Numerical Study on Large Span Pre-Stressed Modified Warren Truss

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***Abstract*—This paper deals with the analysis and design of long-span pre-stressed modified warren trusses under gravity load. An in-house finite element code is developed in MATLAB using the beam element and the trusses are analyzed. The steel trusses are designed using rectangular hollow sections and their linear buckling loads are checked. The advantages of pre-stressing, curvature and additional layers of parallel chord on the member force, vertical deflection, bearing movement and steel consumption of such trusses are studied.**

***Index Terms*—modified warren truss, pre-stressing, up-camber, layers, buckling analysis**

I. Introduction

Parallel chord trusses are increasingly used in pedestrian bridges, highway and railway bridges and large-span flat roofs of auditoriums, stadiums, gymnasiums, industrial buildings and other structures. Commonly used flat truss configurations are *Warren truss, modified Warren truss, Howe truss, Pratt truss*, and *K-truss*. Several research works are reported in the literature over the decades to examine the relative performance [1]-[4] and optimization [5] of different trusses under different static or dynamic lateral loads. It was observed that the *modified Warren truss* is economical beyond span-length of 40 m [1]-[3]. The effects of secondary stresses due to connection rigidity of the members were studied by Smith [4]. Vibration and buckling analyses of several trusses are also available in the literature considering as pin jointed or continuous frame members [6]-[8]. The member forces in such trusses increases rapidly with the increase of span-length and lateral load, which introduces complexities in the design of members and their connections.

The Rectangular Hollow Sections (RHS) with higher flexural stiffness and torsional rigidity are widely used to support higher loads in truss bridges [9], [10]. Such RHS members are either connected by gusset plates or through monolithic welding [11]-[14].

In an attempt to reduce the tensile forces in the bottom parallel chord, pre-stressing cables are used [3], [15]. However, for spans more than 100*m*, *arch-type trusses* and *cable suspension trusses* are preferred to reduce compressive force from top parallel chord [9], [10], [16] and [17].

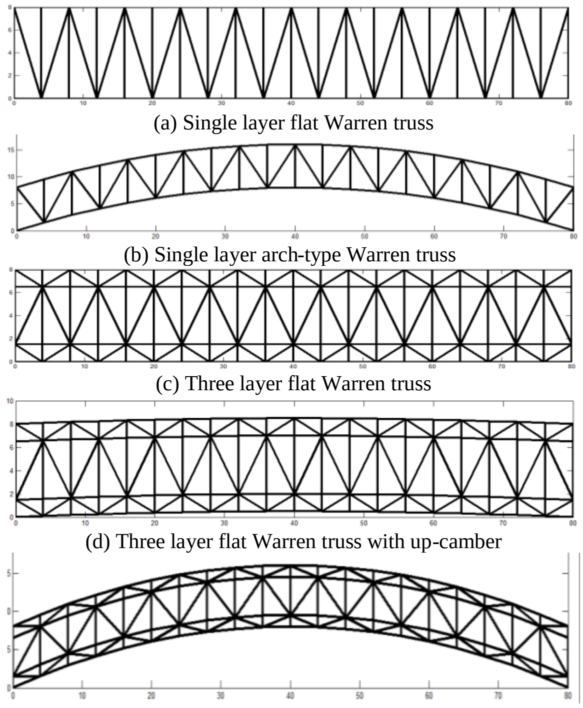
(a) Single layer flat Warren truss
(b) Single layer arch-type Warren truss
(c) Three layer flat Warren truss
(d) Three layer flat Warren truss with up-camber
(e) Three layer arch-type Warren truss

Figure 1. Modified Warren truss of span 80 m and height 8 m

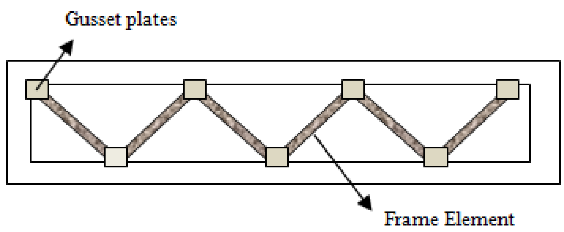

Figure 2. Truss members

Several review articles on existing bridges and roofs demonstrate the efficiency of different long-span bridge trusses. However, the design of large-span flat roof trusses with restricted bearing movement and transverse displacement within limited resources (steel consumption) is a challenge to the designer. It is observed that mostly single-layer (Fig. 1a-b) are used in bridges and roof. Agrawal and Singha [18] have analyzed multi-layer trusses using bar elements. This paper attempts to examine the effectiveness of introducing additional

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parallel chords (Fig. 1d-f), curvature and pre-stressing cables in medium-to-large span *modified warren trusses under static loads*. The truss members are modeled with two-node beam elements and the joints are assumed to be either pinned or fixed (monolithic) as shown in Fig. 2. The trusses are analyzed and are designed with steel hollow rectangular sections to compare their relative advantages.

II. Finite Element Analysis of Frame Element

Two noded beam element (members are connected by rigid joints) with three degrees of freedom (u, w and φ) per node is used to model the members of the truss. Here, u is the axial displacement; w is the transverse displacement and φ is rotation of the node. The axial and transverse displacement components within the element of length "a" may be expressed as:

$$u=\left\{1-\frac{x}{a}\right\}u_1+\left\{\frac{x}{a}\right\}u_2 \tag{1}$$

$$w=N_1w_1+N_2\theta_1+N_3w_2+N_4\theta_2=[N]\{\delta_e\} \tag{2}$$

where,

$$N_1=\left[1-\frac{3x^2}{a^2}+\frac{2x^3}{a^3}\right]; \qquad N_2=\left[x-\frac{2x^2}{a}+\frac{x^3}{a^2}\right];$$

$$N_3=\left[\frac{3x^2}{a^2}-\frac{2x^3}{a^3}\right]; \qquad N_4=\left[\frac{x^3}{a^2}-\frac{x^2}{a}\right]$$

$$\{\delta_e\}=\{u_1\,w_1\,\varphi_1\,u_2\,w_2\,\varphi_2\}^T$$

The derivatives of axial and transverse displacements may be written as

$$\frac{\partial u}{\partial x}=[B_1]\{\delta_e\}$$

$$\frac{\partial w}{\partial x}=[G]\{\delta_e\}; \quad \frac{\partial^2 w}{\partial x^2}=[B_2]\{\delta_e\} \tag{3}$$

Using Euler-Bernoulli's beam theory the element level stiffness matrix [K_e] and geometric stiffness matrix [K_{GM}] may be written as:

$$K_e=\int B_1^T[EA]B_1\,dA+\int B_2^T[EI]B_2\,dA \tag{4}$$

$$K_{GM}=\int G^TPG\,dA \tag{5}$$

where, E is the modulus of elasticity; A is the cross-sectional area of the member; I is the second moment of area (moment of inertia), P is the pre-buckling axial stress resultant due to unit external load.

The displacement vector, element stiffness matrix and geometric stiffness matrix may be transformed to global coordinate system by the following relationship

$$\{\delta_e\}^G=[T]\{\delta_e\} \tag{6}$$

$$[K_e]^G=[T]^T[K_e][T] \tag{7}$$

$$[K_{GM}]^G=[T]^T[K_e][T] \tag{8}$$

where the transformation matrix {T} transforms the local displacement components (axial and transverse) to global displacement components (horizontal and vertical) and may be written as:

$$T=\begin{bmatrix} l & m & 0 & 0 & 0 & 0\\ -m & l & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & l & m & 0\\ 0 & 0 & 0 & -m & l & 0\\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

where θ is the angle between the member and the global X-axis and $l=\cos\theta$, $m=\sin\theta$

The finite element equations of static equilibrium of the truss may be written as

$$[K]\{\delta\}=\{F\} \tag{10}$$

where, {δ} is the displacement vectors consisting the horizontal and vertical displacement of nodes (joints of truss); {F} is the load vector.

The buckling load may be obtained by solving the following eigen-value problem,

$$[K_G-\lambda K_{GM}]\{\delta\}=0 \tag{11}$$

where, the eigen-value λ is the global buckling load.

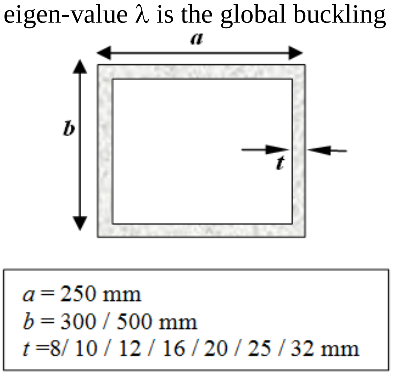


Figure 3. Rectangular hollow cross section (E = 200 $\times 10^6$ kN /m^2, density 78.5 kN/m^3 and yield stress σ_y = 310 $\times 10^3$ kN /m^2).

III. Design of Truss Elements

Static analysis is carried out on five types of steel modified warren trusses (shown in Fig. 1) of two different spans (80 m and 100 m) subjected to a uniformly distributed load of 50 kN/m. The overall height of 100 m truss is 10 m, while the height of 80 m truss is 8 m. One end of the truss is simply supported, while the other end is roller supported. Hollow rectangular sections with width "a" and depth "b", as shown in Fig. 3 are used to design the members. The dimensions of the members (a, b) are fixed at the beginning, while thicknesses "t" of the members are evaluated in the design process. The pre-stressed cables of length 79.95 m and 99.95 m and cross-section of area 50 cm^2 and 70 cm^2 are used for 80 m and 100 m span of trusses.

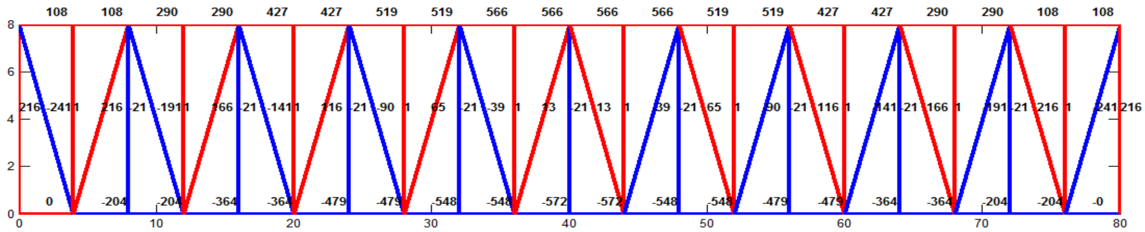
***For single layer of modified warren* truss:** Width (a) of the member is 250 mm, depth (b) of top and bottom parallel chord is 500 m, while the depths of inclined and vertical members are 300 mm and 250 mm respectively.

***For three layer of modified warren* truss:** Width (a) of the member is 250 mm, depth (b) of top and bottom parallel chord is 400 m while the depth of other members is 300 mm and 250 mm.

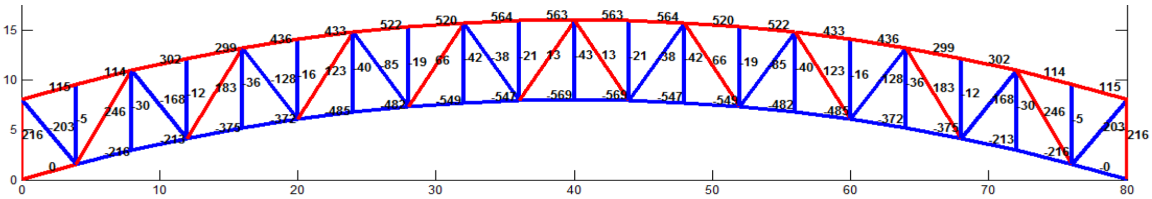
Equation (1) is solved for nodal displacements and the member forces (F_m) are calculated. The member thickness is selected from the available dimensions (8/ 10 / 12 / 16 / 20 / 25 / 32 mm) based on the minimum cross-sectional area requirement

$$A_{\min} = \frac{F_m}{0.6\sigma_y} \tag{12}$$

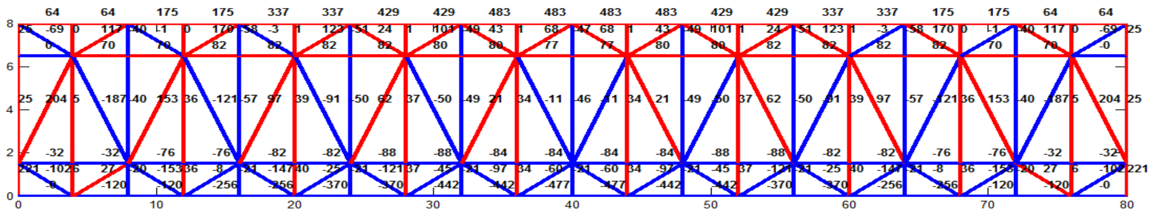
After member thicknesses are designed, the self-weight is added to the dead load and the analysis is repeated to evaluate member force and design is performed again.



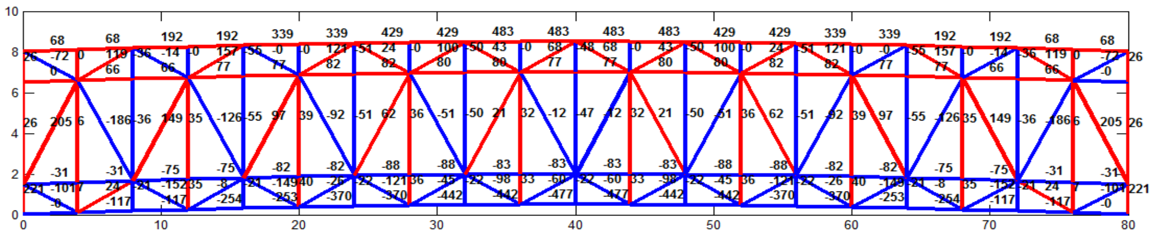
(a) Single layer Warren truss with up-camber of 0.5m



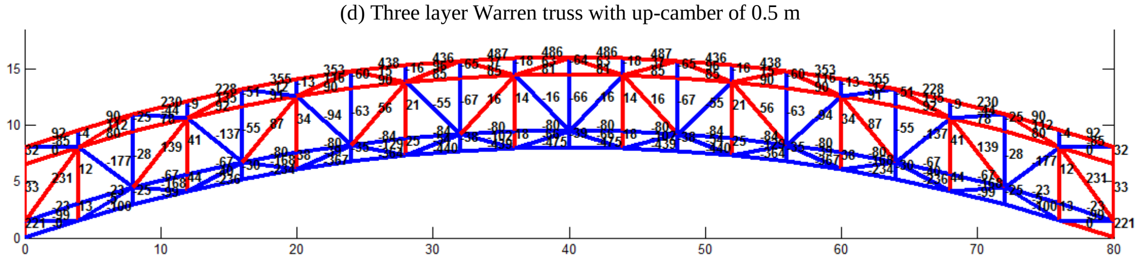
(b) Single layer arch-type Warren truss (up-camber 8 m)



(c) Three layer flat Warren truss



(d) Three layer Warren truss with up-camber of 0.5 m



(e) Three layer Warren truss with up-camber of 8m

Figure 4. Member forces of modified warren truss of span 80 m and height 8 m

Member forces of 80 m long single-layer and three-layer warren type trusses with or without curvature are presented in Fig. 4, when the truss is subjected to a superimposed load of 50kN/m. The maximum tensile and compressive forces in the members are also listed in Table I. The maximum member force in the bottom tension layer of the single-layer and three-layer 80 m long flat trusses are 570 ton and 477 ton respectively, which marginally reduces for the arch-type of trusses. The member forces further reduces to 443 ton and 349 to when a prestressing cable of area 50 cm^2 is provided at the bottom tension layer. It is observed that the member forces of three-layer trusses are less compared to single-layer trusses. The requirement of steel quantity, vertical deflection and horizontal bearing movements are also listed in Table I.

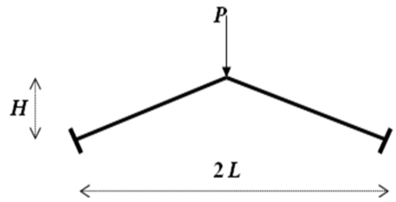
TABLE I. EFFECT OF PRE-STRESSED CABLE (CROSS-SECTION AREA 50 CM^2) AND LENGTH 79.95 M IN DESIGN OF PARALLEL CHORD TRUSSES OF SPAN 80 M, HEIGHT 8.0 M SUBJECTED TO A DISTRIBUTED LOAD OF 50KN/M

Modified Warren truss	Max Weight (ton)	Deflection (cm)	Bearing Movement (cm)	Member Forces Tension (Ton)	Member Forces Compression (Ton)
Single Layer Flat truss					
Without Pre-stressing	54	23	6	570	565
With Pre-stressing	53	23	5	443	560
Single Layer with up-camber					
Without Pre-stressing	54	23	7	572	566
With Pre-stressing	52	23	5	427	551
Pre-stressed cable is in between mid span	51	23	7		
Single Layer with arch					
Without Pre-stressing	53	25	20	570	565
With Prestressing	40	19	11	137	338
Pre-stressed cable is in between mid span	49	23	19		
Three-Layers Flat truss					
Without Pre-stressing	62	21	6	477	483
With Pre-stressing	61	21	5	349	456
Three- Layers with up-camber					
Without Pre-stressing	62	21	7	477	483
With Pre-stressing	61	21	5	339	449
Pre-stressed cable is in between mid span	59	21	6		
Three- Layers with arch					
Without Pre-stressing	63	22	18	475	485
With Prestressing	56	16	10	128	254
Pre-stressed cable is in between mid span	59	21	17		

TABLE II. EFFECT OF PRE-STRESSED CABLE (CROSS-SECTION AREA 70 CM^2) AND LENGTH 99.95 M IN DESIGN OF PARALLEL CHORD TRUSSES OF SPAN 100 M, HEIGHT 10.0 M SUBJECTED TO A DISTRIBUTED LOAD OF 50KN/M

Modified Warren truss	Max Weight (ton)	Deflection (cm)	Bearing Movement (cm)	Member Forces Tension (Ton)	Member Forces Compression (Ton)
Single-layers Flat truss					
Without Pre-stressing	84	29	8	736	729
With Pre-stressing	84	29	6	574	721
Pre-stressed cable is in between mid span	81	29	7		
Three –layer truss					
Without Pre-stressing	91	27	7	599	605
With Pre-stressing	92	27	6	496	602
Pre-stressed cable is in between mid span	89	26	7		
Single-layers with up-camber					
Without Pre-stressing	84	29	8	736	729
With Pre-stressing	84	29	7	552	708
Pre-stressed cable is in between mid span	81	29	8		
Three –layer truss with up-camber					
Without Pre-stressing	91	27	8	598	605
With Prestressing	91	27	6	424	561
Pre-stressed cable is in between mid span	88	27	8		
Single-layers with arch					
Without Pre-stressing	84	31	24	737	730
With Pre-stressing	62	23	13	182	435
Pre-stressed cable is in between mid span	75	29	23		
Three –layer truss with arch					
Without Pre-stressing	94	28	22	596	610
With Prestressing	78	21	12	128	318
Pre-stressed cable is in between mid span	89	26	7		

The analysis and design procedure is performed for 100 m long 10 m height warren type of truss and the corresponding results are presented in Table II. Similar observations are made in Table I and Table II, while evaluating the relative advantages of different truss configurations.



E = 100000, A = 84.16, I = 265.74, L = 328.56, H = 38.6

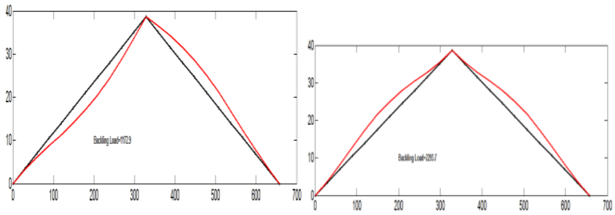


Figure 5. Buckling analysis of toggle frame.

IV. Buckling Analysis

At the beginning, buckling analysis of toggle frame (Fig. 5) is carried out to validate the present finite element code. The dimensions and material properties of the two member frame are taken from Ref [19] and shown in Fig. 5. Each member of toggle frame is discretized into six beam elements and the buckling loads are calculated from equation (11). The buckling modes are also shown in the figure. The first and second buckling loads from the linear analysis are found to be 1172.9 N and 2293.7 N while, the buckling loads reported in Ref [19] are 1113.65 N and 2044.52 N respectively.

Thereafter, same buckling analysis is repeated with single-layer and three-layer modified warren trusses of span length 100 m and height 10 m. The joints are either assumed to be perfectly rigid or perfected pinned. Analysis is performed for flat trusses as well as curved trusses. Mode shapes of modified warren truss and the buckling loads are plotted in Fig. 6.

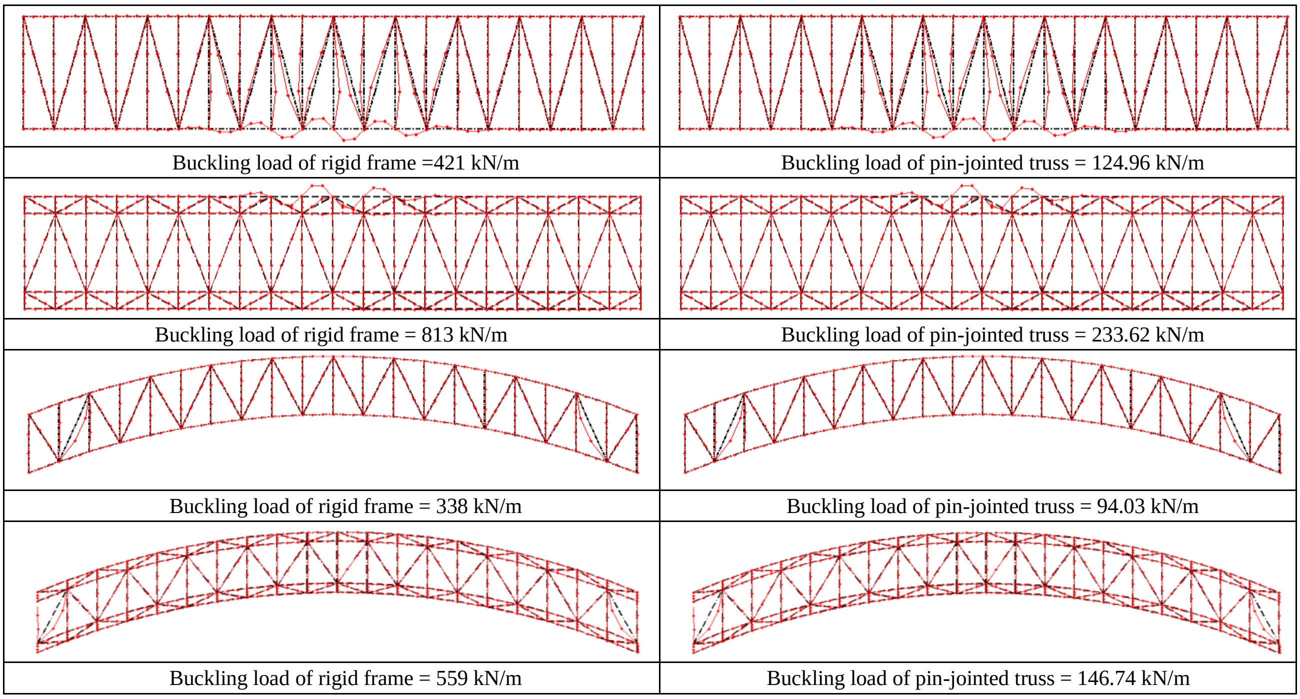


Figure 6. Buckling mode shapes of modified warren truss

V. Conclusions

From the limited parametric study on the single-layer and three-layer warren type of steel trusses, the following observations are made:

- For 100 m long truss, steel consumption of three-layered trusses is marginally more than single layer trusses. But the deflection and bearing movement of three-layer trusses is less (7 % to 12.5 %) compared to single layer trusses.
- The use of pre-stressing cable at the bottom chord is beneficial. Pre-stressing cables in curved trusses significantly decreases the steel consumption. For the particular cases of 100 m long single-layer modified warren truss, the requirement of steel reduces by 24.1%, while the vertical deflection reduces by 23%.
- The higher flexural stiffness of hollow rectangular sections and assumed perfect monolithic joints between the members has ensured safety against member buckling. Buckling load of three-layered truss is nearly double of the corresponding single layer truss
- Arch type of truss has comparatively lower buckling load then flat trusses.

References

[1] B. Schmits, “Design consideration of parallel chord one way steel trusses,” MS. Thesis, Department of Architectural Engineering, College of Engineering, Kansas State University, 2008.

[2] J. Heath, "Economical design consideration for one way 300 foot span, steel parallel top and bottom chord warren trusses, " M.S. thesis, Department of Architectural Engineering College of Engineering, Kansas State University, 2012.

[3] S. Agrawal, "Analysis and design of long span parallel chord trusses," M.Tech. thesis, Department of Applied Mechanics, Indian Institute of Technology Delhi, 2015.

[4] M. C. Smith, "An analysis of secondary stresses in steel parallel chord pratt trusses M.S. thesis, Department of Architectural Engineering, Kansas State University, 2009.

[5] H. Rahami, A. Kaveh, and Y. Gholipour, "Sizing geometry and topology optimization of trusses via force method and genetic algorithm," *Engineering Structures*, vol. 30, pp. 2360-2369, 2008.

[6] S. Pradelok, "Test of local vibration in the node of truss bridge," *Architecture Civil Engineering Environment*, pp. 61-68, 2012.

[7] P. Dumonteil, "Inplane buckling of trusses," *Can. J. Civ. Eng.*, vol. 16, pp. 504-518, 1989.

[8] W. Dunaiski, "Investigation into the effective length of web compression element of parallel chord trusses," M.S. thesis, Stellenbosch University, 2008.

[9] N. Hu, G. L. Dai, B Yan, and K. Liu, "Recent development of design and construction of medium and long span high-speed railway bridges in China," *Engineering Structures*, pp. 233-241, 2014.

[10] J. Mason and K. Ghavami, "Development in brazilian steel bridge construction," *Journal of Construct Steel Research*, vol. 28, pp. 81-100, 1994.

[11] J. A. Packer, J. Wardenier, X. L. Zhao, G. J. van. der. Vegte, and Y. Kurobane, "Design guide for Rectangular Hollow Section (RHS) joints under predominantly static loading," *International Committee for the Development and Study of Tubular Structures*, Second Edition, 2009.

[12] R. M. Korol, H. S. Mitri, and F. A. Mirza, "Finite element model of RHS double chord k-joint," *Journal of Structural Engineering*, vol. 115, no. 5, pp. 1038-1053, 1989.

[13] D. Timosidis and S. J. Pantazopoulou, "Experimental evaluation of bridge monolithic joints under seismic loading," in *Proc. 14th World Conference on Seismic Loading, Graduate Research Engineer*, Demokritus University of Thrace, 2008.

[14] B. Cheng, Q. Qian, and H. Sun, "Steel truss bridges with welded box-section members and bowknot integral joints, Part I: Linear and non-linear analysis," *Journal of Constructional Steel Research*, vol. 80, pp. 465-474, 2013.

[15] P. Albrecth and A. Lenwari, "Design of pre-stressing tendons for strengthening steel truss bridges," *Journal of Bridge Engineering*, vol. 13, no. 5, pp. 449-454, 2008.

[16] A. Bensalem, A. Sibbald, and C. A. Fairfield, "The use of dynamics characteristic for the optimum use of arch bridges," *Computers and Structures*, vol. 68, pp. 461-472, 1998.

[17] M. A. Bradford, B. Uy, and Y. L. Pi, "In plane elastic stability of arches under a central concentrated, load," *Journal of Engineering Mechanics*, vol. 128, pp. 710-719, 2002.

[18] S. Agrawal and M. K. Singha, "A comparative study on long span 2-d parallel chord trusses," in *Proc. Indian National Conference on Applied Mechanics*, IIT Delhi, July13-15, 2015.

[19] S. Lopez, "Post critical analysis of structures with a non linear pre-buckling state in the presence of imperfections," *Comput. Methods Appl. Mech. Eng*., vol. 191, pp. 4421-4440, 2002.

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