Development and Optimization of Artificial Intelligence-Based Concrete Compressive Strength Predictive Models

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***Abstract*—Accurate prediction of the compressive strength of High-Performance Concrete (HPC) is crucial in concrete design and construction. However, HPC is a very complex material, as the inter-relationship between its constituent materials is highly nonlinear and its property is affected by several interacting factors. Hence, existing conventional empirical and statistical methods are limited in their ability to accurately predict the compressive strength of HPC. In this study, the application of three artificial intelligence techniques, namely, the Artificial Neural Network (ANN), Fuzzy Inference System (FIS), and Adaptive Neuro-Fuzzy Inference System (ANFIS) techniques, are explored. A data-driven approach based on fuzzy c-means clustering (FCM) is employed to generate both the Mamdani and Sugeno FIS models. Different model structures and parameters—such as number of neurons and choice of transfer function for the ANN technique, and number of clusters and choice of fuzzification coefficient and inference methods for the FIS and ANFIS techniques—are optimized to improve the accuracy of each technique. Results of this study indicate that ANFIS and ANN perform better than the FIS models in predicting the compressive strength of HPC. The main contributions of this paper are: (1) providing accurate concrete compressive strength prediction models that represent the complex, nonlinear relationship between the constituent materials and concrete compressive strength; (2) presenting a data-driven methodology for the development of FIS concrete compressive strength models; and (3) subjecting artificial intelligence-based concrete compressive strength models to structure and parameter optimization to improve prediction accuracy.**

***Index Terms*—high-performance concrete, compressive strength, artificial neural network, fuzzy inference system, adaptive neuro-fuzzy inference system**

I. Introduction

Concrete is the most versatile and widely used construction material. The reasons for concrete's dominance are varied, but among the most important are: the economy and widespread availability of its constituent materials; its ability to be molded into any desired shape; its adoptability and sustainability; and its high compressive strength, stiffness, and durability [1], [2]. Concrete is categorized according to purpose, range of compositions, finishes, and performance characteristics. Lightweight, heavyweight, high-strength, high-performance, self-compacting, and fiber-reinforced are among the most widely available concrete types.

According to the American Concrete Institute (ACI), High-Performance Concrete (HPC) is that "meeting [a] special combination of performance and uniformity requirements that cannot always be achieved routinely using conventional constituents and normal mixing, placing, and curing practices" [3]. Most prevailing definitions for HPC emphasize properties such as high strength, high workability, dimensional stability, and durability [4]. In addition to common concrete ingredients (aggregates, sand, and cement), supplementary cementitious materials—principally, fly ash and blast furnace slag—and chemical admixtures, such as superplasticizer, are used in preparation of HPC to improve performance and economic return [2], [4].

Compressive strength is the most important mechanical property of concrete, since it is primarily used as quality control and compliance criteria in standards and specifications. Moreover, most of the important properties of concrete, including flexural strength, direct tensile strength, splitting tensile strength, and modulus of elasticity, are directly related to compressive strength [5]. Thus, proper prediction of concrete compressive strength is vital to schedule and manage concrete works such as formwork removal and pre- or post-tensioning activities [6]. In the past, several techniques based on either empirical methods (statistical evaluation of relationships) or computational modeling have been tested, and empirical methods based on Multi-Linear Regression (MLR) have been commonly proposed to predict compressive strength. However, most of the available empirical models do not account for the mineral and chemical admixtures used in HPC [7], [8]. Moreover, the numbers of interacting factors influencing the compressive strength of HPC are very high and the relationship between these factors is not precisely known because it is considered to be highly complex and nonlinear [6], [8]. Therefore, the empirical methods are limited in their ability to accurately predict the compressive strength of HPC. Alternative modeling methods employing Artificial Intelligence (AI) techniques, such as the Artificial Neural Network (ANN), Fuzzy

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Inference System (FIS), and Adaptive Neuro-Fuzzy System (ANFIS) techniques, provide a flexible environment better suited to dealing with such a complex, nonlinear relationship.

In the past two decades, AI-based modeling methods have been extensively used in wide-ranging civil engineering applications including modeling of material behavior, determination of concrete mix proportion, and prediction of strength [9], [10]. Yeh [4] applied ANN to predict the compressive strength of HPC, and found that these predictions are more accurate than those obtained from an MLR model. Similarly, Ozturan, Kutlu, and Ozturan [11] compared the prediction accuracies of ANN- and MLR-based models, and concluded that using ANN provides the best result. Topcu and Saridemir [9] developed ANN and FIS models to predict the compressive strength of concrete containing fly ash at different strength-gain ages. Similarly, Ozcan, Atis, Karahan, Uncluoglu, and Tanyildizi [12], Topcu and Saridemir [13], and Aggrawal and Aggrawal [14] adopted ANN and FIS to predict the compressive strength of silica fume concrete, recycled aggregate concretes containing silica fume, and self-compacting concrete, respectively. Overall, their findings affirmed that ANNs and FIS models have very promising potential to accurately predict the compressive strength of concrete. Vakshouri and Najadi [5] applied different optimization methods and membership functions in an ANFIS to predict the compressive strength of High-Strength Concrete (HSC) based on splitting tensile strength and modulus of elasticity. Badde, Gupta, and Patki [15] used FIS- and ANFIS-based models to predict the 28-day compressive strength of Ready-Mixed Concrete (RMC) and concluded that for this application, the ANFIS approach has a better predictive capability than the FIS. Aydin, Tortum, and Yavuz [16] employed an ANFIS to predict the elastic modulus of normal- and high-strength concrete based on compressive strength and compared the results with the values obtained from codes.

The accuracy of ANN models largely depends on the architecture, function, and parametric properties of the network. However, the effect of these properties on model accuracy was not thoroughly investigated in most of the aforementioned research. In addition, most of the FIS and ANFIS models developed for prediction of compressive strength heavily rely on expert knowledge to establish the fuzzy inference rules, rather than using a data-driven approach. Expert-based models have a critical shortcoming: their rules are highly prescriptive, very general, and difficult to develop due to the high-dimensionality of the problem [17]. However, a FIS's inability to learn from data and develop and optimize model parameters is a major limitation. Thus, hybridizing a model by combining the FIS technique with other AI techniques, such ANNs, could improve learning capabilities; however, this approach has rarely been used to predict compressive strength of HPC.

The major objective of this paper is to develop and compare three artificial intelligence models based on ANN, FIS approach using fuzzy C-means clustering (FCM), and ANFIS approach to predict the compressive strength of HPC. The different model parameters, such as number of neurons and type of transfer function in the case of ANN, and number of clusters, fuzzification coefficient (*m*-value), and selection of inference methods for FIS and ANFIS will be optimized to improve model accuracy and to overcome challenges associated with each approach.

Section II of this paper gives an overview of AI modeling techniques and introduces the structure and components of each model. In Section III, the data set used to develop the AI models is explained and the details of each model's implementation, including model structure and parameters, are demonstrated. The results obtained by adopting these models are presented and their performances in prediction accuracy are compared and contrasted. Finally, conclusions and recommendations are presented in Section IV.

II. Artificial Intelligence Modeling Techniques

In the following sections the structure and components of the three Artificial Intelligence (AI) modeling techniques used in this study, namely, Artificial Neural Networks (ANNs), Fuzzy Inference System (FIS), and Adaptive Neuro-Fuzzy System (ANFIS) are briefly discussed.

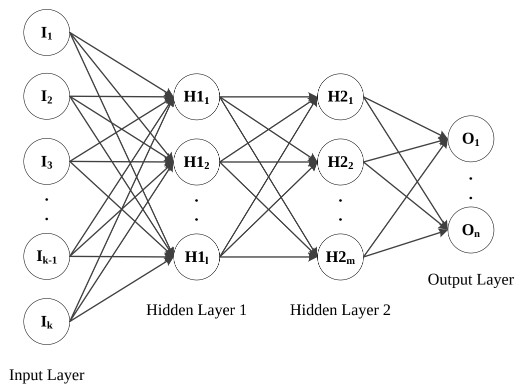


Figure 1. Typical architecture of an ANN with two hidden layers.

A. Artificial Neural Networks

ANNs are information-processing systems whose architecture imitates the learning capability of the human brain [8], [9]. Neurons are the fundamental building blocks of ANNs and they are logically arranged into a single or multiple layers. Fig. 1 shows the architecture of a two-hidden-layer network with *k* inputs and *n* outputs. The neurons in each layer are linked to all neurons in the next layer through weighted connections. The output of each neuron in the initial layer is communicated to the neurons in the next layer through an activation function [11], [14]. According to Boussabaine [18], even though there are a range of ANN types differing in architecture and mode of operation, ANNs generally include the following components: (1) a set of processing neurons, (2) a state of activation for each neuron, (3) a pattern of connectivity among the neurons, (4) a propagation method, (5) an activation rule, (6) an external environment, and (7) a learning method.

Most practical applications of ANNs are based on multi-layer feedforward architecture comprised of an input layer, one or more hidden layers, and an output layer (Fig. 1) with a back-propagation learning algorithm that adopts a gradient-descent method to minimize the margin of error between the neurons of the desired target and those of the outputs [4], [14]. References [14] and [19] discuss ANN theory and the mathematical formulation of the back-propagation algorithm in detail. ANNs provide learning capability, robustness, generalization, parallel processing, and non-linearity, making them advantageously able to accurately model the mechanical behavior of concrete [9], [10].

B. Fuzzy Inference System

Fuzzy sets were first introduced by Lotfi Zadeh in 1965 to deal with uncertainty and imprecision, which are commonly encountered in real world applications [12], [17]. The underlying notion in fuzzy sets is that an object belongs to different classes/subsets of the universal set with unsharp boundaries in which membership is a matter of degree of belongingness. This is unlike set theory, which deals with only two possibilities, i.e., 0 (non-membership) or 1 (full-membership). The partial belongingness to a set is easily described numerically by using a Membership Function (MF), which assumes values between 0 and 1, inclusively [20]. FISs are models composed of conditional if-then rules, where a collection of fuzzy sets represented by MFs provides a system for reasoning about a certain problem. In the case of Mamdani FIS models, the conclusion is represented as a fuzzy set, and defuzzification is employed to obtain a crisp output value. In Sugeno FIS models, the conclusion is represented using a function [17]. Interpretability, ability to represent complex relations, and capability to deal with both subjective and objective variables are some of the most important advantages of FISs. Fuzzy rules are capable of capturing all possible relationships between input and output variables, and are useful to construct models of complex systems using domain knowledge, experience, and experimental data [17]. A typical FIS architecture has five basic components: the input interface; the rule base, which contains the fuzzy if-then rules; the database, which defines the MFs used in the fuzzy rules; fuzzy inference, which performs the inference procedure based on the rules; and the output interface. Multiple-input single-output fuzzy rules generally assume the following form: If input_1 is A_i and input_2 is B_j …and input_n is C_t then output is D_u, where A_i, B_j,…, C_t, and D_u are fuzzy sets defined in the corresponding input and output spaces, respectively.

According to Pedrycz and Gomide [17], expert-based and data-driven are the two fundamental approaches available for constructing FIS models. In expert-based FISs, the rules are formulated by experienced experts fluent in the basic concepts and variables associated with a problem under investigation. Expert-based FIS models have access to readily available and easily quantified knowledge, facilitate easy addition and modification of rules, and are easily communicated and interpreted, as natural language is used as descriptors for variables [17]. However, there are notable shortcomings associated with this approach: the rules are highly prescriptive and very general; it is difficult to establish rules when dealing with high-dimensionality problems, as for a problem of n input variables with p linguistic values, the complete rule base will require $N = p^n$ rules; and it is challenging to ensure completeness and consistency of the rules when the number of rules increases [21].

The data-driven approach captures the main structure and relationship existing in the data by automatically transferring the numeric data into fuzzy sets, which contribute to the construction of the rule-based system [22]. This can be achieved by employing different clustering techniques, such as k-means clustering, subtractive clustering, and fuzzy c-means clustering (FCM). In this study, the FCM technique is adopted to generate the fuzzy if-then rules. In contrast to the expert-based approach, the data-driven approach results in a reduced number of rules, enables the use of existing data to establish the rules, and is suitable for high-dimensionality problems. However, loss of information and semantics, and marginalization of the role of the output are some of the drawbacks of data-driven approach [17], [22].

FCM is one of the most frequently used fuzzy clustering algorithms. In this technique, the data set is partitioned into the required number of clusters and each data point belongs to the clusters to some degree, as specified by the membership grade; thus, the degree of membership decreases when a data point is further from the cluster center, and vice versa. The main components of a FCM algorithm are: number of clusters (c), objective function (Q), distance function, fuzzification coefficient (m), and termination criteria [17]. For n-dimensional data set $\{\boldsymbol{x}_k\}, k = 1,2,\dots,n$ FCM develops n-dimensional prototypes or cluster centers $\boldsymbol{v}_i (i = 1, 2, \dots, c)$ by minimizing the objective function Q, defined as:

$$Q = \sum_{i=1}^{c} \sum_{k=1}^{n} u_{ik}^m d_{ik}^2 \quad (1)$$

subject to

$$0 < \sum_{k=1}^{n} u_{ik} < n \text{ and } \sum_{i-1}^{c} u_{ik} = 1 \quad (2)$$

where u_{ik} is the membership degree of data $\boldsymbol{x}_k$ in the i^{th} cluster, m is a fuzzification coefficient, and d_{ik} denotes the Euclidean distance from data $\boldsymbol{x}_k$ to cluster center $\boldsymbol{v}_i$. In FCM clustering, the values of u_{ik} and $\boldsymbol{v}_i$ are iteratively updated using equations 3 and 4, respectively, until the termination criteria is met. The information obtained from FCM clustering can be directly used to generate a FIS (i.e., either a Mamdani or Sugeno FIS model) that best represents the underlying relationship of the data set.

$$\boldsymbol{v}_i = \frac{\sum_{j=1}^{n} u_{ik}^m x_k}{\sum_{k=1}^{n} u_{ik}^m} \quad (3)$$

$$u_{ik} = \frac{1}{\sum_{j=1}^{c} \left(\frac{d_{ik}}{d_{jk}}\right)^{2/(m-1)}} \quad (4)$$

C. Adaptive Neuro-Fuzzy Inference System

The ANFIS was first proposed by Jang [23] for modeling highly nonlinear functions. An ANFIS is a hybrid and advanced FIS system that combines the linguistic interpretability and fuzzy reasoning of FIS and learning capability of ANN to map inputs into an output [5], [24]. An ANFIS is a FIS implemented in the framework of adaptive networks. According to Amani and Moeini [25], adaptive networks are "multi-layered feedforward structures whose overall output behavior is determined using the value of a collection of modifiable parameters". The main feature of ANFIS is its ability to tune the modifiable parameters of membership functions in the antecedent and consequent through the learning process so that the system output better matches the training data. Fig. 2 depicts the architecture of a typical ANFIS with two inputs, each with two membership functions, two rules, and one output.

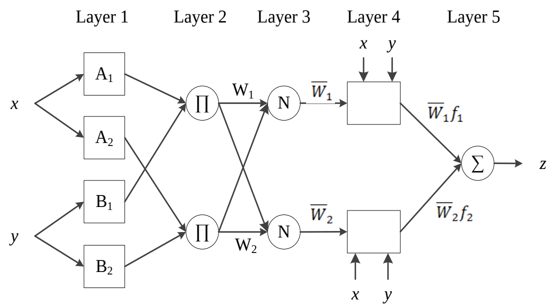


Figure 2. ANFIS structure with two inputs and two rules.

The ANFIS architecture shown in Fig. 2 comprises five different layers. The nodes in Layer 1 generate the membership functions of the inputs. The nodes in Layer 2 perform as a simple multiplier and determine the firing strength of each rule, whereas the nodes in Layer 3 normalize the firing strengths. The nodes in Layer 4 compute the consequent parameters by taking the product of the normalized firing strength and a first order polynomial (in case of a first-order Sugeno FIS model). Finally, the overall output of the ANFIS is computed using the single node in Layer 5 by taking the summation of all outputs of Layer 4 [26], [27]. Each node in Layer 1 and Layer 4 is adaptive, while the nodes in the rest of the layers are all fixed. For a first-order Sugeno FIS model, the two fuzzy rules can be expressed as:

Rule 1: If x is A_1 and y is B_1, then $f_1 = p_1x + q_1y + r_1$

Rule 2: If x is A_2 and y is B_2, then $f_2 = p_2x + q_2y + r_2$

where, A_1 , A_2 and B_1 and B_2 are the membership functions for input x and y, respectively; p_1, q_1, r_1 and p_2, q_2 , r_2 are the parameters of the rule 1 and rule 2 consequents, respectively [23].

III. Model Implementation and Results

For the purpose of developing and optimizing the AI-based concrete compressive strength predictive models, published data in HPC studies was reviewed. The most complete data set for HPC were provided by references [4] and [28]. The data set has 425 samples of the 28-day compressive strength of HPCs. HPC 28-day compressive strength is a function of seven input variables, namely, cement, fly ash, Blast Furnace Slag (BFS), water, superplasticizer, coarse aggregate, and fine aggregate. Details of the input variables and descriptive statistics of the data set are presented in Table I. For model development and optimization, the data set was randomly divided into two: 70% of the data (300 records) were considered part of the training data set, and the remaining 30% (125 records) were used to verify the accuracy of the trained models.

TABLE I. Descriptive Statistics of the Data Set

Attributes	Minimum	Maximum	Mean	Standard Deviation
Cement (kg/m^3)	102.00	540.00	265.44	104.67
BFS (kg/m^3)	0.00	359.40	86.28	87.83
Fly ash (kg/m^3)	0.00	200.10	62.79	66.23
Water (kg/m^3)	121.75	247.00	183.06	19.33
Superplasticizer (kg/m^3)	0.00	32.20	6.99	5.39
Coarse Aggregate (kg/m^3)	801.00	1,145.00	956.06	83.80
Fine Aggregate (kg/m^3)	594.00	992.60	764.38	73.12
Compressive Strength (MPa)	8.54	81.75	36.75	14.71

In line with the major objective of this paper, three AI models based on ANN, FIS approach using fuzzy C-means clustering (FCM), and ANFIS models are developed and optimized so as to come up with a model that can accurately predict the compressive strength of high performance concrete. In the following sections, model implementation and results using the three AI techniques are presented.

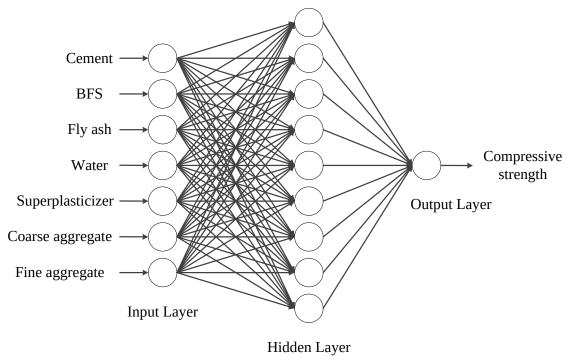


Figure 3. Architecture of an ANN for compressive strength prediction.

A. ANN for Modeling Concrete Compressive Strength

1) ANN architecture and parameters

The predictive accuracy and generalization capability of ANNs are mainly affected by the selected architecture, and its associated network parameters. In this study, ANN models were developed using MATLAB NN Toolbox™ and different structures were examined by varying network parameters such as number of neurons in the hidden layer (*n*) and transfer functions in both the hidden and output layers. The ANN models have seven input or independent variables (cement, fly ash, BFS, water, superplasticizer, coarse aggregate, and fine aggregate) in the input layer and one output or dependent variable (compressive strength) in the output layer (Fig. 3).

In developing the ANN models, a multi-layer feedforward back-propagation network with a single hidden layer was selected for its ability to approximate any function provided that sufficient neurons are used in the hidden layer [9], [12]. Ozturan, Kutlu, and Ozturan [11] summarized the different empirical criteria (as a function of the number of input and output variables) proposed by researchers to determine the number of neurons in the hidden layer.

However, in this study, the effect of number of neurons (*n*) on the performance of the networks is investigated by sequentially increasing the number of neurons (from 2 to 25 neurons). Basically, any type of differentiable transfer functions can be employed by neurons to generate their output. The effect of using the commonly employed transfer functions such as Log-Sigmoid (LOGSIG), Tan-Sigmoid (TANSIG), and Linear (PURELIN) in the hidden and output layer is examined by considering different combinations.

TABLE II. SUMMARY OF NETWORK PARAMETERS

No.	Properties	Types/Values
1	Architecture properties	
1.1	Network type	Multilayer feed forward back propagation
1.2	Number of inputs	7
1.3	Number of network outputs	1
1.4	Number of hidden layers	1
2.	Function properties	
2.1	Network adaption function	Gradient descent method (LEARNGDM)
2.2	Network initialization function	Randomized
2.3	Network performance function	Mean square error
2.4	Network training function	Levenberg-Marquardt (LM)
3	Parameter properties	
3.1	LM training parameters ▪ Minimum gradient ▪ Validation checks ▪ Maximum training epochs ▪ Performance goal ▪ Initial mu ▪ mu decrease factor ▪ mu increase factor ▪ Maximum mu	 $1x10^{-5}$ 6 1000 0 0.001 0.1 10 $1x10^{10}$

The data set (300 records) used to train the ANNs was randomly divided into three subsets, namely, training, validation, and test sets with a ratio of 0.7, 0.15 and 0.15, respectively. The Levenberg-Marquardt (LM) training algorithm, which uses a gradient descent method with momentum weight, was selected as training function, since it is the fastest and performs better on function fitting [29]. The performances of the networks during training were assessed based on the mean square error (MSE) performance function and the trainings are terminated using minimum gradient magnitude and validation checks (based on the number of successive iterations that the validation performance fails to decrease). The network parameters and values considered common for all the networks are summarized in Table II.

The results of ANN models are presented and discussed in the following subsection.

2) Results of ANN models

Once the training of the ANNs was completed, the validation data set (125 records) was introduced to the networks to evaluate their predictive accuracy using the following error measures: Mean Absolute Error (MAE), Mean Square Error (MSE), root mean square error (RMSE), and coefficient of determination (R^2). These are computed based on the predicted (compressive strength predicted by the networks) and actual values. The error measures used to compare the performance of the ANN models are defined as follows:

$$MAE = \frac{1}{N}\sum_{i=1}^{N}|t_i - y_i| \qquad (5)$$

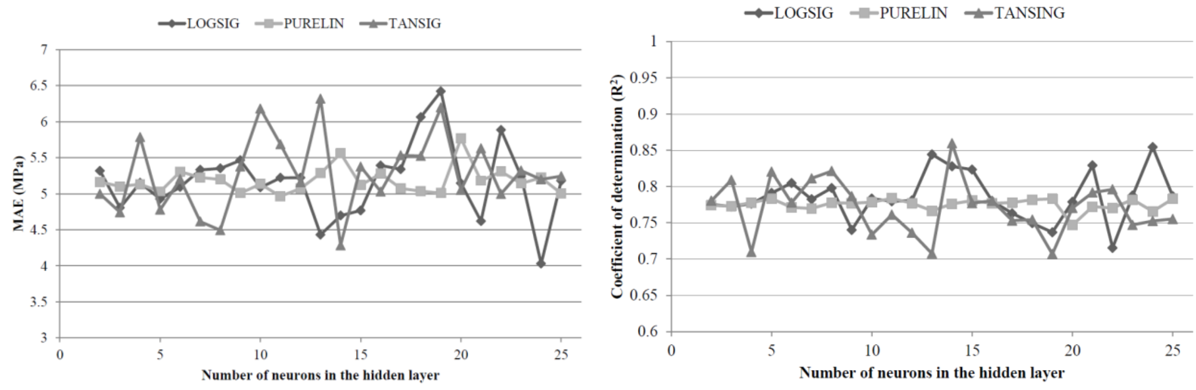
$$MSE = \frac{\sum_{i=1}^{N}(t_i - y_i)^2}{N} \qquad (6)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(t_i - y_i)^2}{N}} \qquad (7)$$

$$R^2 = 1 - \frac{\sum_{i=1}^{N}(t_i - y_i)^2}{\sum_{i=1}^{N}(t_i - \overline{t_i})^2} \qquad (8)$$

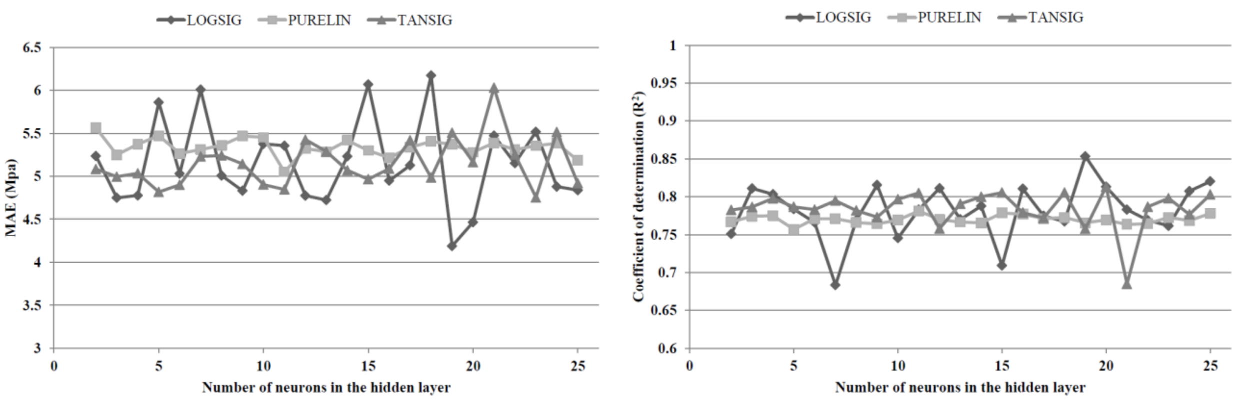
where t_i and y_i are the i^{th} actual and predicted compressive strengths, respectively; $\overline{t_i}$ is the average of actual compressive strength; and N is the total number of validation data instances. A value of $R^2 = 1$ indicates an exact linear relationship between the predicted and actual values. Thus, the network with minimum error values and maximum R^2 can be selected as the optimum network for modeling the compressive strength.

A total of 216 ANN models were developed by varying the number of neurons in the hidden layer (*n*) and the type of transfer function in the hidden and output layers. Since the predictive capability of ANN models developed using Log-Sigmoid transfer function on the output layer were extremely poor (R^2 values ranging between 2.04×10^{-31} and 0.53), only the MAE and R^2 values of ANN models with Linear and Tan-Sigmoid transfer functions on the output layer are shown in figures 4a and 4b, and Fig. 5a and 5b, respectively. As can be seen from these figures, ANN models developed using the LOGSIG and TANSIG transfer functions show better predictive performance based on MAE and R^2 compared to models that use the PURELIN transfer function in the hidden layer. Moreover, the improvement in model accuracy performance due to the increase of the number of neurons in the hidden layer—where PURELIN was used for transfer function in the hidden layer—is not that significant (the maximum percentage increment attained in R^2 was only 4%).



(a) (b)

Figure 4. (a) MAE and (b) R values of ANN models with linear transfer function on the output layer.



(a) (b)

Figure 5. (a) MAE and (b) R^2 values of ANN models with Tan-Sigmoid transfer function on the output layer.

TABLE III. MODEL PERFORMANCE RESULTS OF BEST PERFORMING ANN MODELS

Rank	No. of neurons in the hidden layer (n)	Type of transfer function		Error measures			R^2
		Hidden layer	Output layer	MAE	MSE	RMSE	
1	24	LOGSIG	PURELIN	4.03	30.41	5.51	0.86
2	14	TANSIG	PURELIN	4.18	30.65	5.54	0.85
3	19	LOGSIG	TANSIG	4.19	30.70	5.56	0.85
4	13	LOGSIG	PURELIN	4.43	34.43	5.87	0.84
5	21	LOGSIG	PURELIN	4.62	35.62	5.97	0.83
6	14	LOGSIG	PURELIN	4.70	37.45	6.12	0.83
7	15	LOGSIG	PURELIN	4.77	37.87	6.15	0.82
8	8	TANSIG	PURELIN	4.49	39.46	6.28	0.82
9	5	TANSIG	PURELIN	4.78	38.38	6.20	0.82
10	25	LOGSIG	TANSIG	4.83	38.19	6.18	0.82

Table III shows error measures and R^2 values of the top ten ranked ANN models along with their architecture and parameters. The best predictive ANN model (MAE = 4.03 and R^2 = 0.86) is achieved when 24 neurons are considered in the hidden layer and LOGSIG and PURELIN transfer functions are used in the hidden and output layers, respectively.

The R^2 values in Table III indicate that the correlation between the predicted and actual compressive strengths is high enough to give a very good prediction. Overall, a better predictive accuracy is achieved when the PURELIN transfer function is used in the output layer.

B. FIS for Modeling Concrete Compressive Strength

1) FIS model structure and parameters

Similar to the ANN models, two data sets were employed: the training set (300 records) was used to construct FIS models using FCM clustering, while the validation set (125 records) was used to evaluate the predictive accuracy of the models. Generally, development of FIS models from existing data is carried out in two major stages: structure identification and parameter estimation [30]. Structure identification deals with determining the input and output variables, choosing the type of FIS, deciding on the type and number of membership function for the input and output variables, and deciding on the number of fuzzy rules, whereas, parameter estimation addresses the following FIS properties: number of clusters which also determine the number of rules, the fuzzification coefficient, and iteration information [14], [30]. In this study, FIS models were generated using “genfis3” function of MATLAB Fuzzy Toolbox™ by varying the model structure and parameters. The “genfis3” is a built-in function that generates an FIS structure of a specified type, using FCM clustering to capture a set of rules that best represent the data behavior [31].

The input and output variables used in developing the FIS models are the same as those used in ANN models. In FCM clustering, the number of clusters determines both the number of membership functions of the input and output variables, and the number of rules. For instance, if the number of clusters is three, there will be three rules, where each cluster represents a rule, and each input and output variable has three membership functions (Fig. 6). Pedrycz and Gomide [17] suggested that the number of clusters should be kept quite low (5 to 9) to ensure the interpretability of developed FIS models. However, in this study an attempt was made to investigate the effect of number of clusters on the predictive accuracy of FIS models; thus, the number of clusters varies from 3 to 30. The fuzzification coefficient (m) is another essential parameter that affects the geometry of the membership function generated by FCM algorithm. The most commonly assumed value of m equals 2. While lower values of m (closer to 1) result in localized membership values around 0 or 1, higher values of m (m = 3, 4, etc.) yield spiky membership functions [17]. In this study, the following m values were considered in developing the FIS models: 1.5, 2.0, 2.5, 3.0, 3.5, and 4.0. Additionally, the FCM clustering process is set to terminate when the maximum number of iterations reaches 1000 or when the minimum amount of improvement between two consecutive iterations is less than 1×10^{-5}.

TABLE IV. SUMMARY OF INFERENCE METHODS USED FOR MAMDANI AND SUGENO FIS MODELS

Inference methods	Mamdani	Sugeno
Fuzzy operator (AND)	Min	Product
Implication	Min	Product
Aggregation	Max	Sum
Defuzzification	Centroid	Weighted average

For comparison purposes, two sets of FIS models were developed using Mamdani and Sugeno inference types. In Mamdani-type inference, after the aggregation process, each output variable is expressed as a fuzzy set that needs to be defuzzified, whereas in Sugeno-type, the output membership functions are either linear or constant [14]. A Gaussian membership function is employed for the input and output variables of Mamdani FIS (Fig. 6), and for the input variables of Sugeno FIS. Also, the output function of the Sugeno FIS is represented using linear functions. A Gaussian membership function was adopted because of its continuity and smoothness, simplicity in representation (it needs only two parameters, modal value μ representing the typical value and σ representing the spread), ease of construction using a data-driven approach, faster convergence during optimization of membership functions, and suitability for models that seek high-control accuracy [5], [31].

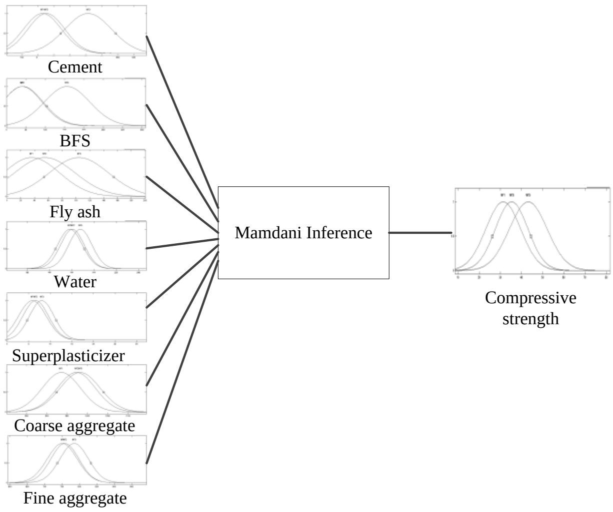


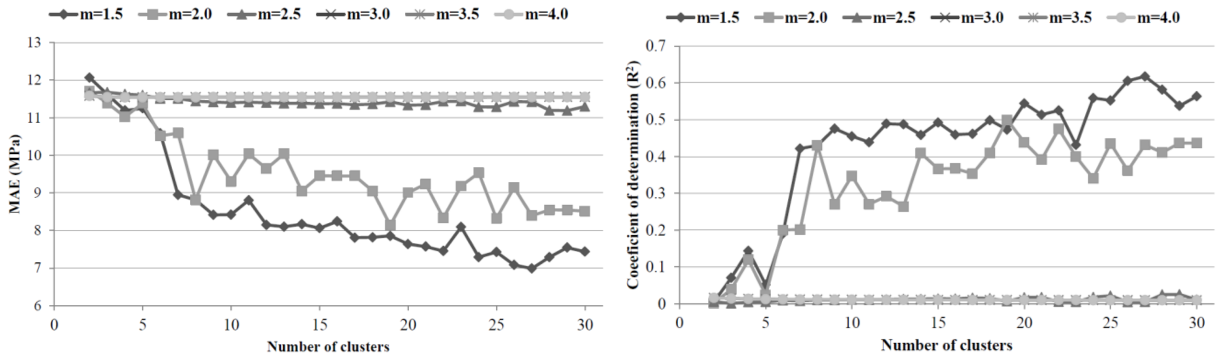
Figure 6. Mamdani FIS models with three membership functions (cluster centers).

Table IV summarizes the inference methods selected in developing the Mamdani and Sugeno FIS models, specifically the fuzzy operator used in the antecedent, the type of implication employed (from the antecedent to the consequent), the method adopted for aggregation of the consequent across the rules, and the defuzzification method chosen to get a single crisp predicted value from the fuzzy output set.

Sensitivity analysis was carried out for the best performing FIS models by considering different combinations of inference methods (fuzzy operator, implication, aggregation, and defuzzification). The results of this analysis are presented and discussed in the next subsection.

2) Results of FIS models

A total of 348 FIS models with different combinations of inference types (Mamdani and Sugeno), numbers of clusters (c), and fuzzification coefficients (m) were developed and trained. The predictive accuracies of the trained models were evaluated using the validation data set (125 records). Fig. 7 illustrates the MAE and R^2 values of the Mamdani FIS models, which each have different number of clusters and m values. As shown in Fig. 7, the performance of the FIS models with m values of 2.5, 3.0, 3.5, and 4.0 is exceptionally poor. A gradual increment in MAE and R^2 values appears for models with m values of 1.5 and 2.0 as the number of clusters increases. Even though the R^2 values are low, models with m values of 1.5 perform better in terms of MAE than those models with m values of 2.0 when an identical number of clusters is used.



(a) (b)

Figure 7. (a) MAE and (b) R^2 values of Mamdani FIS models.

Among the Mamdani FIS models the best performance (MAE = 6.99 and R^2 = 0.62) is achieved when the numbers of clusters (*c*) and *m* values are 27 and 1.5, respectively. The error and R^2 values of the best ten Mamdani FIS models are presented in Table V.

TABLE V. STATISTICAL RESULTS OF BEST PERFORMING MAMDANI FIS MODELS

Rank	Number of clusters (*c*)	Fuzzification coefficient (*m*)	Error measures			R^2
			MAE	MSE	RMSE	
1	27	1.5	6.99	81.89	9.05	0.62
2	26	1.5	7.08	83.88	9.16	0.61
3	28	1.5	7.29	92.37	9.61	0.58
4	24	1.5	7.29	93.61	9.68	0.56
5	30	1.5	7.43	95.83	9.79	0.56
6	25	1.5	7.43	96.29	9.81	0.55
7	20	1.5	7.64	96.19	9.81	0.54
8	29	1.5	7.54	99.11	9.96	0.54
9	22	1.5	7.45	101.39	10.07	0.53
10	21	1.5	7.57	103.40	10.17	0.51

TABLE VI. OPTIMIZATION OF FUZZY OPERATORS AND DEFUZZIFICATION METHODS FOR SUGENO FIS

Rank	Fuzzy operators and defuzzification methods				Error measures			R^2
	Input aggregation	Implication methods	Rule aggregation	Defuzzification method	MAE	MSE	RMSE	
1	MIN	MIN	PROBOR	CENTROID	6.90	79.41	8.91	0.63
2	MIN	MIN	SUM	CENTROID	6.90	79.43	8.91	0.63
3	MIN	MIN	PROBOR	BISECTOR	6.93	80.62	8.98	0.62
4	MIN	MIN	PROBOR	BISECTOR	6.95	80.86	8.99	0.62
5	MIN	MIN	MAX	CENTROID	6.99	81.88	9.05	0.62
6	MIN	MIN	SUM	MOM	7.66	95.24	9.76	0.62
7	MIN	MIN	PROBOR	MOM	7.66	95.27	9.76	0.62
8	MIN	PROD	SUM	CENTROID	6.84	83.22	9.12	0.61
9	MIN	PROD	SUM	CENTROID	6.85	83.24	9.12	0.61
10	MIN	PROD	MAX	CENTROID	6.91	85.66	9.25	0.60

To conduct the sensitivity analysis of the best performing Mamdani FISs, the following options of fuzzy operators and defuzzification methods were tested: (i) for input aggregation, MIN (minimum) and PROD (product); (ii) for implication, MIN (minimum), and PROD (product); (iii) for rule aggregation, MAX (maximum), SUM (sum of each rule's output set), and PROBOR (probabilistic OR); and (iv) for defuzzification, CENTROID, BISECTOR, MOM (middle of maxima), LOM (largest of maxima), and SOM (smallest of maxima). The options were varied one at a time, and a total of 60 unique combinations were tested. The results of the ten best performing fuzzy operators and defuzzification methods are summarized in Table VI. Comparing the final optimized Mamdani FIS model against the best performing model indicated that the optimization process improved the accuracy of the model in terms of MAE and R^2 by only 1.29% and 1.60%, respectively.

In Sugeno FIS models, the error and R^2 values obtained were the same irrespective of the number of clusters and *m* values considered in the models (i.e., the models were found to be insensitive to the optimization parameters *c* and *m*). The MAE, MSE, RMSE, and R^2 values of all Sugeno FIS models were 5.08, 45.62, 6.75 and 0.78, respectively. Sensitivity analysis was carried out by varying the input aggregation method (MIN and PROD) and defuzzification methods (Wtaver, or weighted average and Wtsum, or weighted sum). A total of four combinations were tested for each Sugeno FIS, as shown in Table VII. The results of the sensitivity analysis confirm that the performance of FIS models is highly influenced by applying different defuzzification methods. According to the results, in all the Sugeno FIS concrete compressive strength models, the Wtaver defuzzification method (R^2 = 0.78) should be adopted instead of the Wtsum ($R^2 = 4.5 \times 10^{-4}$) for a better predictive accuracy.

TABLE VII. OPTIMIZATION OF FUZZY OPERATORS AND DEFUZZIFICATION METHODS FOR SUGENO FIS

Option	Fuzzy operators and defuzzification methods		Error measures			R^2
	Input aggregation	Defuzzification method	MAE	MSE	RMSE	
1	MIN	Wtaver	5.08	45.62	6.75	0.78
2	PROD	Wtaver	5.08	45.62	6.75	0.78
3	MIN	Wtsum	35.30	1461.24	38.23	$2.83\text{x}10^{-7}$
4	PROD	Wtaver	30.92	1217.11	34.89	$4.5\ \text{x}10^{-4}$

Comparisons between Mamdani and Sugeno FIS models based on the validation results show that Sugeno FIS models outperform Mamdani FIS models in predicting compressive strength. Because of the continuity of the output surface and linear dependence of each rule on the input variable [31], Sugeno FIS models give better performance in mapping the relationship between the inputs and output. Thus, Sugeno FIS models with *m* values of 1.5, 2.0, 2.5, 3.0, 3.5, and 4.0 and number of clusters ranging from 3 to 30 were used as an initial FIS for developing the ANFIS models.

C. ANFIS for Modeling Concrete Compressive Strength

1) ANFIS model structure and parameters

As in the case of the ANN models, two data sets were employed when evaluating the ANFIS models: the training set (300 records) was used to construct FIS models using FCM clustering, while the validation set (125 records) was used to evaluate the predictive accuracy of the models.

The ANFIS models were trained and validated using the same training and validation data sets used in both the ANN and FIS models. In this study, ANFIS models were generated using the ANFIS function of MATLAB Fuzzy Logic Toolbox™. Model structures such as the number and type of membership function, type of output membership function, and inference type, and model parameters including learning algorithm, number of training epochs, and training error goal must be carefully selected in order to develop an ANFIS model with better predictive capability. The FIS models developed using the Sugeno inference system were used to provide the ANFIS models with the initial membership functions for training. Using Sugeno rather than Mamdani FISs in ANFIS has the following advantages: (i) computational efficiency in optimization and adaptive processes [20], (ii) "guaranteed continuity of the output surface" [31], and (ii) more reliable results when data driven techniques are adopted [5]. The model structure of the initial Sugeno FIS used for developing the ANFIS models is summarized in Table VIII. In this study, the effects of number of membership functions (number of clusters) and fuzzification coefficient (*m*) on the performance of the ANFIS models were investigated by varying the membership functions from 3 to 30 and considering *m* values of 1.5, 2.0, 2.5, 3.0, 3.5, and 4.0. The schematic representation of the architecture of an ANFIS model with 5 input membership functions based on a Sugeno FIS is shown in Fig. 8.

TABLE VIII. MODEL STRUCTURE OF INITIAL SUGENO FIS USED TO DEVELOP THE ANFIS MODELS

Model structures	Type/value
Number of membership functions	Varying (3–30)
Type of input membership function	Gaussian
Type of inference	Sugeno
Type of output membership function	Linear
Value of fuzzification coefficient (*m*)	Varying

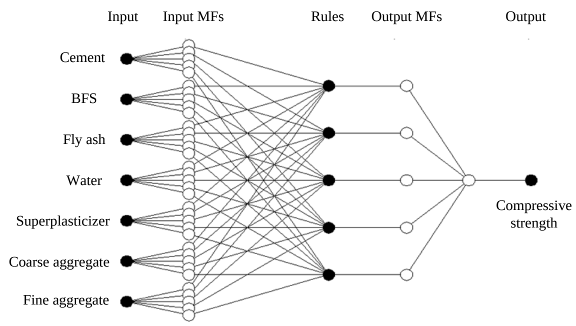
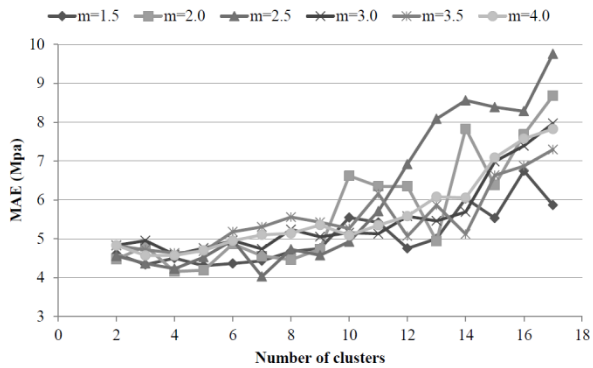


Figure 8. Schematic representation of ANFIS model with 5 input MFs.

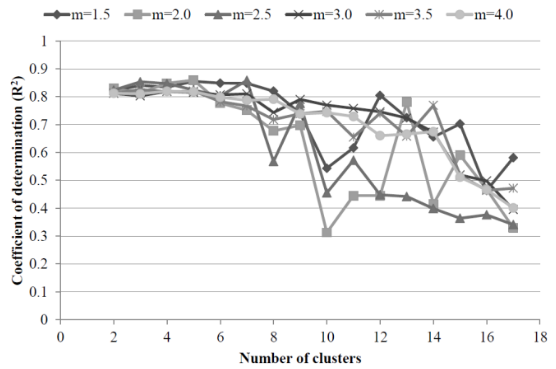
The basic learning rules available to optimize the parameters of membership functions in ANFIS are either back-propagation gradient descent or hybrid learning, which combines the gradient-descent and least-square methods [26]. According to Jang [23], the major limitation of the back-propagation gradient descent method is that the learning process gets trapped in the local minima and takes more time to train. Thus, the hybrid learning method was employed in this study. The hybrid learning procedure works in such a way that the consequent parameters are estimated in the forward pass using the least mean square error procedure by keeping the premise parameters fixed; in the backward pass the back-propagation descent method is used to modify the premise parameters while the consequent parameters are kept fixed [16], [23]. This procedure is repeated until both the premise and consequent parameters are optimized. In this case, the number of training epochs and the training error goal were set to 1000 and 0, respectively. The training process terminates whenever either of these designated values are achieved.

2) Results of FIS models

After successful training, the validation data set was applied to evaluate the predictive capability of the models. The MAE and R^2 values of ANFIS models with different *m* values and numbers of clusters (*c*) ranging from 2 to 17 are presented in Fig. 9 for illustration purposes, as the performance of the models with higher number of clusters were found to be very poor (R^2 values ranging between 8.58×10^{-6} and 0.41). Comparatively, the ANFIS models with *m* values of 1.5, 2.0 and 2.5 perform better in predicting compressive strength than those ANFIS models with higher *m* values. Moreover, higher prediction accuracy is achieved at a lower number of clusters (2 to 7) for all *m* values considered in the models—unlike the Mamdani FIS models, which achieved higher accuracy at a higher number of clusters.



(a)



(b)

Figure 9. (a) MAE and (b) R^2 values of ANFIS models.

The performance results of the top ten ranked ANFIS models in terms of prediction accuracy are presented in Table IX. According to Table IX, the highest prediction accuracy is achieved from an ANFIS model with 5 clusters and *m* values of 2.0. The MAE, MSE, RMSE, and R^2 vales of this model are 4.19, 29.40, 5.42, and 0.86, respectively.

TABLE IX. STATISTICAL RESULTS OF BEST PERFORMING ANFIS MODELS

Rank	Number of clusters (*c*)	Fuzzification coefficient (*m*)	Error measures			R^2
			MAE	MSE	RMSE	
1	5	2.0	4.19	29.40	5.42	0.86
2	5	1.5	4.30	29.99	5.48	0.86
3	7	2.5	4.33	30.36	5.51	0.86
4	3	2.5	4.36	31.25	5.59	0.85
5	6	1.5	4.36	31.52	5.61	0.85
6	7	1.5	4.43	31.66	5.63	0.85
7	4	2.5	4.23	32.59	5.71	0.85
8	4	2.0	4.26	32.90	5.74	0.85
9	3	1.5	4.34	33.73	5.81	0.84
10	4	1.5	4.50	34.65	5.89	0.84

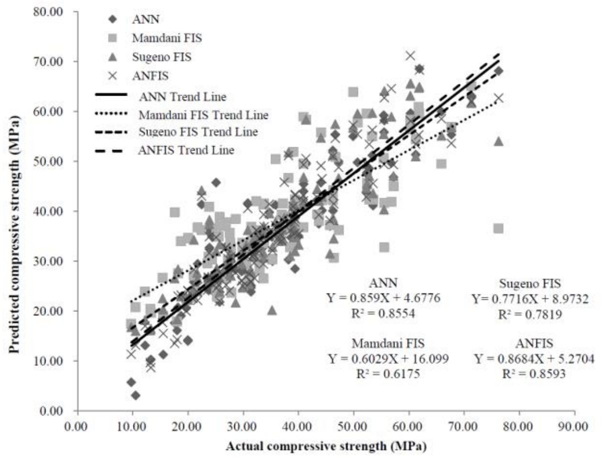


Figure 10. Comparison of actual and predicted compressive strength of the best performing AI models.

D. Comparison of AI Models

To compare the performance of the three AI modeling techniques, a scatter diagram plotting the relationship between actual and predicted compressive strengths was developed for the best performing ANN, Mamdani FIS, Sugeno FIS, and ANFIS models, each of whose model structure and parameters are described in previous sections. The linear least square fit line (trend line) with its corresponding linear equation and R^2 values is depicted in Fig. 10 for each best performing AI model. The prediction accuracy of the Mamdani FIS was relatively low (R^2 = 0.629), and better prediction was achieved with a higher number of clusters (*c* = 27), resulting in limited interpretability. According to Pedrycz and Gomide [17], the number of clusters should be moderately low (5–9) for better interpretability. The prediction accuracy of the Sugeno FIS remained the same (R^2 = 0.782) regardless of the number of clusters and *m* values considered (i.e., it was not sensitive to parameter optimization). The compressive strength values predicted by the ANFIS (R^2 = 0.859) and ANN (R^2 = 0.855) models were reasonably close to the actual compressive strength values as compared to the FIS models. In other words, ANFIS and ANN models were found to be better than the FIS models in mapping the relationship between the input variables and the output. Even though the predictive capability of the ANN is high, the model lacks interpretability, as the logic behind the model cannot be traced. Thus, the best performing ANFIS model should be used for predicting the compressive strength of HPC, as its learning capability contributes by optimizing the membership functions, and the relationship between the input and output can be easily interpreted in the form of if-then rules. The ranking and model performance values of the best performing AI models are shown in Table X.

TABLE X. STATISTICAL RESULTS OF BEST PERFORMING ANFIS MODELS

Rank	AI model type	Error measures			R^2
		MAE	MSE	RMSE	
1	ANFIS	4.19	29.40	5.42	0.86
2	ANN	4.03	30.41	5.51	0.86
3	Sugeno FIS	5.08	45.62	6.75	0.78
4	Mamdani FIS	6.90	79.41	8.91	0.63

IV. CONCLUSION AND FUTURE WORK

High-Performance Concrete (HPC) is a very complex material, as the inter-relationship between the constituent materials is highly nonlinear and the material's property is affected by several interacting factors that are not yet fully understood. Thus, the already available conventional empirical and statistical methods are limited in accurately predicting the compressive strength of HPC. In this study, the potential of AI techniques including ANN, FIS, and ANFIS in predicting the compressive strength of HPC were explored. The AI-based predictive models were trained and verified using a HPC 28-day compressive strength data set obtained from the literature. In order to optimize the AI models, different model structures and parameters were examined. ANN models with a multilayer feedforward back-propagation with a single hidden layer were developed by varying the number of neurons in the hidden layer (from 2 to 25) and by changing the transfer functions (PURELIN, LOGSIG, and TANSING) in the hidden and output layers. A data-driven approach based on fuzzy c-means clustering (FCM) was employed to generate both the Mamdani and Sugeno FIS models. The FIS models developed using Sugeno FIS were used as an initial FIS to train the ANFIS models. The effects of model parameters including number of clusters (varying from 2 to 30) and fuzzification coefficient (m = 1.5, 2.0, 2.5, 3.0, 3.5 and 4.0) on the performance of the FIS and ANFIS models were investigated. After successful training of the AI models, the validation data set was applied to evaluate the predictive capability of the models based on error measures such as MAE, MSE, RMSE, and R^2. Finally, for each AI technique, a model with minimum error values and maximum R^2 was selected as the optimal predictive model of that type.

The conclusions drawn in this study are based on the input factors, model structure and parameters considered, and the training and validation data set used in developing and verifying the models. Comparatively, m values of 1.5 and 2.0 resulted in better predictive accuracy in Mamdani FIS models. The best performing Mamdani FIS model (MAE = 6.99 and R^2 = 0.63) resulted when the number of clusters used was reasonably high (c = 27); by contrast, Sugeno FIS models were insensitive to parameter optimization (m and c) and the MAE and R^2 values of all Sugeno FIS models were 5.08 and 0.78, respectively. The best performing ANN model was achieved when 23 neurons were used in the hidden layer and LOGSIG and PURELIN transfer functions were employed in the hidden and output layers, respectively. It was found that the performance of ANN models is exceptionally poor when the LOGSIG transfer function is used in the output layer, irrespective of the number of neurons and transfer function used in the hidden layer. ANFIS models with low numbers of clusters (2–7) have outperformed those models with high numbers of clusters in predicting compressive strength. The best predictive ANFIS model was attained when the number of clusters (c) and fuzzification coefficient (m) were 5 and 2.0, respectively. Overall, comparisons among the AI models showed that ANFIS and ANN perform better than the FIS models in generalizing the relationship between the input variables (constituent materials) and the output (compressive strength). Though the performances of ANFIS and ANN models were found to be almost the same, the ANFIS model should be preferred over the ANN model for interpretability reasons, as ANN is a "black box" that makes it difficult to explicitly identify possible causal relationships. Moreover, the ANFIS model saves more computational time and eliminates the trial and error procedure required to select the best ANN architecture.

The contributions of this paper can be grouped into three areas. Firstly, the paper presented AI-based HPC compressive strength prediction models. The models accurately represent the complex nonlinear relationship between the constituent materials and compressive strength of HPC, and provide researchers and practitioners with an alternative prediction approach. Secondly, the paper presented an approach for developing FIS models using a data-driven approach, which overcomes the limitations of expert-driven FIS models applied in past concrete compressive strength studies. Lastly, the paper advanced the state of the art in AI modeling for concrete compressive strength prediction by optimizing the structural and parameter components of the three commonly used AI models (ANN, FIS, and ANFIS). The study showed that the structures and parameters for AI models should be carefully examined for better performance of the concrete compressive strength predictive models. The results of this study also show that a data-driven approach based on FCM clustering can be used to generate reliable FIS and ANFIS models to predict the compressive strength of HPC. The developed models will help industry practitioners (i.e., designers and construction engineers) to accurately predict the compressive strength of HPC without having to conduct costly and time-consuming laboratory experiments.

Further research will consist of: (i) investigating the effects of different network types (perceptron, probabilistic) and training algorithms (such as Quasi-Newton, resilient back propagation, etc.) on ANN models; (ii) examining the implications of using different types of fuzzy operators and defuzzification methods, membership functions (triangular, trapezoidal, etc.), and clustering methods (k-means clustering, subtractive clustering, and conditional clustering) on the performance of both FIS and ANFIS models in predicting concrete compressive strength; and (iii) studying the effect of using the back-propagation learning rule to train the ANFIS models and comparing the results to those of ANFIS models employing the hybrid learning rule.

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