Torsional Behavior of Reinforced Concrete Beams Predicted by a Compatibility-Aided Truss Model

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***Abstract*—To avoid brittle torsional failure due to web concrete crushing before yielding of torsional reinforcement on reinforced concrete members, the ACI 318-14 design code and the Eurocode 2 limit the yield strength of torsional reinforcement up to 420 MPa and 600 MPa, respectively. In this study, six beams having different compressive strength of concrete and yield strength of torsional reinforcement were tested. The observed test results were compared with the torsional behavior predicted by a compatibility-aided truss model. Experimental and analytical results showed that torsional strength of reinforced concrete beams did not increase as the yield strength of torsional reinforcement increased. The beams with high strength torsional reinforcement failed due to concrete crushing before yielding of reinforcement. Test results also indicated that the limitation on the yield strength of torsional reinforcement in the ACI 318-14 design code was appropriate but not in the Eurocode 2.**

***Index Terms*—compatibility-aided truss model, failure mode, high-strength reinforcement, reinforced concrete beams, torsional strength**

I. Introduction

The ACI 318-14 design code [1] and the Eurocode 2 [2] limit the yield strength of shear and torsional reinforcement on reinforced concrete (RC) members up to 420 MPa and 600 MPa, respectively. The reasons why the design codes limit the yield strength of shear and torsional reinforcement are as follows.

The first reason is to avoid compression failure of concrete struts. It is because the failure by compression of concrete struts before yielding of shear and torsional reinforcement is brittle. And the second reason is to limit diagonal crack width. When using high-strength reinforcement, spacing of reinforcement is wider at the same level of design shear or torsional strength. Thus the number of diagonal cracks decreases and width of a diagonal cracks increases when using high-strength reinforcement.

In this study, six reinforced concrete beams with high strength materials were tested to investigate the torsional behavior of RC beams. The observed test results were compared with the torsional behavior predicted by a compatibility-aided truss model (CATM). In addition, some simulations are conducted to figure out the torsional behavior of RC beams with high strength materials.

II. Test Program

The torsional strength equation in the design codes were derived based on the thin-walled tube analogy and the space truss model. The design codes determine the torsional strength at the yield strength of the longitudinal and transverse reinforcement (f_{yl} and f_{yt}) [3]. In the ACI 318-14 design code, the torsional strength is less value of the torsional strength of longitudinal reinforcement and that of transverse reinforcement. In the ACI 318-14 design code, the torsional strength is estimated by (1) and the angle of the concrete struts α is recommended to use 45° or can be calculated by (2).

$$T_n = \min\left(\frac{2A_o A_t f_{yt}}{s}\cot\alpha,\ \frac{2A_o A_l f_{yl}}{p_h}\cot\alpha\right) \tag{1}$$

$$\cot^2\alpha = \frac{\rho_l f_{yl}}{\rho_t f_{yt}} \tag{2}$$

where A_o: gross area enclosed by shear flow path; A_t: cross-sectional area of one transverse reinforcement; A_l: total cross-sectional area of longitudinal reinforcement; s: spacing of stirrups in the direction parallel to the longitudinal axis of member; p_h: perimeter of centerline of outermost closed transverse torsional reinforcement; and α: inclination between the direction of principal compressive stress of concrete and the longitudinal direction.

And the ACI 318-14 design code limits shear and torsional strength to avoid compression failure of concrete struts before yielding of torsional reinforcement by empirical and experimental equation (3).

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$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2+\left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d}+\frac{2\sqrt{f_c'}}{3}\right) \quad (3)$$

where V_u: factored shear force at section; b_w: web width ; d: effective depth; T_u: factored torsional moment at section; A_{oh}: area enclosed by centerline of the outermost closed transverse torsional reinforcement; V_c: nominal shear strength provided by concrete; f_c': specified compressive strength of concrete.

By (3), the boundary value of torsional strengths of concrete struts and torsional reinforcement T_{cs} can be calculated as (4).

$$T_{cs}=\frac{5}{6}\sqrt{f_c'}\frac{1.7A_{oh}^2}{p_h} \quad (4)$$

With above equation (4), the failure mode of torsional members can be determined. If the torsional strength of concrete struts is larger than that of reinforcement, the failure mode can be determined as under-reinforced failure. And the opposite case's failure mode can be determined as over-reinforced failure.

A. Testing Plan

Table I shows the details of the test specimens. The experimental program consisted of 6 specimens. The main variables of specimens were the compressive strength of concrete, and the yield strength of reinforcement. Two different compressive strengths of concrete were used (42 MPa and 70 MPa). And three different yield strengths of reinforcement were used (about 300 MPa, 400 MPa, and 600 MPa) [4].

The test specimens had same longitudinal and transverse reinforcement ratio (ρ_l=0.009979 and ρ_t=0.010582). And two different compressive strength of concrete and three yield strengths of reinforcement were used. Thus torsional strengths of the specimens calculated by (1) are different according to the yield strength of reinforcement.

The section and configuration of the specimens are shown in Fig. 1 and Fig. 2, respectively. The cross section of the beam was 300 mm × 350 mm. Closed stirrups were used as transverse torsional reinforcement. Distance between centerlines of the closed stirrups was 260 mm × 310 mm. The amounts of longitudinal and transverse reinforcement ($\rho_l f_{yl}$ and $\rho_t f_{yt}$) were planned to have about 45° of the angle of concrete strut by (2). Total length of specimens was 3,000 mm. The test zone was located at center of the specimens with length of 1,500 mm. And to avoid torsional failure in the end zone, heavier reinforcement (s=50 mm) was placed in the end zone.

B. Testing Method and Measurements

In current design codes, the torsional strength of reinforced concrete beam is determined at yielding of the torsional reinforcement. Thus to use the torsional strength of the design codes, torsional reinforcement must be yielding. To check yielding of torsional reinforcement in the test, two strain gauges ware attached to each longitudinal reinforcement at 600 mm and 900 mm from the end of the test zone. And also strain gauges were attached to upper part and the side of the transverse reinforcement. Five transverse reinforcements attaching strain gauges were located at the center of specimens and about 250 mm and 500 mm away from the center of specimens. Fig. 3 shows the location of strain gauges.

TABLE I. THE DETAILS OF THE TEST SPECIMENS

Specimens	f_c' (MPa)	A_l	ρ_l	f_{yl} (MPa)	$\rho_l f_{yl}$ (MPa)	A_t	s (mm)	ρ_t	f_{yt} (MPa)	$\rho_t f_{yt}$ (MPa)
T-C42S40	42.2	2-D13+4-D16	0.009979	317	3.16	D13	130	0.010582	340	3.60
T-C42S50	42.2	2-D13+4-D16	0.009979	469	4.68	D13	130	0.010582	480	5.08
T-C42S60	42.2	2-D13+4-D16	0.009979	659	6.58	D13	130	0.010582	667	7.06
T-C70S40	68.4	2-D13+4-D16	0.009979	317	3.16	D13	130	0.010582	340	3.60
T-C70S50	68.4	2-D13+4-D16	0.009979	469	4.68	D13	130	0.010582	480	5.08
T-C70S60	68.4	2-D13+4-D16	0.009979	659	6.58	D13	130	0.010582	667	7.06

Note: $\rho_l=A_l/A_c$; $\rho_t=(A_t p_h)/(A_c s)$; p_h=2(260+310) mm; D13: A_s=126.7 mm^2; and D16: A_s=198.6 mm^2.

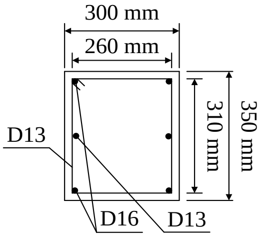


Figure 1. Section of the specimens.

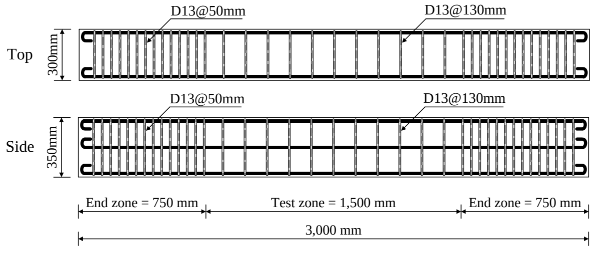


Figure 2. Configuration of the test specimens.

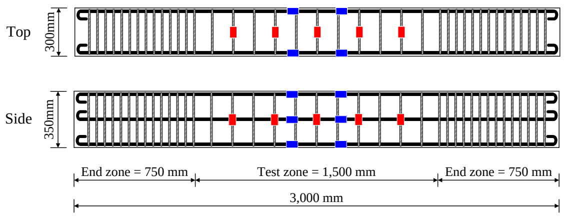


Figure 3. Location of strain gauges.

The loading condition and location of LVDTs are shown in Fig. 4. Four LVDTs were attached to both sides of the test specimen to measure the angle of twist. The torque arms were located apart by 425 mm from the end of test zone. The length of torque arm was 700 mm. The spreader beam was delivering load from Universal Testing Machine (UTM). Spreader beam and torque arms were planned to move vertical direction only. During the test, specimens were loaded monotonically by displacement control method. Loading speed of the test was 0.02 mm/sec. The test was terminated when the resisting force in the post-peak load-deformation curve dropped to about 85% of the peak-recorded strength.

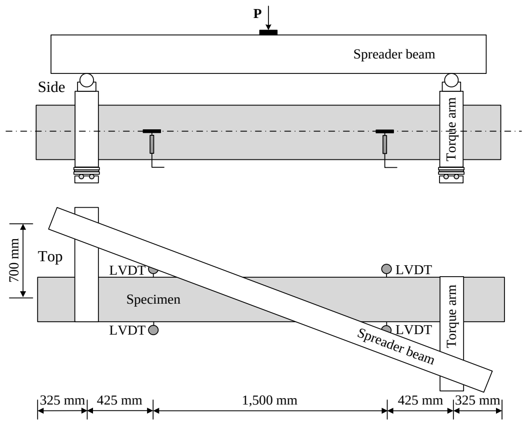


Figure 4. Loading condition and location of LVDTs.

III. Test Results

A. Torque-Twist Curves

The torque versus angle of twist curves of the specimens were shown in Fig. 5. The specimens have same spacing of closed stirrups, but the yield strengths of reinforcement are different. The specimens using higher yield strength of reinforcement expected to have higher torsional strength calculated by (1). But the torsional strengths of the test specimens are almost same from the test results. The torsional strengths are same if same amount of reinforcement are used. However the torsional strengths of C70 specimens were greater than these of C42 specimens in the test. It is because the torsional strength estimated in the ACI 318-14 design code does not consider influence of concrete. The torsional strength from the test and calculated by the ACI 318-14 design code and angle of twist are shown in Table II. Failure of specimens using high-strength torsional reinforcement was more ductile than failure of specimens using normal-strength torsional reinforcements.

TABLE II. Test Result

Specimens	T_{ACI} (kN·m)	T_{exp} (kN·m)	$\theta_{exp,max}$ (rad./m)
T-C42S40	37.43	41.34	0.0161
T-C42S50	56.70	48.03	0.0191
T-C42S60	80.11	45.96	0.0297
T-C70S40	37.43	49.11	0.0193
T-C70S50	56.70	48.03	0.0202
T-C70S60	80.11	48.98	0.0200

Note: T_{ACI}: torsional strength calculated by (1); T_{exp}: maximum torque of the test; and $\theta_{exp,max}$: angle of twist at maximum torque.

B. Failure Modes

From the strain gauges attached to longitudinal and transverse reinforcement, the strain of the torsional reinforcement was obtained. Table III shows whether the torsional reinforcement was yielding or not before reaching maximum torque of each specimen.

From Table III, failure modes of specimens were determined. If torsional reinforcement of test specimens did not yield, the failure modes of these specimens were over-reinforced failure. On the other hand, if torsional reinforcement of test specimens yields, failure modes of these specimens were under-reinforced failure. The failure modes of specimens are shown in Table IV. Except specimens T-C42S50 and T-C70S50, the failure modes from the ACI 318-14 design code are same with the failure modes from the test.

TABLE III. Yielding of Reinforcement

Specimens	Transverse reinforcement		Longitudinal reinforcement
	Side	Top	
T-C42S40	X	X	O
T-C42S50	O	X	X
T-C42S60	X	X	X
T-C70S40	O	O	O
T-C70S50	X	X	X
T-C70S60	X	X	X

Note: O: reinforcement yielded before reaching maximum torque; and X: reinforcement did not yield before reaching maximum torque.

IV. Torsional Behaior Expected by Compatibility-Aided Truss Model

In order to predict the structural behavior of tested six RC beams, a compatibility-aided truss model was used in this study.

A. Governing Equations and Flow Chart

In the RC members subjected to torsion, the shear flow q occupies a zone, called the shear flow zone. This shear flow zone has a thickness td, which is a variable determined from the equilibrium and compatibility conditions. Equilibrium and compatibility equations for torsion analysis are as following [5]:

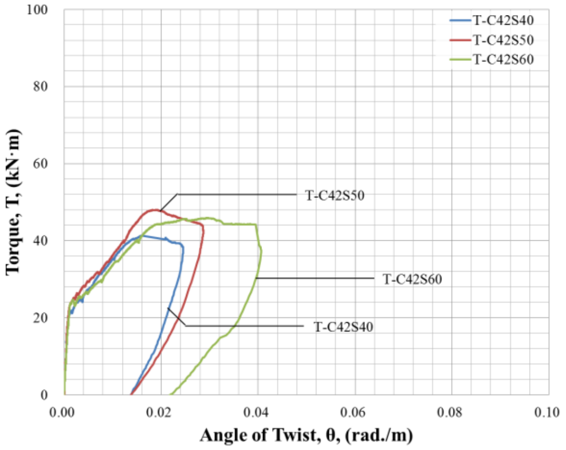
Equilibrium Equations

$$\sigma_l = \sigma_d \cos^2\alpha + \sigma_r \sin^2\alpha + \rho_l f_l \quad (5)$$

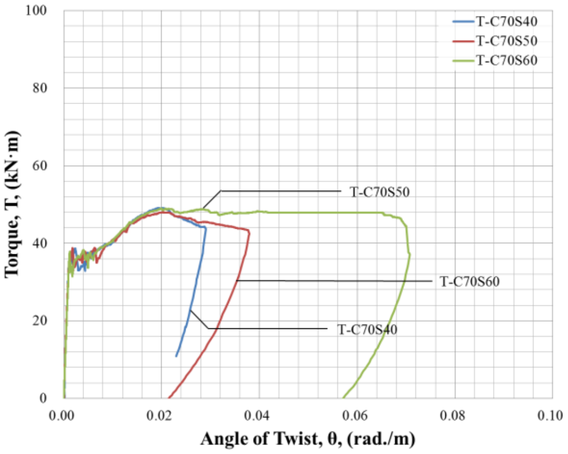
$$\sigma_t = \sigma_d \sin^2\alpha + \sigma_r \cos^2\alpha + \rho_t f_t \quad (6)$$

$$\tau_{lt} = (-\sigma_d + \sigma_r)\sin\alpha\cos\alpha \quad (7)$$

$$T = \tau_{lt}(2A_0 t_d) \quad (8)$$



(a) C42 specimens



(b) C70 specimens

Figure 5. Torque-twist curves of the specimens.

TABLE IV. FAILURE MODE OF THE SPECIMENS

Specimens	Failure mode from the ACI 318-14	Failure mode from the test
T-C42S40	UR	UR
T-C42S50	OR	UR
T-C42S60	OR	OR
T-C70S40	UR	UR
T-C70S50	UR	OR
T-C70S60	OR	OR

Note: UR: under-reinforced failure; and OR: over-reinforced failure.

Compatibility Equations

$$\varepsilon_l = \varepsilon_d \cos^2\alpha + \varepsilon_r \sin^2\alpha \quad (9)$$

$$\varepsilon_t = \varepsilon_d \sin^2\alpha + \varepsilon_r \cos^2\alpha \quad (10)$$

$$\frac{\gamma_{lt}}{2} = (-\varepsilon_d + \varepsilon_r)\sin\alpha\cos\alpha \quad (11)$$

$$\theta = \frac{p_o}{2A_o}\gamma_{lt} \quad (12)$$

$$\psi = \theta\sin 2\alpha \quad (13)$$

$$t_d = \frac{\varepsilon_{ds}}{\psi} \quad (14)$$

$$\varepsilon_d = \frac{\varepsilon_{ds}}{2} \quad (15)$$

where τ_{lt}: shear stress, γ_{lt}: shear strain, θ: angle of twist, p_o: perimeter of the centerline of the shear flow, ψ: curvature of the concrete struts, and ε_{ds}: maximum strain at the surface.

Constitutive Laws of Materials

To solve unknown value of variables, constitutive laws of concrete and steel are needed. Constitutive laws of concrete struts are (16)-(20), and Constitutive laws of steels are (21)-(24).

- *Concrete struts*

For the ratio of the average stress to the peak stress k1, (17-a) and (17-b) were used. (17-a) is for ascending branch, and (17-b) is for descending branch. For a softening coefficient ζ, Belarbi and Hsu's equations (18-a) and (18-b) were used [6]. For tensile stress-strain relationship for concrete, (19-a) and (19-b) were used [7].

$$\sigma_d = k_1\zeta f_c' \quad (16)$$

$$k_1 = \frac{\varepsilon_{ds}}{\zeta\varepsilon_0}\left(1 - \frac{1}{3}\frac{\varepsilon_{ds}}{\zeta\varepsilon_0}\right) \qquad \left(\frac{\varepsilon_{ds}}{\zeta\varepsilon_0} \le 1.0\right) \quad (17\text{-a})$$

$$k_1 = \left[1 - \frac{\zeta^2}{(2-\zeta)^2}\right]\left(1 - \frac{1}{3}\frac{\zeta\varepsilon_0}{\varepsilon_{ds}}\right) + \frac{\zeta^2}{(2-\zeta)^2}\frac{\varepsilon_{ds}}{\zeta\varepsilon_0}\left(1 - \frac{1}{3}\frac{\varepsilon_{ds}}{\zeta\varepsilon_0}\right) \qquad \left(\frac{\varepsilon_{ds}}{\zeta\varepsilon_0} > 1.0\right) \quad (17\text{-b})$$

$$\zeta = \frac{0.9}{\sqrt{1+400\varepsilon_r}} \qquad (f_c' \le 41.5\ \text{MPa}) \quad (18\text{-a})$$

$$\zeta = \frac{5.8}{\sqrt{f_c'}}\frac{1}{\sqrt{1+400\varepsilon_r}} \qquad (f_c' > 41.5\ \text{MPa}) \quad (18\text{-b})$$

$$\sigma_r = E_c\varepsilon_r \qquad (\varepsilon_r \le \varepsilon_{cr}) \quad (19\text{-a})$$

$$\sigma_r = f_{cr}\left(\frac{\varepsilon_{cr}}{\varepsilon_r}\right)^{0.4} \qquad (\varepsilon_r > \varepsilon_{cr}) \quad (19\text{-b})$$

$$f_{cr} = \frac{1}{3}\sqrt{f_c'} \quad (20)$$

where ε_0: strain at the peak stress f_c' (usually taken as 0.002), E_c: modulus of elastic of concrete, ε_{cr}: strain at cracking of concrete, and f_{cr}: stress at cracking of concrete.

- *Steels*

For stress-strain relationship of steels, the equations for steel bars embedded in concrete were used. (21)-(24) are equations for the stress-strain relationship of steel bars [8].

when $f_s \le f_y^{'}$,

$$f_s = E_s \varepsilon_s \tag{21}$$

when $f_s > f_y^{'}$,

$$f_s = (0.91 - 2B) f_y + (0.02 + 0.25B) E_s \varepsilon_s \tag{22}$$

$$f_y^{'} = (0.93 - 2B) f_y \tag{23}$$

$$B = \frac{1}{\rho}\left(\frac{f_{cr}}{f_y}\right)^{1.5} \tag{24}$$

where f_s= f_l or f_t when applied to longitudinal reinforcement or transverse reinforcement, respectively, ε_s= ε_l or ε_t when applied to longitudinal reinforcement or transverse reinforcement, respectively, and ρ= ρ_l or ρ_t when applied to longitudinal reinforcement or transverse reinforcement, respectively.

In this study, torsional analysis was performed by Hsu's flow chart for torsional analysis [9]. Fig. 6 shows the flow chart of Rotating Angle-softened Truss Model (RA-STM) for torsion by Hsu. For the initial values of the principal compressive strain ε_d, the principal tensile strain ε_r, and the thickness of shear flow zone t_d, -0.00001, 0.0001, and 0.0001 in. were chosen, respectively. And the increments of these variables were chosen -0.00001, 0.00002, and 0.0001 in., respectively.

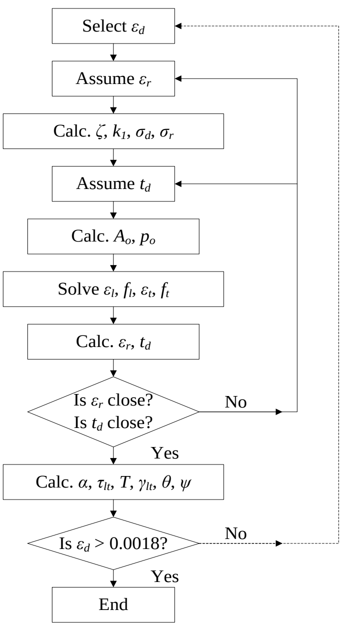


Figure 6. Flow chart of CATM for torsion.

B. Torsional Strength

The torsional strength and the angle of twist at maximum torque expected by CATM are shown in Table V. The torsional strengths expected by CATM were similar to the torsional strengths obtained by the test in case of specimens T-C42S40 and T-C42S40. However the torsional strengths expected by CATM of other specimens were overestimated in comparison with the torsional strength obtained by the test. So expecting torsional strength of RC beams using high-strength reinforcement by CATM can be overestimated.

C. Failure Modes

By checking whether torsional reinforcement was yielding before reaching maximum torque or not, failure mode can be expected by CATM. Except specimen T-C70S50, the failure modes expected by CATM were same with the failure modes obtained by the test. Unlike the failure mode by the test, the expected failure mode of specimen T-C70S50 was under-reinforced failure.

TABLE V. ANALYSIS RESULT

Specimens	T_{CATM} (kN·m)	$\theta_{CATM,max}$ (rad./m)	T_{exp} (kN·m)	$\theta_{exp,max}$ (rad./m)	T_{CATM} / T_{exp}
T-C42S40	47.53	0.0547	41.34	0.0161	1.150
T-C42S50	63.72	0.0500	48.03	0.0191	1.327
T-C42S60	64.80	0.0535	45.96	0.0297	1.410
T-C70S40	49.08	0.0682	49.11	0.0193	0.999
T-C70S50	67.29	0.0449	48.03	0.0202	1.401
T-C70S60	76.42	0.0573	48.98	0.0200	1.560

Note: T_{CATM}: torsional strength expected by CATM; $\theta_{CATM,max}$: angle of twist at maximum torque expected by CATM; T_{exp}: maximum torque from the test; and $\theta_{exp,max}$: angle of twist at maximum torque from the test.

TABLE VI. YIELDING OF REINFORCEMENT AND FAILURE MODE EXPECTED BY CATM

Specimens	Yielding of reinforcement		Failure mode expected by CATM
	Transverse reinforcement	Longitudinal reinforcement	
T-C42S40	O	O	UR
T-C42S50	O	O	UR
T-C42S60	X	X	OR
T-C70S40	O	O	UR
T-C70S50	O	O	UR
T-C70S60	X	X	OR
VT-C90S60	X	X	OR
VT-C100S60	X	X	OR
VT-C110S60	O	X	UR

Note: O: reinforcement yielded before reaching maximum torque; X: reinforcement did not yield before reaching maximum torque; UR: under-reinforced failure; and OR: over-reinforced failure.

In the other literatures, the torsional failure modes expected by CATM tended to match with the torsional failure modes by the tests. Thus torsional analysis by CATM on three more virtual specimens was performed. The virtual specimens VT-C90S60, VT-C100S60, and VT-C110S60 have same yield strength of steels (f_{yl}=659 MPa and f_{yt}=667 MPa) and three different compressive strength of concrete (f_c'=90, 100, and 110 MPa, respectively). The torque versus angle of twist curve of torsional analysis of six test specimens and three virtual specimens are shown in Fig. 7. In case of virtual specimens VT-C90S60 and VT-C100S60, torsional reinforcements were not yielding before reaching maximum torque. But in case of virtual specimen VT-C110S60, longitudinal torsional reinforcement was yielding before reaching maximum torque, and transverse torsional reinforcement was yield after reaching maximum torque. The reason why transverse reinforcement was not yielding before reaching maximum torque is the amount of longitudinal reinforcement ($\rho_l f_{yl}$) was less than that of transverse reinforcement ($\rho_t f_{yt}$). Table VI shows the failure modes of the test specimens expected by CATM.

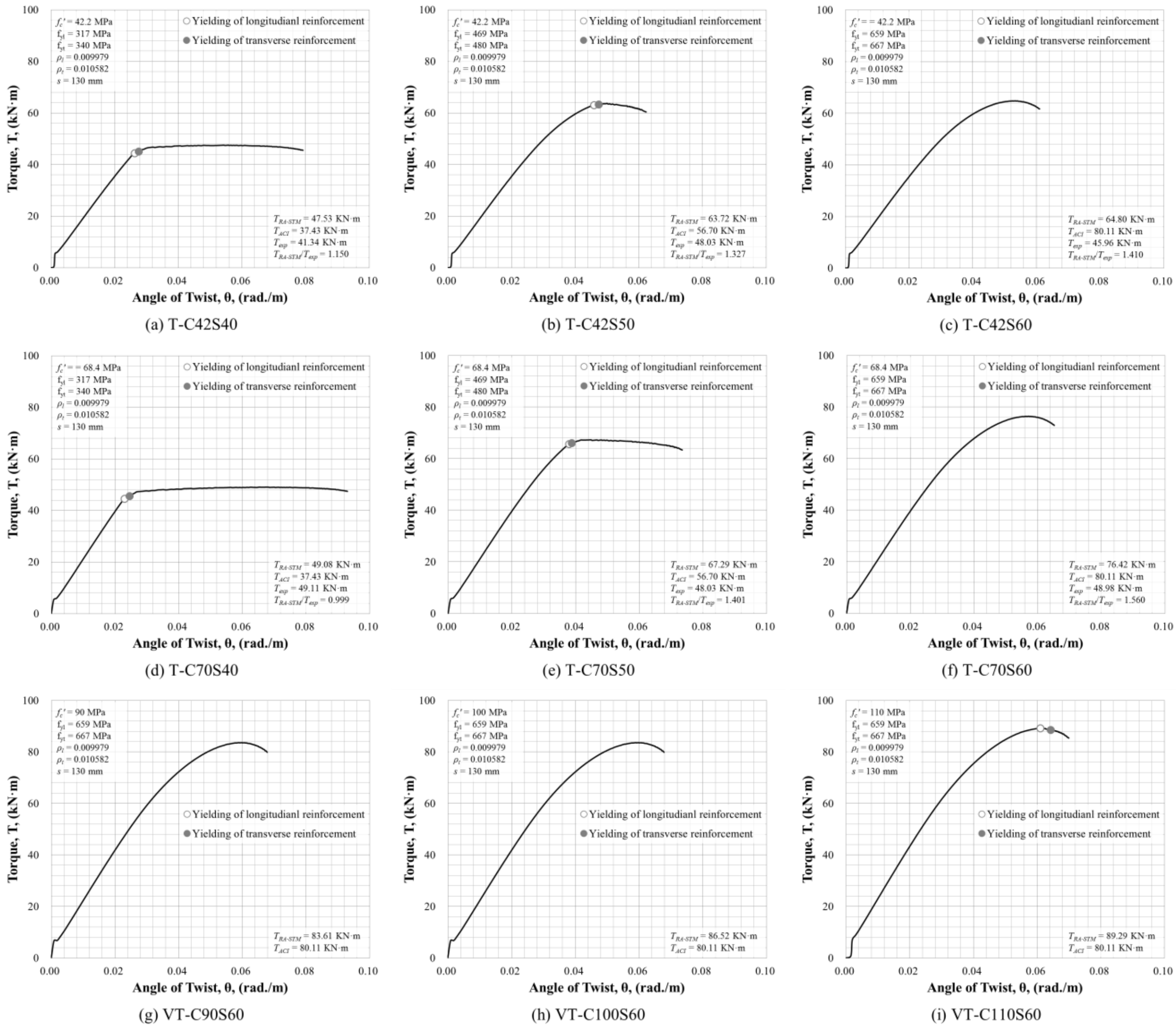


Figure 7. Torque-twist curves expected by CATM.

TABLE VII. TORSIONAL BEHAVIOR OF REINFORCED CONCRETE SOLID BEAMS IN THE LITERATURES

Specimens	Torsional strength					Failure mode		
	T_{ACI} (kN m)	T_{CATM} (kN m)	T_{exp} (kN m)	T_{ACI}/ T_{exp}	T_{CATM} / T_{exp}	ACI 318-14	CATM	Test
T-C42S40[a]	37.43	47.53	41.34	0.906	1.150	UR	UR	UR
T-C42S50[a]	56.70	63.72	48.03	1.180	1.327	OR	UR	UR
T-C42S60[a]	80.11	64.80	45.96	1.743	1.410	OR	OR	OR
T-C70S40[a]	37.43	49.08	49.11	0.762	0.999	UR	UR	UR
T-C70S50[a]	56.70	67.29	48.03	1.180	1.401	UR	UR	OR
T-C70S60[a]	80.11	76.42	48.98	1.636	1.560	OR	OR	OR
T1-1[b]	23.67	31.10	32.86	0.720	0.946	UR	UR	UR

T1-2[b]	35.16	44.88	45.89	0.766	0.978	UR	UR	UR
T1-3[b]	47.34	56.68	54.05	0.876	1.049	UR	UR	UR
T1-4[b]	68.87	72.27	62.41	1.103	1.158	OR	UR	UR
T2-1[b]	20.03	24.23	26.05	0.769	0.930	UR	UR	UR
T2-2[b]	36.80	41.26	38.11	0.966	1.083	UR	UR	UR
T2-3[b]	54.78	54.22	50.16	1.092	1.081	OR	UR	UR
T2-4[b]	64.45	59.67	56.39	1.143	1.058	OR	UR	UR
C24SD30-mid[c]	40.10	50.80	34.91	1.149	1.455	OR	UR	UR
C24SD30-EC[c]	60.53	61.05	40.60	1.491	1.504	OR	OR	OR
C24G60-EC[c]	48.65	55.32	36.64	1.328	1.510	OR	OR	OR
C42G60-ACI[c]	38.04	60.01	39.70	0.958	1.512	UR	UR	UR
H-06-06[d]	76.00	88.41	92.00	0.826	0.961	UR	UR	UR
H-06-12[d]	95.69	108.03	115.10	0.831	0.939	UR	UR	UR
H-12-12[d]	119.50	148.53	155.30	0.770	0.956	UR	UR	UR
H-12-16[d]	181.17	172.45	196.00	0.924	0.880	OR	UR	UR
H-20-20[d]	261.77	210.98	239.00	1.095	0.883	OR	OR	OR
H-07-10[d]	100.21	111.90	126.70	0.791	0.883	UR	UR	UR
H-14-10[d]	131.18	143.14	135.20	0.970	1.059	UR	UR	UR
H-07-16[d]	129.37	123.68	144.50	0.895	0.856	UR	UR	UR
N-06-06[d]	76.00	84.04	79.70	0.954	1.054	UR	UR	UR
N-06-12[d]	95.69	97.93	95.20	1.005	1.029	UR	UR	UR
N-12-12[d]	119.50	128.46	116.80	1.023	1.100	OR	OR	UR
N-12-16[d]	162.82	134.87	138.00	1.180	0.977	OR	OR	UR
N-20-20[d]	261.77	148.04	158.00	1.657	0.937	OR	OR	OR
N-07-10[d]	100.21	96.31	111.70	0.897	0.862	OR	UR	UR
N-14-10[d]	131.18	121.05	125.00	1.049	0.968	OR	UR	UR
N-07-16[d]	129.37	102.86	117.30	1.103	0.877	OR	UR	UR
B5UR1[e]	16.31	20.66	19.40	0.841	1.065	UR	UR	UR
B7UR1[e]	15.77	22.03	18.90	0.834	1.166	UR	UR	UR
B9UR1[e]	16.31	21.55	21.10	0.773	1.021	UR	UR	UR
B12UR1[e]	15.77	22.27	19.40	0.813	1.148	UR	UR	UR
B12UR2[e]	15.58	22.45	18.40	0.847	1.220	UR	UR	UR
B12UR3[e]	20.95	25.52	22.50	0.931	1.134	UR	UR	UR
B12UR4[e]	23.66	27.98	23.70	0.998	1.181	UR	UR	OR
B12UR5[e]	24.74	32.09	24.00	1.031	1.337	UR	UR	OR
Mean.				1.019	1.110			
C.V.				0.241	0.181			

[a]this paper [4].
[b]J.-Y. Lee and S.-W. Kim 2010 [3].
[c]S.-C. Lee 2013 [10].
[d]I-K. Fang and J.-K. Shiau 2004 [11].
[e]N.E. Koutchoukali and A. Belarbi 2001 [12].

D. *Torsional Analysis of Specimens in the Literatures*

To check accuracy and precision of torsional analysis by CATM, torsional behavior of forty-two RC solid beams [3], [4], [10]-[12] subjected to pure torsion including these test specimens was analyzed by CATM. Table VII shows torsional behavior by the ACI 318-14 design code, CATM, and the test. The torsional strengths and failure modes expected by CATM showed good agreement with the test results of RC beams with normal strength torsional reinforcement. However, in case of RC beams with high strength torsional reinforcement, such as S50 specimens (f_{yl}=469 MPa and f_{yt}=480 MPa) and S60 specimens (f_{yl}=659 MPa and f_{yt}=667 MPa), the torsional strengths expected by CATM overestimated the observed ones up to 42.5 %.

V. CONCLUSION

This study investigated the torsional behavior of RC beams with normal- and high-strength torsional reinforcement. The observed test results were compared with the torsional behavior predicted by a CATM. The results obtained from the experimental and analytical study are as follows:

1) Test results indicated that the torsional strength of RC beams did not increase with the increase of yield strength of torsional reinforcement.

2) The beams with high strength torsional reinforcement failed due to concrete crushing before yielding of reinforcement.
3) Test results indicated that the limitation on the yield strength of torsional reinforcement in the ACI 318-14 design code was appropriate, while the limitation in the Eurocode 2 was too high.
4) The CATM predicted the torsional strength and failure mode of RC beams with normal strength torsional reinforcement with good agreement, while over-estimated the torsional strength of RC beams with high strength torsional reinforcement up to 42.5%.

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REFERENCES

[1] ACI Committee 318, *Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary*, Farmington Hills, MI: American Concrete Institute, 2014, p. 520.
[2] Comité Européen de Normalisation (CEN), Eurocode 2: Design of Concrete Structures - Part 1-1: General Rules and Rules for Buildings, EN 1992-1-1, 2004, p. 227.
[3] J. Y. Lee and S. W. Kim, "Torsional strength of RC beams considering tension stiffening effect," *Journal of Structural Engineering*, vol. 136, no. 11, pp. 1367-1378, Nov. 2010.
[4] S. K. Yoon, "Failure modes of reinforced concrete beams with high-strength steel bars subjected to pure torsion," M.S. thesis, Dept. of Civil, Architectural and Environmental System Engineering, Sungkyunkwan Univ., Suwon, Republic of Korea, 2012.
[5] T. T. C. Hsu, *Unified theory of Reinforced Concrete*, Boca Raton, FL: CRC Press, 1993, p. 313.
[6] A. Belarbi and T. T. C. Hsu, "Constitutive laws of softened concrete in biaxial tension-compression," *ACI Structural Journal*, vol. 92, no. 5, pp. 562-573, Sep. 1995.
[7] A. Belarbi and T. T. C. Hsu, "Constitutive laws of concrete in tension and reinforcing bars stiffened by concrete," *ACI Structural Journal*, vol. 91, no. 4, pp. 465-474, July 1994.
[8] S. Tamai, H. Shima, J. Izumo, and H. Okamura, "Average stress-strain relationship in post yield range of steel bar in concrete," *Concrete Library of JSCE*, no. 11, pp. 117-129, June 1988.
[9] T. T. C. Hsu, "Softened truss model theory for shear and torsion," *ACI Structural Journal*, vol. 85, no. 6, pp. 624-635, Nov. 1988.
[10] S. C. Lee, "Torsional failure modes of RC beams with high-strength reinforcement," M.S. thesis, Dept. of Mega Building and Bridges, Sungkyunkwan Univ., Suwon, Republic of Korea, 2013.
[11] I. K. Fang and J. K. Shiau, "Torsional behavior of normal- and high-strength concrete beams," *ACI Structural Journal*, vol. 101, no. 3, pp. 304-313, May-June 2004.
[12] N. E. Koutchoukali and A. Belarbi, "Torsion of high-strength reinforced concrete beams and minimum reinforcement requirement," *ACI Structural Journal*, vol. 98, no. 4, pp. 462-469, July 2004.

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