. 8 of the second circumstance, the relationship is showed between the crushing state parameter b and shear strength index φ of quartz sand. Parameter b affects the frictional angel obviously: as reducing of the slope b of PSD curve which means particle crushing gets worse, quartz sand internal frictional angel accessed after shear experiment presents a tendency of increasing first and then reducing, and it is not a simple monotonic relationship .

As shown in Fig.8, as the crushing of particles, the development of frictional angel can be divided into 3 phrases：the rising period(③in Fig. 8), stationary phase (②in Fig. 8) and decline period(①in Fig. 8). At phrase of ③, b is much more bigger, and sample are mainly made of maximum particle group. Due to less volume of fine particles, although, big particles are closely connected to each other and formed certain strength, but there still exist many spaces among them. Under the effects of external force, the occlude effect sliding. It tends to be found that as the exacerbating of crushing, the internal frictional angel is becoming large when the particle crushing experiment is done at this PSD phrase. among particles is damaged and produced relative With the crushing of the particles and increasing of the fine particles, b value decreases to the stage②, in which phrase, PSD particles have both stress-bearing frame formed by specific volumes of big particles and ample fine particles to fill the space. Therefore, the material with this kind of structure has a better compactness, and shear strength also has a significant improvement; Therefore, the frictional angle is found no obvious changes when particles crushing experiment is done during this PSD phrase. In the period ③，the frame function of big particles would disappear because continuity was lost and the big particles are hanging among fine particles as long as the particles continue to crush, in which way the coarse particles will be further reduced and fine particles volume is increasing. Thus the shear strength is lowered, and it will show a decreasing trend for internal frictional angle of the material got in particle crushing experiment done in this period.

TABLE IV. BASIC PARAMETERS OF DIRECT SHEARING TESTS

Test	PSD	side（mm）	e	φ
ρ^2=1	b=25	2.5-3	0.693	38.27
	b=20	2.0-3	0.693	39.35
	b=15	1.43-3	0.692	43.26
	b=10	1.0-3	0.693	44.42
	b=5.0	0.5-3	0.694	39.35
	b=0.5	0.075-3	0.695	36.9
b=0.5	ρ^2=0.9	0.075-3	0.695	37.4
	ρ^2=0.8	0.075-3	0.694	37.2
	ρ^2=0.6	0.075-3	0.693	37.3
b=1.5	ρ^2=0.9	0.16-3	0.695	38.7
	ρ^2=0.8	0.16-3	0.694	40.7
	ρ^2=0.6	0.16-3	0.693	39.7

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Actually, for samples in any state of the crushing, it is due to the particle PSD parameter *b* distributing in various zone that makes inner frictional angel present different evolve law as the exacerbating of particle crushing. Parameters *b* and ρ^2 jointly reflected the crushing state of particle, the evolve possibilities and tendency of the crushing. When the *b* value is bigger and ρ^2 is smaller, particle crushing are more prone to happening; when *b* tends to 0.5 and ρ^2 tends to 1, the crushing will be terminated. And with the development of the particle crushing, inner frictional angel presents a tendency of increasing at first place and then reducing. Strength envelope is not ideal linear relation, that is, particle crushing is one of cause to make strength line present nonlinear evolution. Therefore, it is reasonable to use slope *b* of fitting line and linear related parameter ρ^2 to stand for particle PSD.

V. Conclusions

Targeted at the trail phenomenon of geotechnical material particle crushing, this paper puts forward a new index to measure the particle crushing state, explored the relations between the concerning indexes and the frictional angle and then researched the effect of particle crushing having on the strength of geotechnical materials, from which major conclusions are drew as follows.

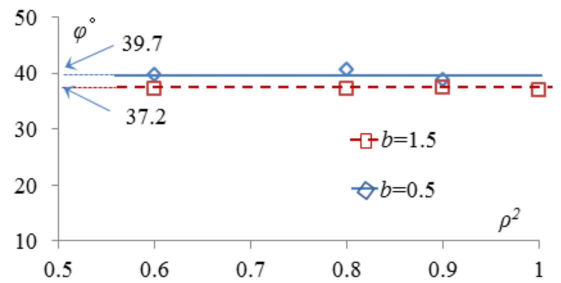

Figure 7. The relationship between the parameters of particle crushing stateρ2 and strength index φ

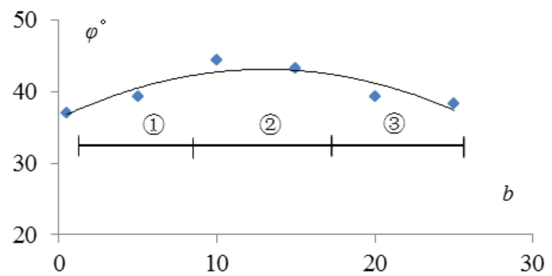

Figure 8. The relationship between the parameters of particle crushing state b and strength index φ

(1) By the fitting line，this paper proposed a method of using the linear regression line slope b and relating coefficient $\rho2$ to describe the revolution characters of particle crushing state. A evolved process of the quantitative description of fractured state can be defined by the changes of the fitting line parameters.

(2) The related crushing tests and theories analysis illustrated a changing tendency that the parameters b and ρ^2--particle crushing state change with the particle crushing: as the development of the particle crushing, correspondingly, the parameters *b* is reduced and ρ^2 is more close to 1, fitting line revolved round the fixed point(0, 4.605)countering clockwise rotation. In this way, intrinsic properties of the particle crushing state can be reflected.

(3) Two sets direct shear tests of quartz sand with specific PSD parameters *b* and ρ^2 make certain that particle crushing state will produce an significant effect on material strength: as the crushing evolved, strength parameter φ will present a tendency of rising first and then reducing, and gives a unified explanation to various experimental phenomenon from aspect of micro-mechanism. Though both parameters *b* and $\rho^2$are needed to define the particle PSD state, they have the definite physical meaning: the strength is solely related to b, but b and ρ^2 jointly determined the possibilities and tendency of particle crushing.

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