Seismic Mitigation in Civil Structures Using a Fractional Order PD Controller

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***Abstract*—In this paper, a systematic tuning procedure for a fractional order PD controller for seismic mitigation is proposed. The tuning is based upon reducing the magnitude of the compensated system at the resonance frequency as compared to the magnitude of the uncompensated structure. For simplicity, a laboratory scale 1DOF (one degree of freedom) steel structure is used as the case study, The simulation results considering the El Centro earthquake accelerograms show that the designed control strategy is highly suitable for solving seismic mitigation of steel structures and ensures improved response in comparison with steel structures equipped with passive protection.**

***Index Terms*—seismic mitigation, fractional order controller, robustness**

I. Introduction

Structural control methods attempt to mitigate the structural response induced by various environmental dynamic loadings, such as powerful wind gusts and earthquakes, and to enhance the safety and quality of structures [1], [2]. Passive structural control techniques have been used intensively, especially in the form of Tuned Mass Dampers (TMD) [2], which add damping to the structure, in the event of an environmental dynamic loading. The major disadvantage of TMDs, which are tuned only to the fundamental frequency of the structure [1], is that they have a limited control capacity, suppressing only a reduced number of vibrations. Therefore, these passive devices may have little effect during earthquakes which stimulate other modes instead of the one that is used to tune the particular TMD [3]. Active control strategies, on the other hand, have the advantage that they require an external energy source that is used to suppress any type of vibrations that may occur through the use of an actuator. Active Tuned Mass Dampers (ATMD) are in fact TMDs equipped with an actuator that is used to apply the control force in real time. Different control algorithms have been proposed over the years in order to yield the control force for the actuator, such as optimal, robust, sliding mode control, fuzzy logic control [2], [4]-[6], mainly used due to the increased robustness they confer to the closed loop system. Different types of proportional-integrative-derivative (PID) controllers have also been employed, due to their simplicity, but tuned to be optimal and robust against uncertainties and modelling errors [1], [7], [8].

An emerging control strategy that has been little considered for seismic mitigation is the fractional order control that is based on combining traditional PID control strategies with the theory of fractional calculus [9]. In fact, fractional order PID controllers are generalizations of the traditional PID controllers, since they involve integrators of order $\mu\epsilon(0,1)$ and differentiators of order $\lambda\epsilon(0,1)$. Among the fractional order control strategies that have been proposed for solving the vibration suppression problems, is the fractional order disturbance observer [10], an enhanced version of the linear quadratic regulator with fractional order filters [11] or the fractional order difference feedback [12]. The choice for such a control algorithm resides in its ability to enhance the closed loop performance, stability and robustness despite uncertainties and modelling errors, while being significantly easier to tune and implement as compared to the more complex advanced control algorithms [13], [14].

The present paper presents a fractional order controller designed to suppress the vibrations that may occur in a structure. For simplicity, a laboratory scale 1DOF (one degree of freedom) steel structure is used as the case study, but the results may be easily extended to a high-rise building and to multiple DOF systems. The structure is equipped with an ATMD, but for comparison purposes a TMD device is also used. Previous research include the tuning of a simple fractional order control algorithm for a similar case study, in which the steel structure has been equipped with viscoelastic mass dampers [15], as well as a trial and error design of a fractional order PD controller, where the fractional order and the derivative gains are pre-selected and the influence of the proportional gain upon vibration attenuation is solely analysed [16]. In this paper, a systematic tuning procedure for a fractional order PD controller for seismic mitigation is proposed. The tuning is based upon reducing the magnitude of the compensated system at the resonance frequency as compared to the magnitude of the uncompensated structure. The simulation results considering the El

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Centro earthquake accelerograms show that the designed control strategy is highly suitable for solving seismic mitigation of steel structures and ensures improved response in comparison with steel structures equipped with passive protection.

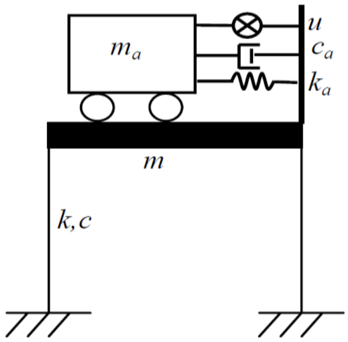

Figure 1. Schematic representation of the ATMD.

II. The Laboratory Scale 1 DOF Structure with Active and Passive TMD

The schematic representation of the ATMD and the structure is given in Fig. 1, where *m*, *k* and *c* are the mass, stiffness and damping coefficients of the structure, while m_a, k_a and c_a are the mass, stiffness and damping coefficients of the ATMD, *u* is the control force acting upon the actuator. The system is modeled in a simplified manner as:

$$\begin{pmatrix} m & 0 \\ 0 & m_a \end{pmatrix}\begin{pmatrix} \ddot{x} \\ \ddot{x}_a \end{pmatrix}+\begin{pmatrix} c+c_a & -c_a \\ -c_a & c_a \end{pmatrix}\begin{pmatrix} \dot{x} \\ \dot{x}_a \end{pmatrix}+\begin{pmatrix} k+k_a & -k_a \\ -k_a & k_a \end{pmatrix}\begin{pmatrix} x \\ x_a \end{pmatrix}=\begin{pmatrix} F_g(t) \\ u(t) \end{pmatrix} \quad (1)$$

with $F_g(t)=m\cdot\ddot{x}_g$ and $\ddot{x}_g$ the seismic excitation. If, $u(t)=0$, then (1) describes the equations for a structure equipped with a simple TMD. The structural parameters are: m=120 N*s^2/m, k=3147 N/m and c=3.886 N*s/m. To tune the TMD, the mass ratio is chosen as $\mu=\frac{m_a}{m}=2\%$, yielding a mass of the TMD, m_a=2.4 N*s^2/m. Next, the natural frequency of the structure and its damping ratio are computed: $\omega_n=16.2$, $\xi=0.01$. The generalised Den Hartog equation [17] is used to determine the damping ratio of the TMD:

$$\xi_a=\sqrt{\frac{3\mu}{8(1+\mu)}}+\frac{0.1616\xi}{1+\mu}=0.0873 \quad (2)$$

The frequency ratio of the TMD and the structure is then computed using the same generalised Den Hartog equation [17]:

$$q=\frac{1}{1+\mu}\left(1-1.5906\xi\sqrt{\frac{\mu}{1+\mu}}\right)=0.9613 \quad (3)$$

The natural frequency of the TMD is determined using (3), while the damping and stiffness coefficients are computed based on (2) and (4):

$$\omega_a=q\omega_n=15.56 \quad (4)$$

$$\begin{cases} k_a=58.1654 \\ c_a=0.6526 \end{cases} \quad (5)$$

III. Design of the Fractional Order PD Controller for Vibration Suppression

The transfer function of the proposed fractional order PD controller is given as:

$$C_{FO}(s)=k_p\left(1+s^{\lambda}\right) \quad (6)$$

where k_p is the proportional gain, k_d is the derivative gain, λ is the fractional order of differentiation, with λϵ(0,1) and s is the Laplace variable. The closed loop system with the fractional order PD controller is given in Fig. 2. The controller receives the measured structural displacement and, according to the proposed algorithm, generates the control force for the actuator which is then applied to the structure. The controller will treat the seismic excitation, $\ddot{x}_g$, as a disturbance and will attempt to maintain the structural displacement, *x*, at its reference position, 0.

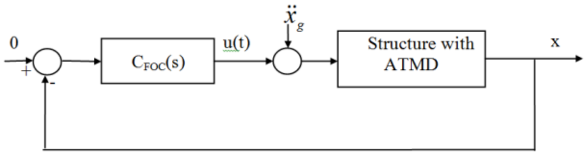

Figure 2. Schematic diagram of the fractional order PD controller applied on the laboratory scale structure with ATMD

Fig. 3 shows the Bode diagram of the reference structure (uncompensated structure), as well as the Bode diagram of the passively controlled structure using the TMD and the actively controlled structure using the ATMD. To tune the fractional order PD controller the following conditions are imposed for the closed-loop system:

$$H_{cl}(j\omega_1)\cong -48\text{ dB},\ \omega_1=0.1\text{ rad/sec} \quad (7)$$

$$H_{cl}(j\omega_2)=-46\text{ dB},\ \omega_2=\omega_r=16.2\text{ rad/sec} \quad (8)$$

$$H_{cl}(j\omega_3)\cong -80\text{ dB},\ \omega_3=100\text{ rad/sec} \quad (9)$$

where $H_{cl}(s)$ is the closed loop transfer function:

$$H_{cl}(s)=\frac{P(s)}{1+P(s)C_{FOC}(s)} \quad (10)$$

with $P(s)$ the transfer function of the steel structure with the ATMD, as determined from (1), and the fractional controller in the frequency domain is described by:

$$C_{FO}(j\omega)=k_p\left(1+k_d\omega^{\lambda}\left(cos\left(\frac{\lambda\pi}{2}\right)+j\,sin\left(\frac{\lambda\pi}{2}\right)\right)\right) \quad (11)$$

obtained by replacing $s=j\omega$ in (6). Solving the system of equations (7)-(9), the three controller parameters may be obtained using the MATLAB optimization toolbox and the *fmincon*() function, where (8) is used as the main function to be minimized and (7) and (9) are used as constraints. The solution is determined to be: k_p=49.8, kd=0.95 and λ=0.805. The implementation of the controller in (6) will be carried out in Matlab, by using an approximation of the fractional derivative s^{λ}, computed using the Oustaloup Recursive Approximation algorithm [18]. An analysis of the Bode diagram in Fig. 3 shows that the designed fractional order PD controller ensures the closed loop specifications in (7)-(9) and manages to reduce the magnitude peak at the resonant frequency in comparison to the uncompensated or passively controlled case.

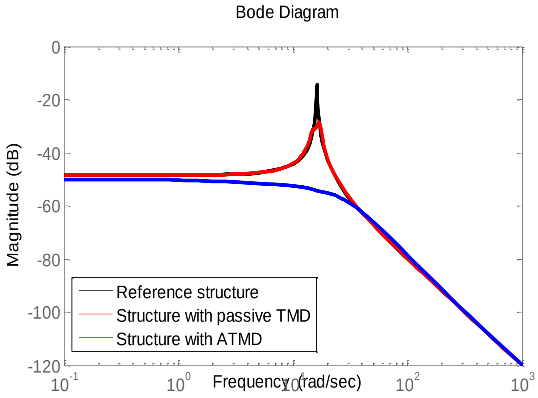

Figure 3. Bode diagram of the structure, passively and actively controlled structure

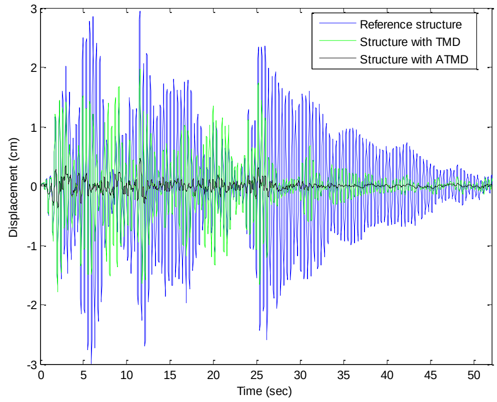

Figure 4. Comparison of time histories of structural displacement subjected to El Centro earthquake in unprotected, passive and active control

IV. Simulation Results and Robustness Evaluation

Fig. 4 presents the comparative simulation results, with the El-Centro earthquake excitation input, for the reference structure without any seismic protection and the passive and active control situations. The simulation results show that the designed fractional order PD controller can actively reject earthquake excitations and suppress the vibrations induced by such phenomena. When compared to the passive protection ensured by the TMD, the attenuation level is significantly increased. To test the robustness of the designed controller, modelling errors are considered in estimating the structural parameters. Thus, the parameters of the structure are modified, such that the resonance frequency is shifted: $\omega_n = 13.4$. Fig. 5 shows the El Centro response, for the reference structure with the new natural frequency, the structure equipped with the previously designed TMD and the structure with the ATMD actuated by the fractional order PD controller. The simulation results show that the proposed controller is indeed robust, being able to attenuate the vibrations within a short time interval. The TMD can still attenuate the vibrations produced by the El Centro earthquake, but the attenuation level and settling time of the oscillations are increased.

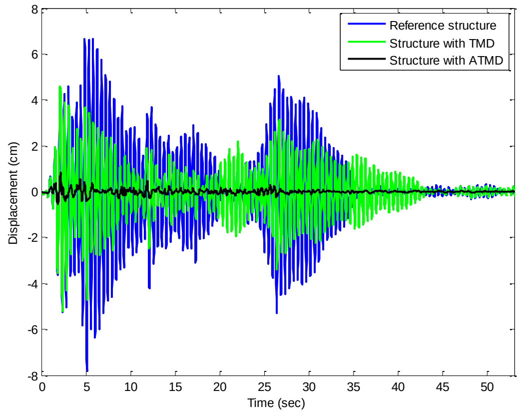

Figure 5. Robustness tests considering modeling errors for the structure subjected to El Centro earthquake in unprotected, passive and active control

V. Conclusions

The purpose of the paper was to design an active control strategy for seismic mitigation based on ATMDs and the emerging theory of fractional calculus. To simplify the modelling task, a simple laboratory scale model of a structure was considered, however, the results can easily be extended to a multiple DOF system. For this model scale structure, a TMD was designed using generalized Den Hartog equations. Next, a fractional order PD controller was tuned in order to actively reject structural excitation inputs. The simulation results show that the designed controller achieves better performance in terms of vibration attenuation, when compared to the passive TMD. Since a simplified description of any structure may lead to an incorrect estimation of the fundamental frequency of vibration of a structure, robustness tests were also considered. The results obtained clearly show that the ATMD with the fractional order PD controller outperforms the passive TMD and ensures vibration attenuation even in the case of modelling errors.

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References

[1] S. Etedali, M. R. Sohrabi, and S. Tavakoli, “An independent robust modal PID control approach for seismic control of buildings,” *Journal of Civil Engineering and Urbanism*, vol. 3, pp. 279-291, 2013.

[2] A. Pourzeynali, H. H. Lavasani, and A. H. Modarayi, “Active control of high rise building structures using fuzzy logic and genetic algorithms,” *Engineering Structures*, vol. 29, pp. 346-357, 2007.

[3] H. R. Owjia, A. H. N. Shirazib, and H. H. Sarvestani, “A comparison between a new semi-active tuned mass damper and an active tuned mass damper,” *Procedia Engineering*, vol. 14, pp. 2779-2787, 2011.

[4] L. Huo, G. Song, H. Li, and K. Grigoriadis, "H_∞ robust control design of active structural vibration suppression using an active mass damper," *Smart Materials and Structures*, vol. 17, 2008.
[5] K. S. Park and W. U. Park, "Min-max optimum design of active control system for earthquake excited structures," *Advances in Engineering Software*, vol. 51, pp. 40-48, 2012.
[6] R. Guclu and H. Yazici, "Seismic-vibration mitigation of a nonlinear structural system with an ATMD through a fuzzy PID controller," *Nonlinear Dynamics*, vol. 58, pp. 553-564, 2009.
[7] N. Aguirre, F. Ikhouane, and J. Rodellar, "Proportional-plus-integral semi active control using magneto-rheological dampers," *Journal of Sound and Vibration*, vol. 330, pp. 2185-2200, 2011.
[8] S. Etedali, M. R. Sohrabi, and S. Tavakoli, "Optimal PD/PID control of smart base isolated buildings equipped with piezoelectric friction dampers," *Earthquake Engineering and Engineering Vibration*, vol. 12, pp. 39-54, 2013.
[9] M. P. Aghababa, "A fractional-order controller for vibration suppression of uncertain structures," *ISA Transactions*, vol. 52, pp. 881-887, 2013.
[10] Y. Q. Chen, B. M. Vinagre, and I. Podlubny, "Fractional order disturbance observer for robust vibration suppression," *Nonlinear Dynam.*, vol. 38, p. 355, 2004.
[11] A. Shafieezadeh, K. Ryan, and Y. Q. Chen, "Fractional order filter enhanced LQR for seismic protection of civil structures," *Journal of Computational and Nonlinear Dynamics*, vol. 3, 2008.
[12] Z. H. Wang and Y. G. Zheng, "The optimal form of the fractional-order difference feedbacks in enhancing the stability of a sdof vibration system," *J. Sound Vib*., vol. 326, no. 3, p. 476, 2009.
[13] A. Cichocki and R. Unbehaven, *Neural Networks for Optimization and Signal Processing*, 1st ed., Chichester, U.K.: Wiley, 1993, ch. 2, pp. 45-47.
[14] C. A. Monje, Y. Q. Chen, B. M. Vinagre, D. Xue, and V. Feliu, *Fractional Order Systems and Controls: Fundamentals and Applications*, London, UK: Springer-Verlag, 2010.
[15] C. I. Muresan, E. H. Dulf, and O. Prodan, "A fractional order controller for seismic mitigation of structures equipped with viscoelastic mass dampers," *Journal of Vibration and Control*, 2015.
[16] O. Prodan and C. I. Muresan, "Seismic mitigation via fractional order active control," in *Proc. Eurosteel 2014 – 7th European Conference on Steel and Composite Structures*, Naples, Italy, September 10-12, 2014.
[17] I. M. Abubakar and B. J. M. Farid, "Generalized den hartog tuned mass damper system for control of vibrations in structures," in *Seismic Control Systems: Design and Performance Assessment*, S. Syngellakis, Ed., London, UL: WIT Press, 2013.
[18] A. Oustaloup, F. Levron, B. Mathieu, and F. M. Nanot, "Frequency band complex non integer differentiator: Characterization and synthesis," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 47, pp. 25-40, 2000.

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