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Optimum Design of Lead-Rubber Bearing System Under the Non-Stationary Random Earthquake Ground Motion

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***Abstract*—In this study, we used a non-stationary random earthquake Clough-Penzien model to describe earthquake ground motion. Using stochastic direct integration in combination with an equivalent linear method, we established a solution for the non-stationary response of Lead-Rubber Bearing (LRB) system to a stochastic earthquake. We used two parameters to develop an optimization method for bearing design: the post-yielding stiffness and the normalized yield strength of the isolation bearing. Using the minimization of the maximum energy absorption level of the upper structure subjected to an earthquake as an objective function, and with the constraints that the bearing failure probability is no more than 5% and the second shape factor of the bearing is less than 5, we present a calculation method for the two optimal design parameters. In this optimization process, the Radial Basis Function (RBF) response surface was applied, instead of the implicit objective function and constraints, and a Sequential Quadratic Programming (SQP) algorithm was used to solve the optimization problems.**

***Index Terms*—seismic isolation structure, optimal design; Lead-Core Rubber Bearing (LRB), stochastic analysis**

I. Introduction

By providing a seismic bearing between the building and the ground, a base isolation system can reduce the seismic response of the upper structure and therefore block seismic ground motion from passing into the upper structure. Through decades of application, base isolation has become the most widely used technique for controlling and reducing the seismic responses of structures. Generally, an isolation bearing must have a lower lateral stiffness to prolong the resonance period and reduce the lateral seismic action. In addition, an isolation bearing needs to have appropriate energy dissipation and high restoration ability to avoid excessive bearing displacement and instability. Numerous studies have shown that the mechanical properties of a bearing will greatly affect its seismic abilities. Thus, in recent years, the optimum design of mechanical parameters for isolation bearings has attracted the attention of researchers in a series of studies [1]-[9].

This paper used two parameters to develop an optimization method for bearing design: the post-yielding stiffness and the normalized yield strength of the isolation bearing. Using the minimization of the maximum energy absorption level of the upper structure subjected to an earthquake as an objective function, and with the constraints that the bearing failure probability is no more than 5% and the second shape factor of the bearing is less than 5, a calculation method for the two optimal design parameters was proposed.

II. An Input Seismic Ground Motion Model

A non-stationary Clough-Penzien [10] stochastic seismic model is used to describe earthquake excitation $a_g(t)$:

$$\begin{aligned} &a_g(t) = -\omega_f^2 x_f(t) - 2\xi_f\omega_f\dot{x}_f(t) + \omega_g^2 x_g(t) + 2\xi_g\omega_g\dot{x}_g(t) \\ &\ddot{x}_f(t) + \omega_f^2 x_f(t) + 2\xi_f\omega_f\dot{x}_f(t) = \omega_g^2 x_g(t) + 2\xi_g\omega_g\dot{x}_g(t) \\ &\ddot{x}_g(t) + 2\xi_g\omega_g\dot{x}_g(t) + \omega_g^2 x_g(t) = -a(t)w(t) \end{aligned} \quad (1)$$

In this equation, $x_g(t)$ and $x_f(t)$ are the responses of the filter, ω_f, ω_g are the characteristic frequencies of the filter, ξ_f and ξ_g are the filter damping ratios, $w(t)$ is the white noise when the power spectral intensity is S_0, $a(t)$ is the time modulation function and the formula from Jennings and Housener [11] was adopted:

$$a(t) = \begin{cases} (t/t_1)^2 & 0 \le t \le t_1 \\ 1 & t_1 \le t \le t_2 \\ e^{-c(t-t_2)} & t \ge t_2 \end{cases} \quad (2)$$

The peak acceleration of ground motion is $PGA = 3\sigma_{a_g}$, so the relationship between S_0 nd PGA is [12]

$$S_0 = \frac{0.222}{\pi}\frac{PGA^2\xi_g\xi_f}{\omega_g^2} \times \frac{\left(\omega_g^4 + 4\xi_g\xi_f\omega_g^3\omega_f + 2(-1+2\xi_g^2+2\xi_f^2)\omega_g^2\omega_f^2 + 4\xi_g\xi_f\omega_g\omega_f^3 + \omega_f^4\right)}{((1+4\xi_g^2)\xi_f\omega_g^3 + \xi_g(1+16\xi_g^2\xi_f^2)\omega_g^2\omega_f + 16\xi_g^4\xi_f\omega_g\omega_f^2 + 4\xi_g^3\omega_f^4)} \quad (3)$$

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III. THE MOTION EQUATION

The nonlinear motion equation for the multi-degree-of-freedom lead-rubber bearing system under horizontal seismic excitation $a_g(t)$ is

$$\boldsymbol{M}_s\ddot{\boldsymbol{X}}_s + \boldsymbol{C}_s\dot{\boldsymbol{X}}_s + \boldsymbol{K}_s\boldsymbol{X}_s = -\boldsymbol{M}_s\boldsymbol{I}(a_g + \ddot{x}_b) \quad (4)$$

$$m_b(a_g + \ddot{x}_b) + \sum_{i=1}^{n} m_i(\ddot{x}_i + \ddot{x}_b + a_g) + F_Q(x_b, \dot{x}_b) = 0 \quad (5)$$

where $\boldsymbol{M}_s$,, $\boldsymbol{C}_s$ and $\boldsymbol{K}_s$ are the mass, damping ratio and stiffness matrices of the upper structure, m_i is the mass of the i-th layer, $\boldsymbol{X}_s = [x_1, \cdots, x_n]'$ is the displacement of the upper structure relative to the base, m_b is the mass of the base, $\boldsymbol{I} = [1, \cdots, 1]'$, s the hysteretic restoring force of the lead- rubber bearing and x_b is the displacement of the base relative to the ground.

In this paper, $F_Q(x_b, \dot{x}_b)$ is treated as a bilinear model (dashed line in Fig. 1). Because the pre-yielding stiffness of the LRB is 10 - 15 times its post-yielding stiffness, under the condition of equal hysteresis area, the bilinear restoring force model simplifies to a rigid-plastic model (solid line in Fig. 1). Thus, $F_Q(x_b, \dot{x}_b)$ can be expressed as

$$F_Q(x_b, \dot{x}_b) = N\alpha k_b x_b + N(1-\alpha) f_y \,\mathrm{sign}(\dot{x}_b) \quad (6)$$

where N is the total number of isolation bearings, k_b is the pre-yielding stiffness of the bearing, f_y is the yield force of the bearing and α is the ratio of pre-yielding and post-yielding stiffness.

Based on a random equivalent linearization method, Equation (6) can be replaced with a linear equation:

$$F_Q(x_b, \dot{x}_b) = N\alpha k_b x_b + N(1-\alpha) f_y c_e(t)\dot{x}_b \quad (7)$$

where $c_e(t)$ is the equivalent time-varying damping coefficient, which can be calculated as

$$c_e(t) = \frac{1}{\sigma_{\dot{x}_b}(t)}\sqrt{\frac{2}{\pi}} \quad (8)$$

In Equation (8), $\sigma_{\dot{x}_b}(t)$ is the time-varying standard deviation of $\dot{x}_b$.

Equation (7) is substituted into Equation (5) to yield the following equation:

$$\sum_{i=1}^{n} r_i\ddot{x}_i + \ddot{x}_b + \omega_b^2 x_b + \mu g c_e(t)\dot{x}_b = -a_g(t) \quad (9)$$

In this equation, $r_i = \frac{m_i}{M_z}$, $M_z = \sum_{i=1}^{n} m_i + m_b$ is the total mass of the isolation structure, $\omega_b = \frac{N\alpha k_b}{M_z}$, $\mu = \frac{N(1\text{-}\alpha) f_y}{M_z g}$ is the ratio of yield force to the mass of the isolation structure and two parameters (ω_b and μ) determine the stiffness k_b and yield force f_y of the bearing.

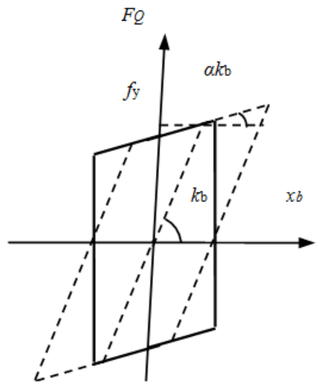


Figure 1. Hysteretic restoring force model of lead-rubber bearing

IV. OPTIMAL BEARING DESIGNS WITH DETERMINISTIC STRUCTURAL PARAMETERS

Based on Equation (9), it can be seen that ω_b and μ determine the post-yielding stiffness and yield force. Therefore, we use ω_b and μ as optimization parameters in this study. The objective function is set to minimize the ratio of the maximum energy absorptions in the upper structure with and without seismic isolation under the influence of an earthquake. The mathematical expression is

Find ω_b, μ

$$\min\ f_{obj}(\omega_b, \mu) = \frac{E_{str}^b}{E_{str}^f} = \frac{\sum_{i=1}^{n} \max_{t\in[0,T]}\left[\sigma_b^2(x_i, t)\right]}{\sum_{i=1}^{n} \max_{t\in[0,T]}\left[\sigma_f^2(x_i, t)\right]} \quad (10)$$

In this equation, $\sigma_b^2(x_i, t)$ is $E[x_i^2(t)]$ when the base is seismically isolated, and $\sigma_f^2(x_i, t)$ is $E[x_i^2(t)]$ when the base is fixed; both values can be calculated by using stochastic direct integration.

While satisfying the above objective function, the following two constraint conditions also need to be satisfied:

Constraint condition 1: The probability of the horizontal displacement of the isolation bearing exceeding the allowable limit under the seismic effect is less than 5%. Mathematically, this is expressed as

$$\boldsymbol{F}^b(\omega_b, \mu) = 1 - \min_{t\in[0,T]} r(t) \le 5\% \quad (11)$$

In this equation, $r(t) = \exp\left\{-\int_0^t 2v_b^t(b,\tau)d\tau\right\}$, and $v_b^t(b,\tau)$ is expressed as follows:[13]

$$v_b^t(b,\tau)=\frac{1}{\pi}\frac{\sigma_{\dot{x}_b}(t)}{\sigma_{x_b}(t)}\left(\frac{1-\exp\left(-\sqrt{\frac{\pi}{2}}\frac{b\left(q(t)\right)^{1/2}}{\sigma_{x_b}(t)}\right)}{\exp\left(\frac{1}{2}\left(\frac{b}{\sigma_{x_b}(t)}\right)^2\right)}\right)v \quad (12)$$

Here, b is the allowable displacement limit. According to the seismic code of China, $b=\min[0.55D, 300\% ntr]$, where D is the bearing diameter, ntr is the total thickness of the rubber in the bearing, $\sigma_{x_b}(t)=\sqrt{E[x_b^2(t)]}$, $\sigma_{\dot{x}_b}(t)=\sqrt{E[\dot{x}_b^2(t)]}$ and q(t) is a bandwidth parameter that can be expressed as follows:

$$q(t)=\sqrt{1-\frac{\lambda_1^2(t)}{\lambda_0(t)\lambda_2(t)}}$$

In this expression, $\lambda_0(t)=E[x_b^2(t)]$, $\lambda_1(t)=E[x_b(t)\dot{x}_b(t)]$, and $\lambda_2(t)=E[\dot{x}_b^2(t)]$.

As shown in the following equation, ntr is related to the second stiffness, $k_2=\alpha k_b$, of a single bearing:

$$Nk_2 = GA/ntr \quad (13)$$

Here, N is the total number of isolation bearings, G is the shear modulus of the rubber, $A=P_d/[\sigma_b]$ is the total area of all bearings, P_d is the total vertical design load, and $[\sigma_b]$ is the allowable stress of the bearing design, usually 10-15MPa.

By substituting $A=P_d/[\sigma_b]$ into Equation (23), the following can be obtained.

$$Nk_2=\frac{GP_d}{ntr[\sigma_b]}$$

Then, after dividing both sides of the above equation by the total mass of the structure, the following equation is obtained:

$$\frac{Nk_2}{M_z}=\frac{GP_d}{ntr\cdot M_z[\sigma_b]}=\frac{GP_d g}{ntr\cdot G_z[\sigma_b]}=\frac{G\beta g}{ntr[\sigma_b]} \quad (14)$$

Here, $G_z=M_z g$ is the representative value of gravity load in the structure, and $\beta=P_d/G_z$ is a number greater than 1.

Because $\frac{Nk_2}{M_z}=\omega_b^2$, Equation (14) can be converted to

$$ntr=\frac{G\beta g}{\omega_b^2[\sigma_b]} \quad (15)$$

Based on Equation (15), it can be seen that b in Equation (12) is a function of ω_b, and when ω_b is determined, the total rubber thickness in the bearing, ntr, is also determined.

Constraint condition 2: To prevent instability under a vertical load, it is required that the second shape factor, S2, of the bearing should be greater than a limiting value m (m usually has a value of 4 - 6):

$$S_2=\frac{D}{ntr}\geq m$$

By substituting Equation (15) into the above equation, we get the following:

$$S_2(\omega_b,\mu)=\frac{D}{ntr}=\frac{D\omega_b^2[\sigma_b]}{G\beta g}\geq m \quad (16)$$

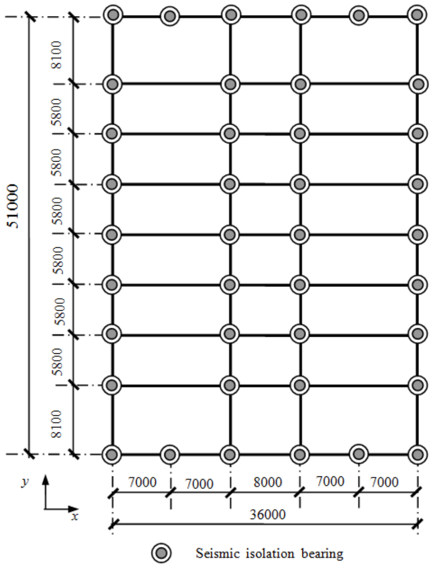
Combining Equations (10), (11) and (16), a mathematical model of optimized bearing parameters can be obtained as follows:

$$\begin{aligned}&Find\quad \omega_b,\mu\\&\min\ f_{obj}(\omega_b,\mu)\\&s.t.\quad \boldsymbol{F}^b(\omega_b,\mu)\leq 5\%\\&\qquad S_2(\omega_b,\mu)\geq m\end{aligned} \quad (17)$$

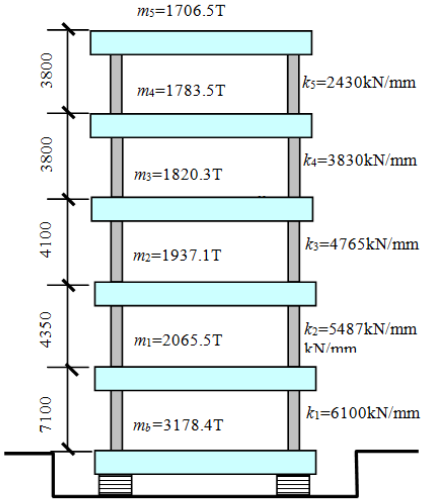
In the process of solving Equation (17), because $f_{obj}(\omega_b,\mu)$ and $\boldsymbol{F}^b(\omega_b,\mu)$ are implicit functions of ω_b,μ, the computational efficiency would be very low if we attempt to solve it directly, resulting in not only long computation times but also non-convergent results. To improve the computational efficiency, we use an RBF response surface to make $f_{obj}(\bullet)$ and $\boldsymbol{F}^b(\bullet)$ explicit [14], and a sequential quadratic programming (SQP) algorithm was used to solve the optimization problems.

V. A Calculation Example

Taking a five-story office building made of reinforced concrete as an example, the plane of the standard floor is rectangular, the short side (x-direction) is 36m and the long side (y-direction) is 51m, as shown in Fig. 2. The building is located on a type II field (as shown in Table I).The earthquake ground motion enters in the x-direction, and the ground motion parameters are listed in Table I. Two conditions are considered for the peak ground acceleration: $PGA=0.5g$ and $PGA=0.8g$ .The basic period of the fixed base in the x-direction is 0.42s, and the limped mass and the inter-layer stiffness of every level are listed in Fig. 2(b).The isolation bearing layout is shown in Fig. 2(a), with a total of 40 lead-core rubber bearings that have a diameter (D) of 0.7m.If the bearing rubber has a shore hardness of 45 degrees, and the shear modulus G is 0.54MPa, then the second shape factor of the bearing is $S_2\geq m=5$.With $\beta=1.3$, the average stress of the bearing is $\sigma=P_d/A=\beta M_z g/(40A_b)=10.6$Mpa (40 is the total number of isolation bearings, A_b is the area of a single bearing).



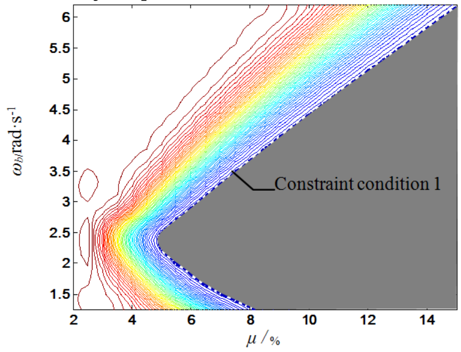
(a) seismic isolation bearing layout



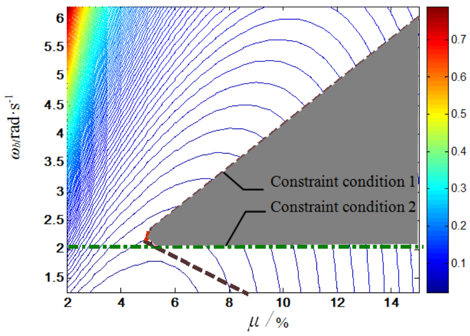
(b) Calculation diagram

Figure 2. Seismic isolation structure

Fig. 3(a) shows the nephogram of constraint condition 1, $\tilde{F}^b(\omega_b, \mu)$, when PGA=0.5g. The thick dashed line indicates the contour line of $F^b(\omega_b, \mu)$ =5% and the shaded area is the region in which constraint condition 1 is satisfied. Fig. 3(b) is the nephogram of the objective function $\tilde{f}_{obj}(\omega_b, \mu)$, the thick dashed line in the figure is the boundary of constraint condition 1. The dotted dashed line is the boundary of constraint condition 2, and the shaded portion is the region where both conditions are satisfied. The ● point is the optimal point of the parameters; the values of the optimized parameters are $[\omega_b^{opt}, \mu^{opt}] = [2.110, 4.776\%]$. It can be seen in the figure that the optimal point is at the boundary of constraint condition 2, where the bearing has a value of S_2=5. Because $S_2 = \dfrac{D}{ntr}$, the total thickness of the rubber is $ntr = D/5 = 0.14m$. In addition, $\mu^{opt} = \dfrac{N(1-\alpha) f_y^{opt}}{M_z g}$, so the optimal yield force of a single bearing is $f_y^{opt} = 150\text{kN}$. When the yield stress of lead is 8.83MPa, we find that the diameter of the lead core is 7.3cm. Fig. 4(a) shows the variation curve of the objective function $f_{obj}(\omega_b^{opt}, \mu)$ with respect to μ when $\omega_b = \omega_b^{opt}$. The figure indicates that the objective function has its minimum value (●point in the figure) when $\mu = \mu^{opt} = 4.776\%$. Fig. 4(b) shows the variation curve of the objective function $f_{obj}(\omega_b, \mu^{opt})$ with respect to ω_b when $\mu = \mu^{opt}$, indicating that $f_{obj}(\cdot)$ monotonously decreases with the decrease in ω_b. However, when $\omega_b < \omega_b^{opt}$, constraint condition 2 is not satisfied (that is, $S_2 < 5$), so the solution does not meet the stability requirement.

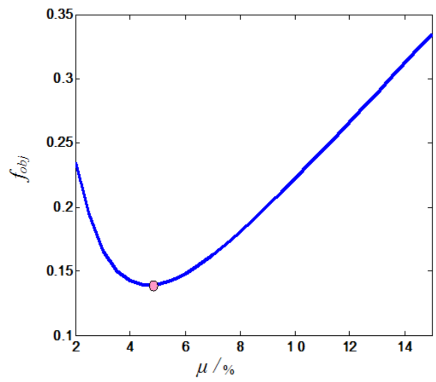


(a)Nephogram of $\tilde{F}^b(\omega_b, \mu)$

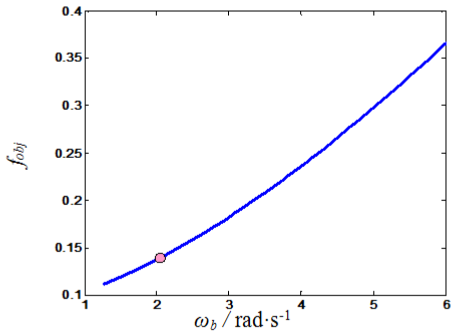


(b)Nephogram of $\tilde{f}_{obj}(\omega_b, \mu)$

Figure 3. Nephograms for $\tilde{F}^b(\omega_b, \mu)$ and $\tilde{f}_{obj}(\omega_b, \mu)$ when PGA=0.5g.



(a)Variation curve of $f_{obj}(\omega_b^{opt}, \mu)$ with respect to μ.



(b) Variation curve of $f_{obj}(\omega_b, \mu^{opt})$ with respect to ω_b.

Figure 4. Variation curves of $f_{obj}(\omega_b^{opt}, \mu)$ with respect to μ and ω_b

VI. Conclusion

In this paper, we proposed an optimization method for mechanical parameters for the design of lead-core rubber bearings system subjected to non-stationary earthquake ground motions. In this method, the post-yielding stiffness and normalized yield force are used as design variables, the minimum value of the maximum energy absorption level of the upper structure during an earthquake is used as an objective function, and the constraint conditions include the probability of bearing failure not exceeding 5% and the second shape factor of the bearing being less than 5. By combining the RBF response surface method and the SQP algorithm, we are able to solve the optimization problem and provide the optimal values for the design variables.

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