

# An Investigation on the Pushover Analysis Based Methods for Compensating the P-Delta Effects

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**Abstract**—Nonlinear Static Analysis (NSA) is currently considered as one of the most commonly used methods to estimate capacity of structural components. It is relatively simple and computationally inexpensive to generate the force-displacement graph using pushover analysis, and unlike nonlinear dynamic analysis the obtained results can easily be interpreted. NSA has been the basis for multiple methods for designing columns with consideration of P-Delta effects. A precise solution for stability problem under dynamic loading requires consideration of the randomness of ground motions and material uncertainties. Static procedures such as NSA ignores the fact that characteristics of the ground motions such as intensity, frequency content, and duration varies from one earthquake record to the other. On the other hand, most engineering firms are reluctant to perform complex nonlinear time-history analysis due to its high computational cost and other inherent complexities. Among different methods which redesign columns for P-Delta effects using pushover analysis two methods which use same energy under force-displacement diagram and same effective stiffness at target ductility have been studied here. Caltrans SDC target ductility for single column bents supported on fixed foundation is considered as the design target. The main objective of this research is to identify the applicability regions of these two methods for designing RC bridge columns in order to compensate for P-Delta effects.

**Index Terms**—Pushover analysis, P-Delta effects, RC bridge column design, Nonlinear Static Analysis, Incremental Dynamic Analysis

## I. INTRODUCTION AND BACKGROUND

The action of vertical loads acting through column's lateral deformations is identified as the P-Delta effects (second-order effects) [1]. For elastic static design it has been proven that a simplistic amplification factor based on the first order analysis is satisfactory to mitigate the P-Delta effects [2]. However, under strong ground motions, the column might experience extensive inelastic responses. Capturing the dynamic P-Delta effects is a key

issue in structural earthquake engineering as it can intensify the structure's seismic responses and trigger instability [3].

Nonlinear static analysis of structures, which informally denoted as pushover analysis has been the basis for multiple methods for designing columns with consideration of P-Delta effects [4], [5]. Force and deformation capacities obtained from pushover analysis of a column can be used to redesign a column to compensate for P-Delta effects. Among these methods Paulay [4] and MacRae [5] will be discussed shortly. Nonlinear static analysis is currently considered as one of the most commonly used methods to estimate capacity of structural components [6]. The process to obtain force versus displacement diagram using nonlinear static procedure is relatively straight forward, and unlike nonlinear dynamic analysis the obtained results can easily be interpreted. Design according to Caltrans-SDC is a displacement based procedure requiring pushover analysis. Caltrans SDC [7] provides a procedure that can be used to evaluate whether P-Delta effects can be ignored in design. In design circumstances, which P-Delta effects can be ignored, structural components are designed based on predefined ductility demands. In cases where the moment induced due to P-Delta effects are greater than 20%, Caltrans SDC recommends to perform nonlinear time-history analysis to verify that the column is capable to resist the P-Delta effects.

Caltrans method for redesigning the column for the P-Delta effects depends on the ratio of the P-Delta induced moment which is the lateral offset between the point of contra-flexure and the base of the plastic hinge multiplied by the axial load, and the idealized plastic moment capacity of a column which is calculated by M- $\phi$  analysis. If this ratio is less than 20%, predefined ductility demands limits the design of structural components. For instance for single column bents supported on fixed foundation Caltrans recommends design target ductility of four.

The P-Delta effects can be more accurately studied using Nonlinear Dynamic Analysis (NDA) with multiple

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ground motion records. Incrementally amplifying the earthquake record to achieve a particular displacement ductility is an application of NDA in studying the structural response considering the P-Delta effects [8], [9]. This process is similar to Incremental Dynamic Analysis (IDA) which is a series of nonlinear dynamic analyses of a particular structure subjected to a suite of ground motions of varying intensities [10]. The intention of such analysis is to provide information on the performance of a structure at various stages, such as, elastic response, inelastic response, and collapse of the structure [8]. Difficulties of performing nonlinear time history analysis such as its high computational cost, lack of available ground motion set for different locations, and inherent complexity of nonlinear dynamic analysis make it less desirable than pushover analysis.

Among different methods which are based on pushover analysis to redesign columns for P-Delta effects following methods are considered in this study.

#### A. Same Energy under Load-Deformation Response (Paulay (1978))

This method is proposed by Paulay (1978), and incorporates NSP to redesign a column for P-Delta effects. P-Delta effects are considered by redesigning RC columns for an increase in capacity. This increase is such that the energy under the load-deformation response of the redesigned column with P-Delta effect is the same as the energy under the original load-deformation response with no P-Delta effect (Fig. 1).

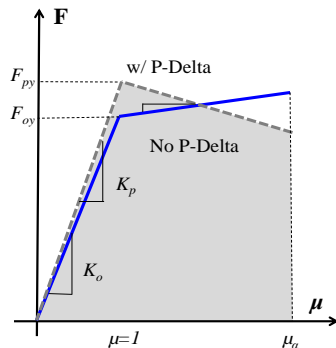


Figure 1. Bilinear force-displacement diagram (Same energy concept)

#### B. Same Effective Stiffness at Target Ductility (Macrae (1990))

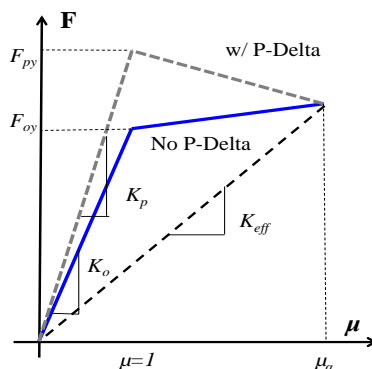


Figure 2. Bilinear force-displacement diagram (Same effective stiffness concept)

This method for considering the P-Delta effect is proposed by MacRae et. al. (1990). In this method the increase in capacity is such that the redesigned column has the same effective stiffness as the original column without P-Delta effect at the targeted displacement (Fig. 2).

## II. METHOD

Throughout this research nonlinear pushover and time history analyses were performed using the open source object-oriented nonlinear structural analysis program, Open System for Earthquake Engineering Simulation (OpenSees) [11].

The procedure to perform NSP is as following; under the nonlinear static procedure, structural model, which directly incorporates inelastic material response, displaces to a target displacement, and resulting internal deformations and forces are determined. The nonlinear load-deformation characteristics of individual components and elements of the building model directly into the mathematical model. Monotonically increasing lateral forces or displacements is applied to the mathematical model until either a target displacement exceeds or the building collapses.

#### A. Iterative Procedure to Redesign the Column

An iterative procedure is used to achieve criteria proposed by Paulay. Reinforcement ratio from the original column (Without P-Delta) is used as the initial reinforcement ratio. Based on the energy absorption capacity of the structure at the target displacement the longitudinal reinforcement ratio is modified. Fig. 3 shows the general procedure to obtain the criterion proposed by Paulay which is having same energy under force-displacement diagrams with and without P-Delta effects (Fig. 4).

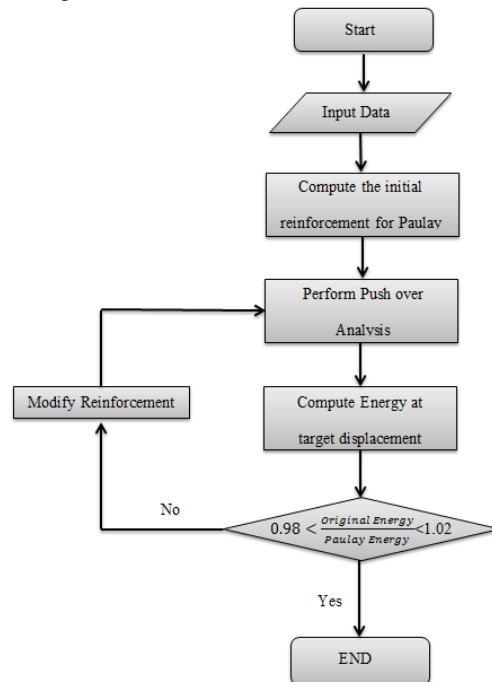


Figure 3. Same energy absorption capacity procedure

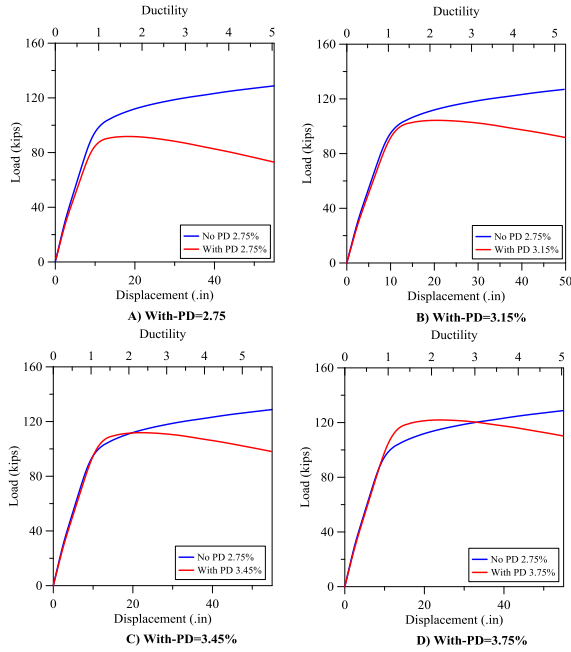


Figure 4. Same energy absorption capacity method sample (Column spec: Col Height=48ft, Axial load=586kips,  $\rho=2.75\%$ )

A similar procedure is used to comply with the criteria proposed by MacRae. Reinforcement ratio from the original column (Without P-Delta) is used as the initial reinforcement ratio. Based on the effective stiffness of the structure at the target displacement the longitudinal reinforcement ratio is modified. Fig. 5 shows the general procedure to redesign columns using the MacRae method which is having same effective stiffness with and without P-Delta effects at target ductility (Fig. 6).

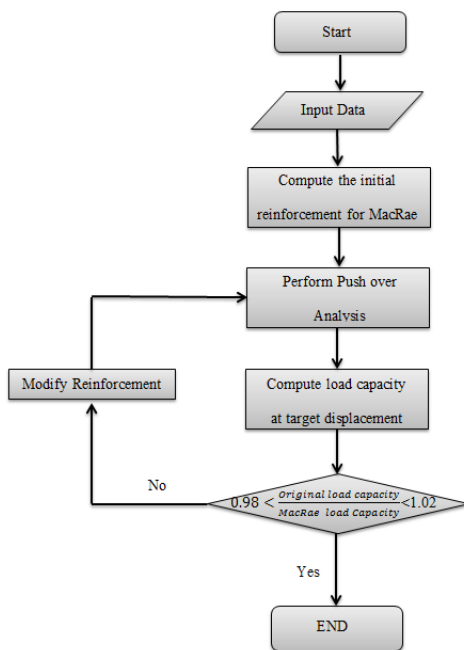


Figure 5. Same effective stiffness procedure

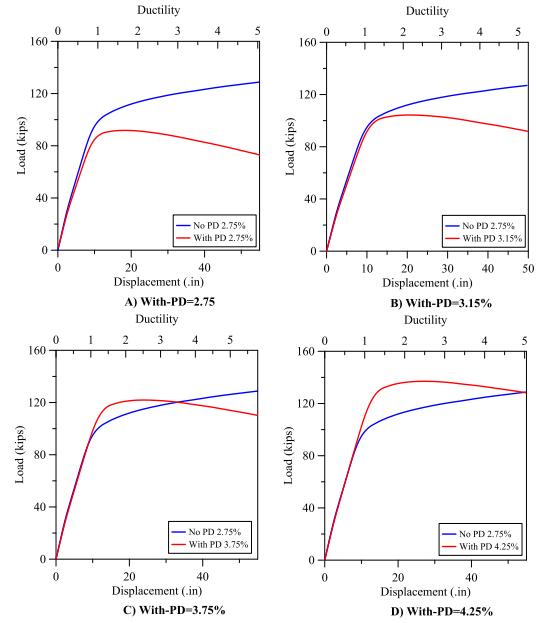


Figure 6. Same effective stiffness method sample (Column spec: Col Height=48ft, Axial load=586kips,  $\rho=2.75\%$ )

Design target ductility suggested by Caltrans SDC is adopted in this research. For single column bents supported on fixed foundation Caltrans recommends design target ductility of four, and for Multi-Column Bents supported on fixed or pinned footings it recommends target ductility of five.

#### B. Column Properties and Specifications

Table I shows the general column properties for the columns subjected to study.

27 columns have been selected based on their axial load and their height to diameter ratio. These columns include low (389 kips), medium (589 kips, 778 kips), and high (973 kips, 1168 kips) axial load columns. Selected columns also include low (16, 20, and 24 ft.), medium (28, 32, and 36 ft.), and high (40, 44, and 48 ft.) column height ratios. Table II tabulates column categories, and Table III presents all the columns subjected to this study.

TABLE I. COLUMN PROPERTIES

Concrete Strength, $f'_c$ (MPa, ksi)	27.5 (4)
yield Strength, $f_y$ (MPa, ksi)	413 (60.0)
Modulus of elasticity, $E_s$ (MPa, ksi)	$2 \times 10^5$ (29,000)
yield strain, $\epsilon_y$	0.0015
Longitudinal reinforcement ratio (%)	1
Column diameter, L (m, ft)	1.21 (4)
Column aspect ratio, (CAR)	4 to 12
Cover concrete (cm, in)	5 (2)
Axial load (kips), (AXL)	389, 589, 778, 973, 1168

TABLE II. AXIAL LOAD AND COLUMN HEIGHT CATEGORIES

	Low	Medium	High
Axial Load (kips)	389	584, 778	973, 1168
Axial load ratio	4	6, 8	10, 12
Column height (ft)	16, 20, 24	28, 32, 36	40, 44, 48
Column height ratio	4, 5, 6	7, 8, 9	10, 11, 12

TABLE III. COLUMNS SUBJECTED TO THIS STUDY

Col.ID	Axial.I (Kips)	Height (ft.)	$\rho_L$ (%)	$\rho_v$ (%)	Target Ductility
1	389	16	1	1	4
2	389	20	1	1	4
3	389	24	1	1	4
4	389	28	1	1	4
5	389	32	1	1	4
6	389	36	1	1	4
7	389	40	1	1	4
8	389	44	1	1	4
9	389	48	1	1	4
10	589	16	1	1	4
11	589	20	1	1	4
12	589	24	1	1	4
13	778	28	1	1	4
14	778	32	1	1	4
15	778	36	1	1	4
16	589	40	1	1	4
17	589	44	1	1	4
18	589	48	1	1	4
19	1168	16	1	1	4
20	1168	20	1	1	4
21	1168	24	1	1	4
22	973	28	1	1	4
23	973	32	1	1	4
24	973	36	1	1	4
25	1168	40	1	1	4
26	1168	44	1	1	4
27	1168	48	1	1	4

### C. Evaluation of the Redesigned Column

Open System for Earthquake Engineering Simulation (OpenSees) [8] is used to investigate the nonlinear load-deformation response of RC bridge columns. The circular cross-section was represented by a fiber-based model and the concrete cover and core sections were modeled with the “Concrete07” uniaxial concrete material class. The procedure to evaluate the redesigned columns using earthquake record and nonlinear time-history analysis is as following.

- 1) Apply the earthquake record and obtain the load-deformation data (without P-Delta).
- 2) Compute the maximum displacement and corresponding ductility level.
- 3) Amplify the earthquake record and perform step 1 and 2 until target ductility of 4 is achieved. (Caltrans recommendation for target ductility)
- 4) Include the P-Delta effect in the analysis and apply the scaled earthquake corresponding to the target ductility.
- 5) Compare the obtained ductility from the analysis with P-Delta and the Target Ductility.

## III. RESULTS

### A. Redesigning Columns Using Pushover Analysis

Iterative pushover analysis as discussed in the method section is performed on the 27 studied columns. Table IV shows the results obtained from the Paulay and MacRae method for redesigning the columns

TABLE IV. REDESIGNED COLUMNS FOR P-DELTA EFFECTS

Col.ID	Initial $\rho_L$ (%)	Paulay method $\rho_L$ (%)	$D_y$ (.in)	MacRae method $\rho_L$ (%)	$D_y$ (.in)
1	1	1.1	2.13	1.1	2.13
2	1	1.1	2.19	1.2	2.30
3	1	1.2	2.35	1.2	2.35
4	1	1.2	2.40	1.3	2.56
5	1	1.2	2.49	1.3	2.63
6	1	1.3	2.67	1.3	2.67
7	1	1.3	2.74	1.4	2.84
8	1	1.4	2.90	1.5	2.97
9	1	1.4	2.95	1.5	3.03
10	1	1.2	2.88	1.2	2.88
11	1	1.2	2.99	1.3	3.17
12	1	1.2	3.00	1.3	3.19
13	1	1.3	4.42	1.5	4.70
14	1	1.4	4.68	1.6	4.88
15	1	1.5	4.88	1.7	5.09
16	1	1.4	3.58	1.6	3.73
17	1	1.4	3.65	1.7	3.91
18	1	1.5	3.79	1.8	4.06
19	1	1.3	7.57	1.5	8.04
20	1	1.4	7.97	1.7	8.61
21	1	1.6	8.47	1.8	8.90
22	1	1.5	6.37	1.7	6.68
23	1	1.6	6.64	1.8	6.98
24	1	1.7	6.90	1.9	7.32
25	1	2	10.00	2.5	10.91
26	1	2.2	10.49	2.7	11.56
27	1	2.4	11.04	2.9	12.05

### B. Evaluation of Redesigned Columns Using Nonlinear Time-History Analysis

The ground motion record is selected from Pacific Earthquake Engineering Research Center (PEER-NGA) database. Table V shows the characteristics of the ground motion record.

TABLE V. EARTHQUAKE RECORD PROPERTIES

EQ ID	Earthquake			PGA <sub>max</sub> (g)	PGV <sub>max</sub> (cm/s.)
	M	Year	Name		
12011	6.7	1994	Northridge	0.52	63

Fig. 7 shows the acceleration versus time for the ground motion file.

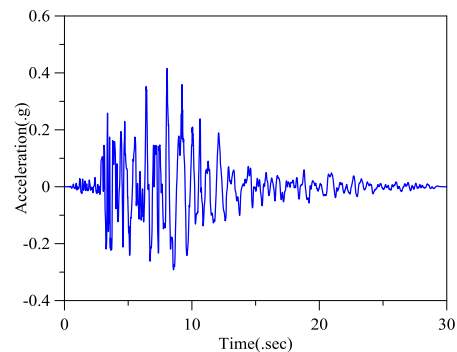


Figure 7. Time-acceleration for EQrecord12011

Typically, shorter columns with smaller axial load don't suffer significantly from P-Delta effects. This is also justified by the results obtained from pushover

analysis performed in this research as the redesigned columns for P-Delta effects have similar longitudinal reinforcement ratio as the original column. As the columns get taller and the axial load increases the P-Delta effects become more significant. Fig. 8 and Fig. 9 shows the result obtained from redesigning the columns (Col.ID=25, 27). these columns have high axial load ratio and high column height ratio. Both MacRae and Paulay methods were unable to redesign these columns for P-Delta effects for target ductility of 4. Paulay and MacRae methods show more promising results for columns which have Medium and low axial loads on top (Fig. 10 and Fig. 11). These methods precisely were able to redesign the columns for target ductility of four. Table VI shows the results obtained from all 27 studied column.

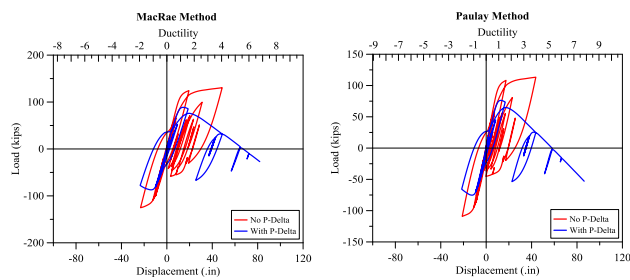


Figure 8. Redesigned Column (Col.ID=27)

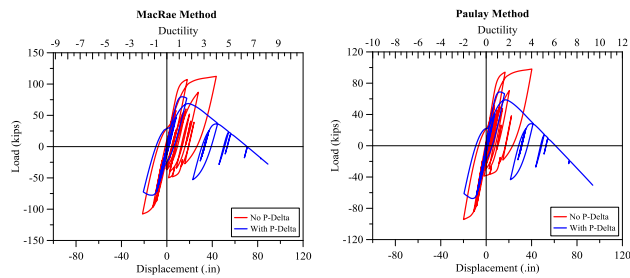


Figure 9. Redesigned Column (Col.ID=25)

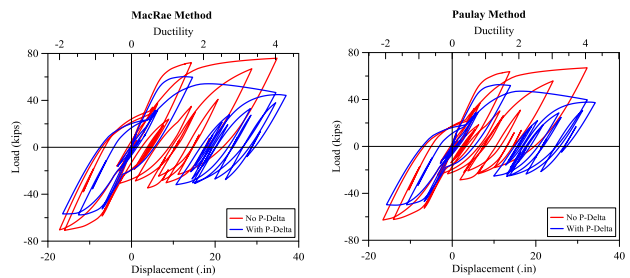


Figure 10. Redesigned Column (Col.ID=20)

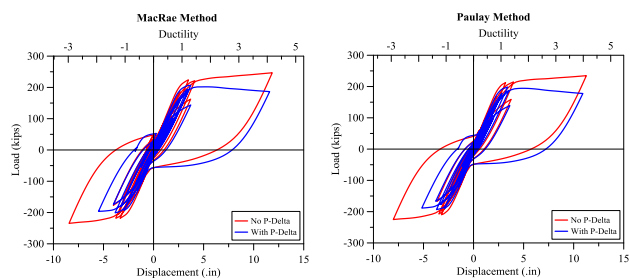


Figure 11. Redesigned Column (Col.ID=7)

TABLE VI. NONLINEAR TIME-HISTORY RESULTS(EQID12011)

Col.ID	AXL	CAR	Paulay max disp .in	$\mu$	MacRae max disp .in	$\mu$
1	LOW	LOW	8.37	3.92	8.37	3.92
2	LOW	LOW	8.90	4.07	9.71	4.22
3	LOW	LOW	9.87	4.20	9.87	4.20
4	LOW	MID	10.54	4.39	11.26	4.40
5	LOW	MID	11.09	4.45	11.86	4.50
6	LOW	MID	11.45	4.29	11.45	4.29
7	LOW	HIG	10.91	3.98	11.59	4.08
8	LOW	HIG	16.33	5.64	14.24	4.79
9	LOW	HIG	16.34	5.54	16.99	5.61
10	MID	LOW	12.68	4.40	12.68	4.40
11	MID	LOW	16.07	5.37	13.43	4.24
12	MID	LOW	15.84	5.27	16.77	5.25
13	MID	MID	19.17	4.34	19.72	4.20
14	MID	MID	18.39	3.93	19.84	4.07
15	MID	MID	18.32	3.76	19.76	3.88
16	MID	HIG	15.97	4.46	16.07	4.30
17	MID	HIG	15.58	4.27	16.36	4.19
18	MID	HIG	18.07	4.76	18.35	4.51
19	HIG	LOW	30.14	3.98	32.22	4.01
20	HIG	LOW	34.20	4.29	36.97	4.30
21	HIG	LOW	37.87	4.47	39.45	4.43
22	HIG	MID	24.90	3.91	26.37	3.95
23	HIG	MID	24.30	3.66	29.13	4.18
24	HIG	MID	28.87	4.18	30.66	4.19
25	HIG	HIG	93.88	9.39	89.11	8.17
26	HIG	HIG	88.22	8.41	86.62	7.49
27	HIG	HIG	85.83	7.77	81.18	6.74

#### IV. CONCLUSIONS

Columns with low and medium axial load which were redesigned using both Paulay and MacRae method for compensating for P-Delta effects showed promising results in terms that nonlinear time-history analysis on redesigned columns with inclusion of P-Delta effects achieved damage ductility very close to target damage ductility. For single column bents supported on fixed foundation Caltrans recommends design target ductility of four which is used in this research.

Obtained results for columns with high axial load ratio and high column aspect ratio showed that the redesigned columns collapsed under nonlinear time-history analysis. For columns with high axial load ratio and high column aspect ratio, which suffer the most from P-Delta effects it is suggested to perform time-history analysis with multiple earthquake records to study the instability effects of P-Delta effects instead of using static nonlinear analysis which is unable to fully capture the dynamic nature of P-Delta effects under dynamic loading. The stability problem under seismic loading is dynamic by nature, and using static procedures such as pushover analysis especially for cases with high P-Delta effects is discouraged.

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