A Comparative Study Between Least Square Support Vector Machine (Lssvm) and Multivariate Adaptive Regression Spline (Mars) Methods for the Measurement of Load Storing Capacity of Driven Piles in Cohesion Less Soil

Mohit Chugh[1], Shraddha Satish Thumsi[2] and Vivek Keshri[1]
[1]School of Mechanical and Building Sciences, VIT University, Vellore, Tamil Nadu, India
[2]School of Computer Science and Engineering VIT University Vellore, Tamil Nadu, India
Email: mohit.chugh123@gmail.com

***Abstract*—Many investigations have been done in last few years to predict the load bearing capacity of driven piles in cohesion less soil .There are many factors on which the load bearing capacity of pile depends, the calculation involved in load bearing capacity of a pile is very complex in general. Hence some assumptions are made in the calculation of load bearing capacity which either oversimplify the problem or are considered improperly, which causes increase in the error in measured pile load bearing capacity. In this paper, LEAST SQUARE SUPPORT VECTOR MACHINE (L.S.S.V.M) method and MULTIVARIATE ADAPTIVE REGRESSIONS SPLINE (Mars) are used to predict load bearing capacity of driven piles in cohesion less soil by mat lab software and the results obtained from both the methods are compared with the actual measured capacities.**

***Index Terms*—cohesion less soil, load bearing capacity, lssvm method, mars method**

I. Introduction

Deep foundation is used in case the soil at or near the ground surface is not capable of supporting the structure, hence the load is to be transferred to shallower strata. Pile foundation is the most common type of deep foundation. The pile foundations are generally adopted where soil has low bearing capacity or proper bearing stratum is not available at shallow depth and is also used when heavy load is applied, where shallow foundation may not be feasible.

There are several types of methods, both experimental and theoretical to calculate the load-bearing capacity of piles but the mechanism involved in them is not so correct, especially in case of cohesion less soil driven piles. The possible aspects that affect the behavior of the pile includes stress-strain history of the soil, the soil fabric installation effects, compressibility and the difficulty of obtaining undisturbed soil samples.

Various in situ tests such as the cone penetration test, the pressure meter and the standard penetration test are being used to overcome the complications causing the problems but these tests reflect natural soil conditions up to a limited extent only i.e. there are few drawbacks associated with each of these tests, for example, in the cone penetration test performed in fine grained soil containing granular inclusion, there is sharp reduction in pore water pressure and also the depth of penetration is limited to 150 to 200 feet.

There are many empirical formulae developed between pile capacity and soil parameters, in both end-bearing as well as friction piles so as to overcome the limitations obtained in the in situ tests, for providing quick but sufficient accuracy in estimates of pile capacity. Several empirical methods are proposed by Meyerhof Coyle and Castello (1981), the American Petroleum Institute (RP2A 1984, 1991) and Randolph (1985, 1994).In these methods ,several factors like the residual stresses effect, stress history and actual soil parameters that exists after pile driving are either oversimplified or their effects have not been considered properly. Therefore there is a lot of inconsistency between the physical processes that dictate actual pile capacity and the designed guidelines of these methods for example Meyerhof's analysis for deep footings gives the value of bearing capacity much greater than Terzaghi's analysis. Thus there is a need for an alternative method in order to overcome the uncertainties in predicting pile capacities.

Recently, LEAST SQUARE SUPPORT VECTOR MACHINE (L.S.S.V.M) method and MULTIVARIATE ADAPTIVE REGRESSIONS SPLINE (Mars) have been successfully applied in geotechnical engineering for example LSSVM is used for determination of evaporation losses in reservoir [1] and MARS is used for prediction of elastic modulus of jointed rock mass [2].

LEAST SQUARE SUPPORT VECTOR MACHINE (L.S.S.V.M) method ,contains sets of related supervised learning methods which are used for analyzing the data

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and recognizing pattern and hence classification and regression analysis of data is done. The solutions are derived by solving a set of linear equations instead of a complex quadratic programming .

MULTIVARIATE ADAPTIVE REGRESSIONS SPLINE (M.A.R.S) method is another method of regression, non-parametric approach which models non linarites and interactions between variables automatically and is also used to estimate general functions of high dimensional arguments.

II. LEAST SQUARE SUPPORT VECTOR MACHINE (L.S.S.V.M)

An alternate formulation of SVM regression (Vapnik and Lerner, 1963), LEAST SQUARE SUPPORT VECTOR MACHINE (L.S.S.V.M) method proposed by Suykens et al (2002) . A training set of N data points $\{x_k, y_k\}_{k=1}^{N}$ with input data $x_k \in R^N$ and output $y_k \in r$ where R^N is the N-dimensional vector space and r is the one-dimensional vector space is considered. In this study, diameter of pile (D), embedded length of pile (L), load eccentricity (e) and shear strength of clay (C_u) are used as input variables for the LSSVM model. The ultimate bearing capacity of driven pile in clay is the output of LSSVM model. So, in this study, $x = [D, L, e, C_u]$ and $y = Q$. In feature space, LSSVM models is expressed as

$$y(x) = w^T \varphi(x) + b \tag{1}$$

where the nonlinear mapping $\varphi(.)$ maps the input data into a higher dimensional feature space; $w \in R^n$; $b \in r$; w = an adjustable weight vector; b = the scalar threshold.

In LSSVM, for estimation of function, the formulation of following optimization problem is done:

Minimize: $\frac{1}{2} w^T w + \gamma \frac{1}{2} \sum_{k=1}^{N} e_k^2$

Subjected to:

$$y(x) = w^T \varphi(x_k) + b + e_k, \text{ k=1,…,N.} \tag{2}$$

where is the regularization parameter, determining the trade-off between the fitting error minimization and smoothness and e_k is error variable.

The Lagrangian $L(w, b, e; \alpha)$ for the above optimization problem (2) is

$$L(w,b,e;\alpha) = \frac{1}{2} w^T w + \gamma \frac{1}{2} \sum_{i=1}^{N} e_i^2 - \sum_{i=1}^{N} \alpha_i \{ w^T \varphi(x_i) + b + e_i - y_i \} \tag{3}$$

with k Lagrange multipliers. The conditions for optimality are given by

$$\frac{\partial L}{\partial w} = 0 \Rightarrow w = \sum_{k=1}^{N} \alpha_k \varphi(x_k)$$

$$\frac{\partial L}{\partial b} = 0 \Rightarrow \sum_{k=1}^{N} \alpha_k = 0$$

$$\frac{\partial L}{\partial e_k} = 0 \Rightarrow \alpha_k = \gamma e_k, \text{ k=1,…,N.}$$

$$\frac{\partial L}{\partial \alpha_k} = 0 \Rightarrow w^T \varphi(x_k) + b + e_k - y_k = 0, \text{ k=1,…,N.} \tag{4}$$

After elimination of e_k and w, the solution is given by the following set of linear equations

$$\begin{bmatrix} 0 & 1^T \\ 1 & \Omega + \gamma^{-1} I \end{bmatrix} \begin{bmatrix} b \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 \\ y \end{bmatrix} \tag{5}$$

where $y = [y_1, ..., y_N], 1 = [1, ..., 1]$, $\alpha = [\alpha_1, ..., \alpha_N]$ and Mercer's theorem (Vapnik, 1998; Smola and Scholkopf, 1998), is applied within the Ω matrix, $\Omega = \varphi(x_k)^T \varphi(x_l) = k(x_k, x_l)$, k,l=1,…,N

where $k(x_k, x_l)$ is the kernel function. Choosing ensures the matrix $\Phi = \begin{bmatrix} 0 & 1^T \\ 1 & \Omega + \gamma^{-1} I \end{bmatrix}$ is invertible. Then the analytical of and b is given by

$$\begin{bmatrix} b \\ \alpha \end{bmatrix} = \Phi^{-1} \begin{bmatrix} 0 \\ y \end{bmatrix} \tag{6}$$

Radial basis function has been used as a kernel in this analysis. Radial basis function is given by

$$K(x_k, x_l) = \exp\left\{ -\frac{(x_k - x_l)(x_k - x_l)^T}{2\sigma^2} \right\} \text{ k,l=1,…,N} \tag{7}$$

where is the width of radial basis function.

The resulting LSSVM model for Q prediction becomes then

$$y(x) = \sum_{k=1}^{N} \alpha_k K(x, x_k) + b \tag{8}$$

III. MULTIVARIATE ADAPTIVE REGRESSIONS SPLINE (M.A.R.S)

MULTIVARIATE ADAPTIVE REGRESSIONS SPLINE (M.A.R.S) method - Among various Artificial Intelligence methods, this method is much more flexible, innovative, accurate and fast. This technique requires less input parameters and less computational efforts for accurate and quick prediction of output. This method unlike conventional methods can learn complicated relationships easily between the input and output implicitly from sample training datasets. MARS method was developed by statistician Jerrie Friedman as a method for fitting the relationship between predictors and dependent variables, continuous or binary targets [3]. This method presents a non-parametric statistical method for the model development. Its use is obvious from the fact that it automatically searches for non-linearity in the input/output relationship present in a large dataset and hence creates an exclusive model. The MARS algorithm involves a mechanism in which the data with insignificant inputs or which over fits are eliminated. In this method, basically the training datasets are divided into separate regions and each region gets its own regression line.

Since its development in the early 1990s, this method has been successfully applied to various fields notably in monetary management, financial policies, and on targets difficult for conventional methods. Its application involves forecast for economic recession [4], simulation of time series variations using historical records of variables such as monthly rainfall, Carbon dioxide concentration in gas furnace system [5].

In order to build a MARS (MARS) model, training data, including the input variables and the expected output targets, is required. The MARS model split's the training data into several splines on an equivalent interval basis (Friedman, 1991). In each spline, MARS splits the data further into many subgroups and creates several knots, which can be located between different input variables or different intervals in the same input variable, for separating the subgroups.

The MARS model approximates a regression function, called a basis function (BF), using smoothing splines to generally represent the data in each subgroup (Friedman, 1991; Sephton, 2001). Between any two knots, the model can characterize the data either globally or by using linear regression. The BF is unique between any two knots, and is shifted to another BF at each knot (Abraham & Steinberg, 2001a; Friedman, 1991). The two BFs in two adjacent domains of data intersect at the knot to make model outputs continuous (Sephton, 2001). Thus, in contrast with conventional regression algorithms, MARS creates a bended regression line to fit the data from subgroup to subgroup and from one spline to other spline. For evading over-fitting and over-regressing, the shortest distance between two neighboring knots is pre-determined to prevent too few data in a subgroup.

In such models, the dataset is divided into following two groups:

- Training dataset: Using this the data driven model is developed.
- Testing dataset: It determines the performance of the data driven model.

The general form of MARS model can be written as:

$$y = c_0 + \sum_{i=1}^{N} c_i \prod_{j=1}^{K} \left(x_{v(j,i)} \right)$$

where, y is the output variable, c_0 is constant, c_i is vector of coefficients of the non-constant basis functions, $b_{ji}(x_{v(ji)})$ is the truncated power basis function with v (j,i) being the index of the independent variable used in the i^{th} term of the j^{th} product, and K_i is a parameter that limits the order of interactions. In the present study, the input variables are $F_{\mathbf{shaft}}$, $F_{\mathbf{tip}}$, Length, Area, $\mathbf{s'_{tip}}$. The output of the MARS model is Q. Therefore, for the present study, x = [$F_{\mathbf{shaft}}$, $F_{\mathbf{tip}}$, L, A, $\mathbf{s'_{tip}}$] and y = Q. The spline b_{ji} is defined as:

$$b_{ji}(x) = (x - t_{ji})_+^q = \begin{cases} (x - t_{ji})^q, & \text{If } x < t_{ji} \\ 0, & \text{Otherwise} \end{cases}$$

$$b_{ji+1}(x) = (t_{ji} - x)_+^q = \begin{cases} (t_{ji} - x)^q, & \text{If } x < t_{ji} \\ 0, & \text{Otherwise} \end{cases}$$

where t_{ji} = knot of the spline

Basically, there are two steps in MARS model: A forward process and then a backward process. Basic functions are selected to define Eq.(1) in the forward process while in the backward process the worthless basis functions are removed from the model based on the Generalized Cross-Validation(GCV) criterion (Craven and Wahba 1979).

The criterion for GCV is defined as:

$$GCV = \frac{\frac{1}{N}\sum_{i=1}^{N}\left[y_i - f(x_i)\right]^2}{\left[1 - \frac{C(B)}{N}\right]^2}$$

where, N is the number of data, C(B) is a complexity penalty.

It increases as the number of basic function in the model increases as:

$$C(B) = (B+1) + dB$$

where d is a penalty for each basic function involved in the model. It is also known as smoothing parameter. Friedman (1991) provided more details about the selection of the parameter.

IV. Results and Discussion

The data has been collected from M. A. Abu Kiefa [6]. Variation in soil relative density; stress history; geographic locations; and pile types, lengths and diameters are some factors covered in these records. The main data of the selected pile load test and the references of such database in shown in table 1.The database consists of many parameters that are used for the determination of pile capacity. In total, 59 datasets have been used to obtain both the models. The data has been divided into two sub-sets for the formulation such as :

Normalization has been done between 0 to 1 using the equation

$$k_{normalised} = \frac{(k - k\min)}{(kmax - kmin)}$$

where k = any input or output data, k_{min}= minimum value of entire dataset, k_{max}= maximum value of entire dataset and $k_{normalised}$ is the normalised value of the data.

1) A training dataset: This is required for the development of model. 41 out of the 59 data are considered for training dataset.

2) A testing dataset: this is required for estimating the model performance. 18 data are used as testing dataset.

BOTH the models, L.S.S.V.M & M.A.R.S are developed using MATLAB.

The value of coefficient of Correlation(R) has been obtained using the following formula:

$$R = \frac{\sum_{i=1}^{M}\left(Q_{ai}-\bar{Q}_a\right)\left(Q_{pi}-\bar{Q}_p\right)}{\sqrt{\sum_{i=1}^{M}\left(Q_{ai}-\bar{Q}_a\right)}\sqrt{\sum_{i=1}^{M}\left(Q_{pi}-\bar{Q}_p\right)}}$$

where Q_{ai} and Q_{pi} are the actual and predicted values of Q, respectively while $\bar{Q}_a$ and $\bar{Q}_p$ are mean of actual and predicted values corresponding to n patterns.. The value of R should be close to 1 for a good model. The performance of training dataset is shown in Fig. 1. The R value for training dataset is 0.9737 and for testing dataset it is 0.8694. Therefore the input and output relationship has been captured very well for the developed LSSVM model. Now the developed LSSVM performance has been examined for testing dataset. The testing dataset performance has been depicted in Fig. 2. for the testing dataset, the value of R is 0.8681. Hence the developed LSSVM model can be used as a practical tool for the determination of Pile Capacity.

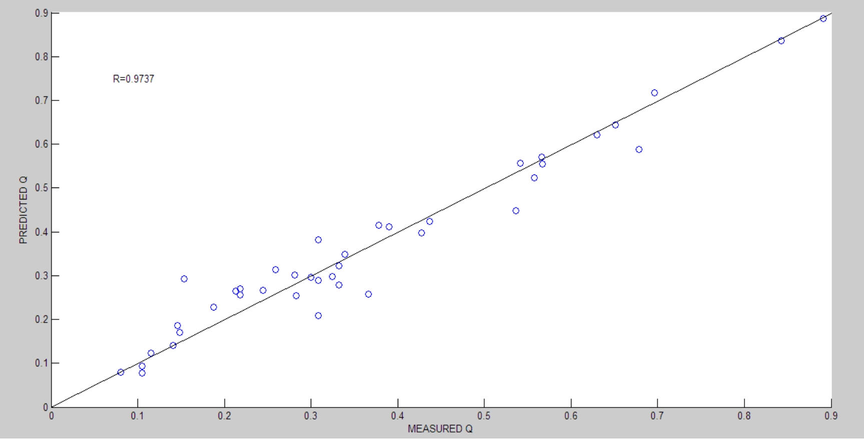

Figure. 1 Performance of training dataset for LSSVM

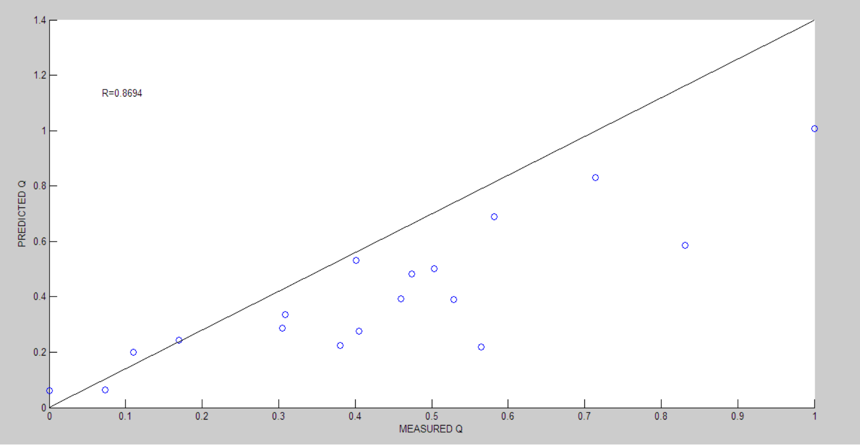

Figure. 2 Performance of testing dataset for LSSVM

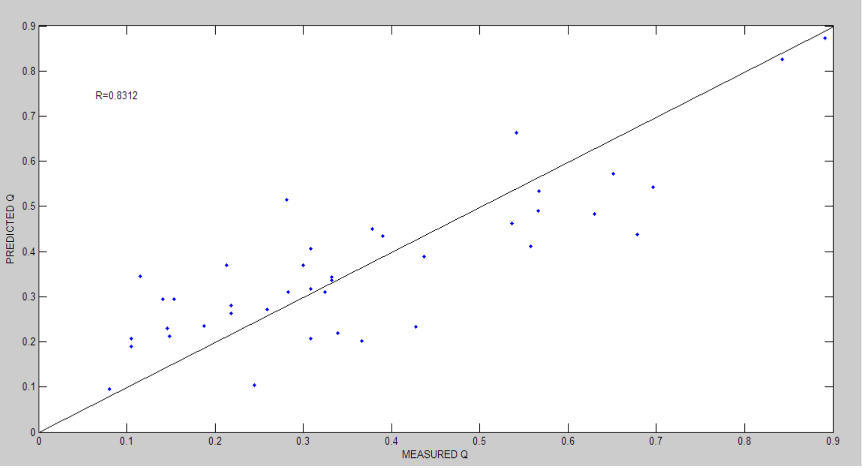

Figure. 3 Performance of training dataset for MARS

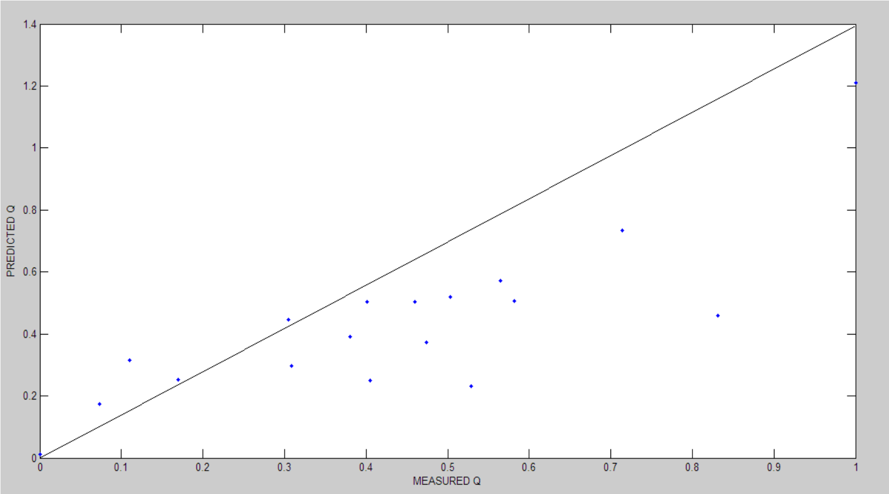

Figure. 4 Performance of testing dataset for MARS

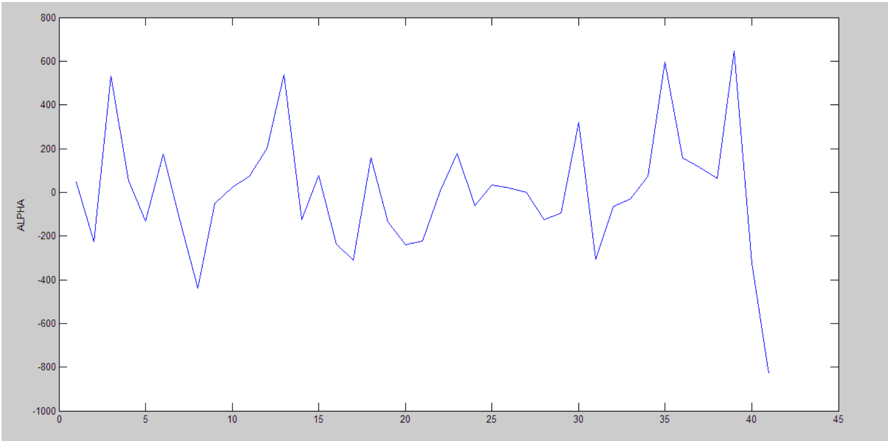

Figure 5. ALPHA

For the MARS model, the value of R for training and testing dataset are 0.8312 and 0.82. As the value of R is close to 1, the developed MARS model can also be used for determination of load bearing capacity of driven piles.. The final equation of prediction of Q of driven piles in cohesionless soil is given by:

$$Q = 0.505 + \sum_{i=1}^{4} a_i B_i$$

Ultimately, 4 Basic functions have been used for the optimization of MARS model as shown in Table I. The values of a_i and B_i are given in Table I. The performance has been assessed by using the coefficient of Correlation(R) formula as given in Eq.1.

The performance of the training and testing dataset (normalised values) has been shown in Fig. 3 and Fig. 4. As the performance of training and testing dataset is almost same, the developed MARS does not show overturning phenomenon.

Basic function, BF_i	Equation	Coefficient, a_i
B_1	max(0, x5 - 0.24327647149)	0.52179547536
B_2	max(0, 0.24327647149 -x5)	-1.72199664654
B_3	max(0, x3 - 0.40503432494)	0.99365841965
B_4	max(0, 0.40503432494 -x3)	-0.31644602368

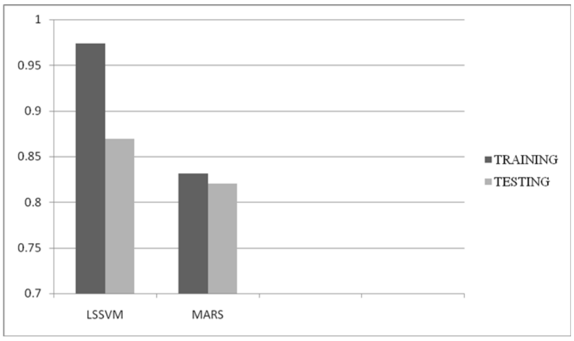

Figure. 6 LSSVM vs. MARS

The results of the LSSVM and MARS model has been compared by using values of coefficient of correlation(R) values in Fig. 5.

V. Conclusion

The implemented LSSVM and MARS models prove that they are effective in measuring the storage capacity of the pile. The graphs prove that within a certain limit of the number of outliers, most of the clustering of dataset has been done effectively for the most part. The complexity of the problem of finding the storage capacity of the pile has been simplified from quadratic to a simpler form, at a good rate of accuracy despite Variation in soil relative density; stress history; geographic locations; and pile types. This proves that the LSSVM and MARS methods indeed have potential for accurate calculations in the related field. Between these two, LSSVM was found to be the better method based on the graphs and nature of clustering that was performed on the data set. Additionally, the value of R factor in LSSVM is closer to 1 than MARS, which is a required parameter for greater accuracy during the calculation of the storage capacity of the pile.

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