Reliability Analysis of Steel Frame Structure

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Abstract—**This paper analyzes the reliability of the steel frame construction loaded with permanent load, variable load and wind. Reliability calculation is carried out by the "First Order Second Moment Method" with the help of the software "Vap" and in accordance with the recommendations of the JCSS 2001 taking into account the geometrical characteristics of the frame as a deterministic size. The analysis included three mechanisms of fracture and for each of the mechanisms their probability event. The results of this research show how much reliability indexes differ from different adopted steel profiles as well as the determining the most reliable steel profile for a given girder.**

Index Terms—**reliability, failure of the construction, reliability index**

I. Introduction

During the design and the execution of the steel construction we are trying to respect the basic characteristics of stability and security and that means for the buildings to be sufficiently rigid, load-bearing and ductile. In order for a steel construction to be reliable within the stipulated period of time none of these mentioned characteristics must not be undermined. More specifically, the resistance of the construction must be greater than the external load to meet the basic function and the purpose of the object. The main task of the reliability analysis is to perceive all the weaknesses of the structure and with the proper design and executions of buildings prevent failure.

Steel as the material is quite ductile and his behavior in the nonlinear area is not as prone to brittle fracture as it is the case with the concrete. Because of this characteristic of steel, these structures are capable a long time after the first plastic hinge and the formation of new joints to survive without the demolition and to serve their purpose. This system of formation of plastic fringes operates until the system becomes unstable and turned into mechanism. When we talk about the failure of buildings sometimes even the smallest failure on the structure can cause complete destruction of the building, but it is also true about the opposite case that the seemingly higher fracture in the structure does not mean the fracture of the entire building. From the above it is clear to conclude that in designing the structure we should be familiar with the most vulnerable, that is the most stressed parts of the buildings to be able to prevent collapse. Reliability analysis helps us that with the corresponding probability we can learn about this and prevent this breakage.

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II. Reliability Analysis

If the failure of the building is expressed with P_f, and the reliability with r follows the relation:

$$r = 1 - P_f \tag{1}$$

Determination of the reliability of the building actually represents the determination of the probability of fracture and from the previous relation is easy to determine reliability. The probability of fracture presents a common integration density distribution (load and resistance) f_{RS} per domain for which it is valid.

$$G(R,S) \le 0 \tag{2}$$

$$\begin{aligned} P_f &= P\left[G(R,S) \le 0\right] = P\left[R - S \le 0\right] = P\left[Z \le 0\right] \\ &= \int_D f_{RS}(R,S)\,dRdS \end{aligned} \tag{3}$$

where the G(r,s) is a limit function, f_{RS} is a joint density distribution of load and resistance (R is resistance and S is stress-load). Mean and standard deviation are marked as μ and σ.

Given the fact that this integration process is sometimes very difficult to perform and it is not easy to define a common density distribution today are frequently used other methods to determine the failure probability and one of them is the First Order Second Moment Method which performs approximation of the limit function with the first Taylor order and for calculating the probability of fracture uses the mean value and standard deviation.

$$P_f = P\left[Z \le 0\right] = P\left[\left(R - S\right) \le 0\right] \tag{4}$$

$$\mu_z = \mu_R - \mu_S, \sigma_z^2 = \sigma_R^2 + \sigma_S^2 \tag{5}$$

Previous formulations are valid under the assumption that resistance and load follow a Normal distribution.

Then the probability of fracture is equal to (6) and (7):

$$P_f = \Phi\left(\frac{Z - \mu_z}{\sigma_Z}\right) \tag{6}$$

$$P_f = \Phi\left(-\frac{\mu_z}{\sigma_z}\right) \tag{7}$$

The term β is called the index of reliability of safety index [1]. The essence of this method is based on

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approximation of the cumulative density distribution of random variables that do not follow a Normal distribution of random variable of Normal distribution and then determining the mean values and standard deviations by which we determine the reliability index β.

The advantage of this method is the ease of the use as that does not require knowledge of the distribution of a random variables, but this method can be inaccurate if the distribution function can not be approximated by a Normal distribution or if this function is not linear that is its approximation of Taylor's first order is not adequate. Hasofer and Lind have modified this index of reliabilty in the way that the random variables converted into standard Normal variables N~(0,1).

For linear boundary function apply the following formulation (8), (9) and (10)

$$R^{'} = \frac{R-\mu_R}{\sigma_R}, S^{'} = \frac{S-\mu_S}{\sigma_S} \quad (8)$$

$$Z = R - S = R^{'}\sigma_R + \mu_R - S^{'}\sigma_S - \mu_S = 0 \quad (9)$$

$$\beta_{HL} = \frac{\mu_R-\mu_S}{\sqrt{\sigma_R^2+\sigma_S^2}}, P_f = \Phi\left(-\beta_{HL}\right) \quad (10)$$

Previous formulations because of the simplicity can be graphically displayed in Fig. 1 in which we can see that the minimum distance from the limit function to the origin of ordinates actually represents β_{HL} that is Hasofer-Lind reliability index.

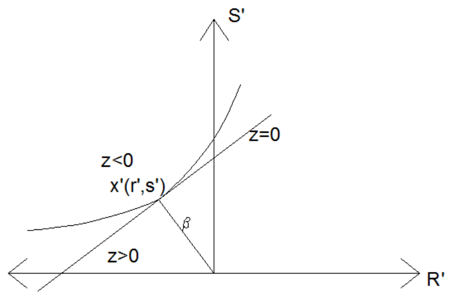

Figure 1. Hasofer-Lind method with nonlinear limit function

For a nonlinear boundary functions the problem of determining the probability of fracture or reliability index becomes iterative, because it is necessary to find a “designe point” x_d that is a point which is at least distanced from the point of ordinates on the basis of the (11):

$$\beta_{HL} = -\frac{\sum_{i=1}^{n} x_{di}^{'}\left(\frac{\partial G}{\partial x_{di}^{'}}\right)}{\sqrt{\sum_{i=1}^{n}\left(\frac{\partial G}{\partial x_{di}^{'}}\right)^2}} \quad (11)$$

III. Reliability Analasys of Steel Structures

The theme of this work is to determine the reliability of one steel frame and to create possible fracture mechanisms due to various loads.

Nowadays is increasingly being used the term reliability of structures and in the Eurocode the reliability and durability of the structures is an important segment and a great attention is devoted to it. According to the [1] the recommended reliability indexes are given in Table I for the return period of one year.

TABLE I. Reliability Index According to [7]

Relative cost of the security measures	Small consequences as a result of cancellation	Secondary consequences as a result of a failure	Great consequences as a result of a failure
High	β=3.1 (P_f≈10^{-3})	β=3.3 (P_f≈10^{-3})	β=3.7 (P_f≈10^{-4})
Normal	β=3.7 (P_f≈10^{-4})	β=4.2 (P_f≈10^{-5})	β=4.4 (P_f≈5*10^{-5})
Low	β=4.2 (P_f≈10^{-5})	β=4.4 (P_f≈10^{-5})	β=4.7 (P_f≈10^{-6})

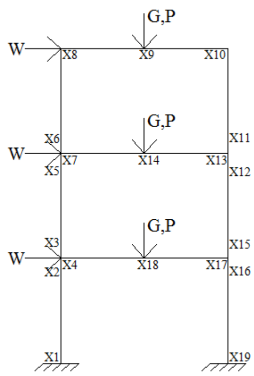

Figure 2. Places of potential plastic joints

The frame in Fig. 2 is nine times statically indeterminate and taking into account the position of the load acting on the frame, the number of the positions of possible plastic hinges is 19th. If n is marked as statically indeterminate, and X represents the number of plastic joints [2], we get a number of basic mechanisms where N is 10, 3 sway, 3 beam and 4 hinge mechanism. In this paper we will analyze the sway, beam on Fig. 3 and combined mechanisms of fracture.

$$N = X - n = 19 - 9 = 10 \quad (12)$$

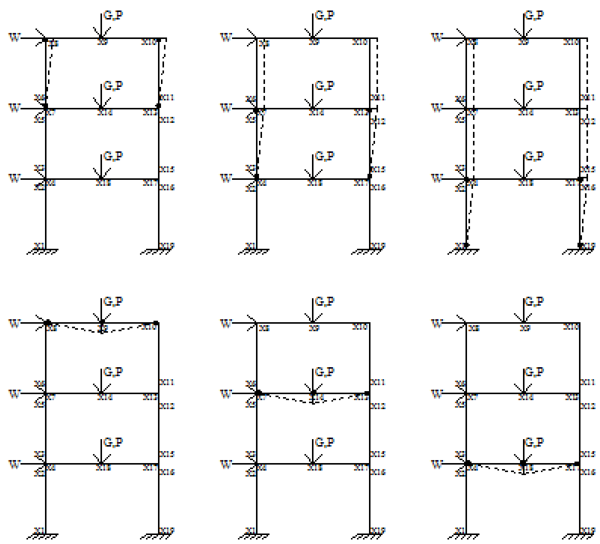

Figure 3. Sway and beam mechanisms of fracture

The reliability calculation is defined by the appropriate limit functions at each mechanism according to the kinematic theorem of boundary analysis using the principle of virtual displacement which states that the work of external forces on virtual displacements is equal to the work of the internal forces [2]:

$$\sum_{i=1}^{k} P_i \delta\varepsilon_i = \sum_{j=1}^{m} M_j^* \delta\varphi_j \tag{13}$$

All loads (fixed, variable and wind) are shown as concentrated forces with the goal of simplifying the problem of calculating the reliability in the Fig. 2 and 3. From the above it is easy to conclude that the force of the wind will work on the virtual moving of the sway mechanism while the vertical forces will participate in the moving of beam mechanism of the frame. When defining boundary function of the combined sway-beam mechanism, both of these loads will participate in a virtual displacement. The equations (14) of the limit functions of three sway, one beam and one combined mechanisms are as follows:

$$G_{s1} = m_r\left(X_6 + X_8 + X_{10} + X_{11}\right) - m_e hW \le 0$$

$$G_{s2} = m_r(X_5 + X_3 + X_{15} + X_{12}) - m_e 3hW \le 0$$

$$G_{s3} = m_r(X_1 + X_2 + X_{16} + X_{19}) - m_e 6hW \le 0$$

$$G_b = m_r(X_8 + 2X_9 + X_{10}) - m_e a_1 0.5(G+Q) \le 0$$

$$\begin{aligned} G_k &= m_r(X_1 + X_{19} + 2X_9 + 2X_{14} + 2X_{18} + X_{10} + X_{13} + X_{17}) \\ &-m_e(6hW + 3a_1 0.5(G+Q)) \le 0 \end{aligned} \tag{14}$$

A. Defining of Random Variables

For the purpose of calculating the limit function, load and structural resistance are expressed through appropriate random variables and deterministic sizes and all according to the JCSS guidelines [3].

Constant load of the steel constructions includes its own weight and the weight of unconstructive elements.

The term for own weight is as follows:

$$G = \int_V \gamma_s dV \tag{15}$$

In the expression (15) γ_s is the volumetric weight of steel and V is the volume of steel section. For steel structures according to [3] the volume is taken as the variable size while the volume weight of the steel is the deterministic size. Variable load is taken according to the recommendations [3] adopting that the building is intended for office work. On the basis of [3] we obtain the standard deviation of the long and short term variable load.

$$\sigma_L = \sqrt{\sigma_V^2 + \sigma_U^2 k \frac{A_0}{A}}$$

$$\sigma_S = \sqrt{\sigma_V^2 k \frac{A_0}{A}} \tag{16}$$

Equation of the wind force acting on the unit area is:

$$w = c_a c_g c_r q_{ref} = c_a c_g c_r 0.5\rho_a v^2 \tag{17}$$

Basic distributions of these factors can follow Log-normal or Normal distribution while the wind speeds (in the case of the maximum rate) follow Gumbel distribution. For wind speed we will take the mean 10 minute value of v=30m/s with coefficient of variation of 0.1, but if we want to determine the wind speed for N years the maximum wind speed also follows Gumbel distribution and mean value and standard deviation of such distribution may be determined on the basis of maximum term of mean values and standard deviation for one year [3]:

$$\mu_N = \mu_1 + 0.75\sigma_1 \ln(N), \sigma_N = \sigma_1 \tag{18}$$

The following Table II shows all the variables that are taken into account when addressing the reliability of the steel frame using the aforementioned border function.

TABLE II. BASIC VARIABLES AND THEIR DISTRIBUTIONS AND DETERMINISTIC SIZES

Variable	Distribution	Parameter (μ,σ)
a_1 (m) width of the frame	Deterministic	5
A_p (m^2) area of the steel profile	Normal	0.0069,0.000138
b (m) distance between the frames	Deterministic	5
c_a aerodynamic factor	Normal	1.1, 0.132
c_g factor of collision	Normal	2.87,0.344
c_r roughness factor	Normal	1.12,0.168
d_c(m) concrete thickness	Deterministic	0.1
f_y (kN/m^2) steel limit of yield	Log-normal	300000, 18000
h (m) height of the floor	Deterministic	3
m_q the unreliability of wind	Normal	0.8,0.2
m_r unreliability resistance	Normal	1.1,0.05
q_l (kN/m^2) long-term load	Gamma	0.5, 0.53
q_s (kN/m^2) short-term load	Exponential	0.2,0.29
ρ_a (kg/m^3) air density	Deterministic	1.25
γ_c (kN/m^3) volume density of concrete	Normal	25,1
γ_s (kN/m^3) volume density of steel	Normal	77,3
V (m/s) wind speed T=5 years	Gumbel	34, 3.4
W_{plb} (m^3) plastic resistant moment of beam	Normal	Variable
W_{plc} (m^3) plastic resistant moment of columns	Normal	Variable

IV. RELIABILITY RESULTS

We took into the account the equation of the limit function assumption that the neglected influence of the normal forces on the moment of the bending of a beam (pure bending) and that the beam is secured to the action of the shearing forces. The term for the moment of the full plasticity of the appropriate steel profile we got on the basis of the next formulation [2]:

$$M_p = f_y W_p \tag{19}$$

W_p and f_y are plastic resistant moment and the boundary of the yield stress of steel and S_x is static moment. Plastic resisting moment for each steel profile is shown in the table.

$$W_p = 2S_x \quad (20)$$

TABLE III. Beam Fracture Mechanism for Different I Profiles

Steel profiles for beams	Plastic resistant moment (m^3)	Reliability index for 5 years	Reliability index for 50 years
INP 300	0.000761	5.00	4.54
INP 280	0.000630	4.52	3.96
INP 260	0.000513	4.01	3.43
IPE 270	0.000484	3.87	3.26
IPE 300	0.000628	4.51	3.99
IPE 330	0.000804	5.15	4.70
IPB1 (HEA) 220	0.000568	4.26	3.71
IPB1 (HEA) 240	0.000744	4.94	4.47
IPB (HEB) 200	0.000642	4.56	4.05
IPB (HEB) 220	0.000827	5.23	4.78
IPBv (HEM) 140	0.000493	3.91	3.32
IPBv (HEM) 160	0.000674	4.69	4.19

In the Table III is clearly shown that for the beam girders are more favorable INP and IPE due to the smaller width of the foot and consequently due to the smaller weight of the carrier. For IPE 300 index of reliability for the period of 50 years is 3.99 which is greater than the recommended 3.8 [4] and its total weight is 42.2 kg/m. The following recommended profile is INP 280 with a total weight of 47.9 kg/m. The use of rolled profiles and profiles with narrow legs is certainly rational when the girder is exposed only to bending around a stronger axis of inertia so it is justified.

Due to the sway and combined mechanism which occur in the plastic joints in which it is necessary to check the stability and the influence of the normal force at the moment of plasticity according to [5]-[7].

If we consider that the normal force can influence in the increase of the moment of plasticity in the columns (second order theory) by the equation:

$$M_{p,II} = M_{p,I} + N_{pl} u$$

$$N_p = A_p f_y \quad (21)$$

$M_{p,I}$, and M_{pII} are moments according to first and second order theory. Normal force is N_{pl} and horizontal deflection is shown as u.

When calculating this frame according to the theory of the second order (Radimpex Tower 6) we get the maximum horizontal deflection u of the frame 2.5 cm for adopted beam profile INP 300 and the profile in the column IPB 300. Normal force affects slightly the increase of the bending moment in the columns of the frame so that in this case we will ignore its impact. As for the sway fracture mechanism the most attention will be given to the right sway mechanism in which the plastic joints primarily appear in the fixed joints X_1 and X_{19} on Fig. 2 and below first floor. On the moment of plasticity we will dismiss the impact of normal and transversal forces that in one part affect the reduction of the moment of plasticity according to the expression.

$$M_{p,N,V} = M_p \left[1-\left(\frac{N}{N_p}\right)^2\right]\sqrt{1-\left(\frac{V}{V_p}\right)^2} \quad (22)$$

In (22) normal force, normal plastic force, vertical and vertical plastic force are shown as N, N_{pl},V and V_{pl}.

TABLE IV. Sway Fracture Mechanism for Different I Profiles

The steel profile of the columns	Plastic resistant moments (m^3)	Reliability index for 5 years	Reliability index for 50 years
IPB1 (HEA)300	0.001383	3.50	3.09
IPB1 (HEA) 320	0.001628	3.85	3.24
IPB1 (HEA) 340	0.001850	4.12	3.55
IPB (HEB) 280	0.001534	3.64	3.00
IPB (HEB) 300	0.001868	4.14	3.57
IPB (HEB) 320	0.002149	4.43	3.91
IPBv (HEM) 220	0.001419	3.55	2.89
IPBv (HEM) 240	0.002116	4.40	3.87

If you look at the Table IV for the least favorable limit function of the sway mechanism G_{B3} it is clear that the IPB1 girders are I- girders which have the smallest weight and as such are quite favorable to take the complete load of the columns of the frame. On the other hand IPBv girders are opposite from the IPB1 because for the lowest height they have the highest consumption of steel.

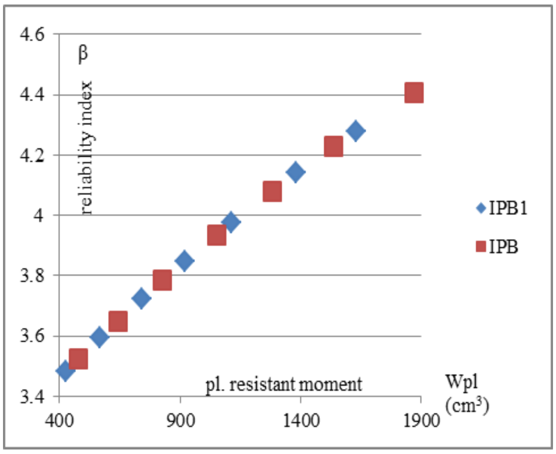

Figure 4. Sway-beam fracture mechanism for different profiles in columns and INP 300 in beams.

Based on the data obtained on profiles which satisfy the sway and the beam mechanism we will take into the consideration two cases. In the first case we will take a constant cross-section of the beam girder while our steel profile in the columns will be changeable but in the second case we will take into the account the reverse situation. Both these situations are shown in the following diagrams Fig. 4 and Fig. 5.

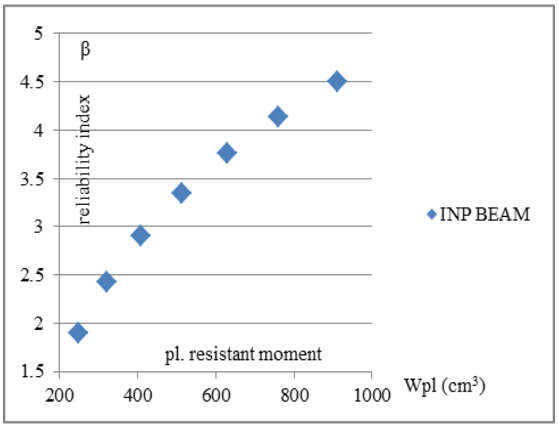

Figure 5. Sway-beam fracture mechanism for different INP profiles in beams and IPB1 300 in the columns

TABLE V. COMPARISON OF DIFFERENT FRACTURE MECHANISMS

Types of fracture mechanism	The probability of fracture for the return period T= 5 years	Reliability index for the period T=5 years	Reliability index for the period T=50years
Beam mechanism	$2.816*10^{-7}$	5.00	4.54
Sway mechanism	$2.363*10^{-4}$	3.50	2.83
Combined mechanism	$1.737*10^{-6}$	4.64	4.14

In the following Table V we will discuss the probability of the fracture for the beam, sway and combined fracture mechanism and for the beams with adopted INP 300 and for the columns which are defined with IPB 300 profile.

V. CONCLUSION

Previous analysis of the reliability has considered one three-storey steel frame that is dimensioned according to the EC3 [8] regulations and with the help of software Radimpex Tower taking into the account only I steel profiles. Of the load on this frame, in addition to the permanent and variable it is taken into the account the horizontal load from the wind and all with the recommendations [3].

Based on the different fracture mechanisms and different I profiles it is got a different reliability indexes which are arranged with a targeted reliability index 3.8 for the returned period of the construction of 50 years. The table clearly shows that the lowest reliability index is for the sway mechanism, what is understandable considering that the first plastic joints in this frame will be created in the places of the maximum impact that is on the place of the biggest bending moment and that is in the fixing of the frame. For the other sway mechanisms that is for the appearance of the plastic joints on the columns of the first and the second floor it is obtained reliability indexes from 4.37 and 6.91 for the return period of 50 years.

If we look at the beam failure mechanism it is best to adopt the profile that meets the prescribed reliability index for a period of fifty years and at a same time consider about the weight of the girders because of the reasons of economy as well as for its own weight of the girder. When designing these structures in addition to the above mentioned criteria, it is clear that such a profile is adopted in relation to the very project task that includes the permitted building height, which depends also on the adopted height of these girders. Optimal girders that are recommended in this case if we take into consideration their weight but not the height are as well as the resistance to bending around the dominant axis of inertia which are INP 300, IPE 300 or IPE 330.

In the columns are certainly preferable one of the IPB girders with respect that the buckling is performed around both of the axis of these girders. And in this case it is recommended the IPB1 340 and bigger with respect to the above table where it is not met the reliability index of 3.8 for the limit state of the capacity for the period of 50 years. The most unfavorable case is the sway mechanism GB3 where the plastic hinges occur in the columns of the ground floor. This case of creating the sway mechanism is avoided a lot and it is known as the case of “the flexible or soft ground floor”.

The most preferred mechanisms are certainly the combined fracture mechanisms in which besides the plastic hinges in the columns they also occur in the beams.

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