Damage is a change introduced into a system that adversely affects current or future performance of that system. Structural Health Monitoring (SHM) is used for detection, localization and extends of damage. This paper is to detect the damage occurred in a structure using time series. It also aims to detect the severity of damage occurred in a structure. Here Mahalanobis distance method is used to detect damage in engineering systems. And thus damage index is obtained corresponding to each sensor. Damage index is found for both simply supported beam and cantilever beams. To prove the accuracy of work experimental data is taken from undamaged and damaged condition. And Damage index is found out. The information obtained from monitoring is generally used to plan and design maintenance, increase the safety, verify hypotheses, reduce uncertainty and to widen the knowledge concerning the structure being monitored.

**Keywords:** Damage index, Mahalanobis distance, Structural health monitoring, Time series

**INTRODUCTION**

Structural health monitoring is a process aimed at providing accurate and in time information concerning structural condition and performance. The information obtained from monitoring is generally used to plan and design. Vibration responses of structures have become one of the most common types of information for damage identification and structural health monitoring of structures, especially bridges. Vibration Responses are measured using Accelerometers. Time series techniques are reliable and economical in damage assessment. The use of model (AR, ARMAX, and ARX) fitted using a set of data measured on the healthy structure can provide parametric information which can be useful for a further prognostic step. Time series is an ordered sequence of values of a variable at equally spaced time intervals. Time series data is normally used to obtain an understanding the underlying forces and structure that produced the observed data and to fit a model and proceed to forecasting,
monitoring or even feedback and feed forward control.

LITERATURE REVIEW
Mustafa Gula, F Necati Catbas, Michael Georgiopoulos found time series analysis is one of the methods, which is implemented in statistical pattern recognition applications to SHM. Auto-Regressive (AR) models are highly utilized for this purpose. In this study, AR model coefficients are used with different outlier detection and clustering algorithms to detect the change in the boundary conditions of a steel beam.

A number of different boundary conditions are realized by using different types and amounts of elastomeric pads. The advantages and the shortcomings of the methodology are discussed in detail based on the experimental results in terms of the ability of it to detect the structural changes and localize them. Time series analysis, i.e., Auto-Regressive models coefficients are extracting using Mahalanobis distance-based outlier detection algorithms is used to discriminate different structural conditions. The methodology is applied to laboratory test data where ambient vibration tests are conducted on a steel beam with different boundary conditions. In this analysis normalization is applied to each channel of the data before constructing the Auto Regressive (AR) models. After normalizing (averaging) the data, AR models are fitted to the averaged data. Then the coefficients of these models are used as the damage indicating features and they are fed to the outlier detection and clustering algorithms.

The random response of a system at a particular time consists of three components, which are the step response due to the initial displacements, the impulse response from initial velocity, and a random part due to the load applied to the structure. By taking average of data segments, every time the response has an initial displacement bigger than the trigger level, the random part due to random load will eventually vanish and become negligible. Additionally, since the sign of the initial velocity can be assumed varying randomly in time, the resulting initial velocity will also be zero leaving a pseudo-free response of the system. It is an important to improve the methodology to pinpoint the location of the change.

METHODOLOGY
The methodology uses auto-regressive models in conjunction with a Mahalanobis distance-based outlier detection algorithm. In the proposed methodology, free vibration response from the ambient vibration data is taken. It’s a modified methodology by implementing the normalizing for the ambient vibration data before constructing the AR models. Ambient vibration (acceleration) data is used to obtain pseudo-free responses by means. Then AR models of these free responses are constructed. The coefficients of these AR models are used as the damage features. Finally, the Mahalanobis distance-based outlier detection algorithm is used to separate different states of the structure damage indicator, defined using the distance between AR models, and is proposed. A reference AR model, which is fitted to the vibration response of the structure under normal operation conditions, represents the healthy structure. If the structure is damaged, a different model is to be used to represent the damaged structure, and the distance
between the two models is correlated with the location and severity of the damage. For efficiently distinguishing between the damaged structure and the undamaged one, it is necessary to define an appropriate distance between AR models of damaged and undamaged structures. The Mahalanobis distance for AR models is adopted here to relate a common distance that is widely accepted in system identification for comparing healthy and damaged AR models and the AR model is shown in Figure 1.

![Figure 1: Obtaining the Feature Sets Using Time Series Modeling](image)

An AR model estimates the value of a function at time \( t \) based on a linear combination of its prior values. The model order (generally shown with \( p \)) determines the number of past values used to estimate the value at \( t \). The basic formulation of a \( p \)th order AR model is defined as follows.

\[
X(t) - \sum_{j=1}^{p} \phi_j X(t-j) = e(t)
\]

where \( X(t) \) is the time signal, \( f \) is the model coefficients and \( e(t) \) is the error term. After obtaining the coefficients of the AR models, they are fed to the outlier detection algorithm where the Mahalanobis distance between the two different data sets is calculated.

Detection can be considered as the detection of damages in the sensors, which deviate from other sensors so that they are assumed to be generated by another system or mechanism detection is one of the most common pattern recognition concepts among those applied to SHM problems. In this study, Mahalanobis distance-based detection is used to detect the novelty in the data. One of the most common is based on deviation statistics and it is given by the following.

\[
d_i = \frac{d_i - \bar{d}}{\sigma}
\]

where \( d_i \) is the potential outlier and, \( \bar{d} \) and \( \sigma \) are the sample mean and standard deviation, respectively. The multivariate equivalent of this discordance test for \( nxp \) (where \( n \) is the number of the feature vectors and \( p \) is the dimension of each vector) data set is known as the Mahalanobis squared distance.

The mahalanobis squared distance will be referred as mahalanobis distance after this point and it is given as

\[
d_i = (x_i - \bar{x})^T \Sigma^{-1} (x_i - \bar{x})
\]

where \( d_i \) is the outlier index form ultivariate data, \( x_i \) is the potential outlier vector and \( x \) is the sample mean vector and \( \Sigma \) is the sample covariance matrix. By using the above equations, the outliers can be detected if the Mahalanobis distance of a data vector is higher than a pre-set threshold level. The
flowchart of the damage detection methodology using the proposed damage indicator is shown in Figure 2.

![Flowchart for the Damage Detection Methodology](image)

Numerical experiments have been conducted to test and verify the damage diagnostic technique using the AR model discussed. It is expected that the proposed algorithm will be more reliable in robustly identifying the damage instant and also the spatial location of damage in the structure. Numerical examples of a simply supported bridge girder and a cantilever beam are presented to demonstrate the effectiveness of the damage indicator derived by comparing AR models of baseline data and the current data.

A simply supported and a cantilever beam girder with a span of 10 m are considered. For the purpose of numerical simulation studies, the beam is discretised into 20 elements. The material and geometrical properties are also given. The beam is excited using random dynamic loading which is stochastic in nature. The acceleration time history response is computed using finite element analysis with Newmark’s time marching scheme. The sampling rate is considered as 1000 samples per second. From the data collected from sensors we create acceleration time histories which are referred as Y data (for damaged and undamaged conditions). Here we use some algorithms in mat lab to get Y data.

Damage is identified by analyzing the errors obtained between the undamaged time series and the curve obtained by auto regression for damaged structure. Y data (undamaged) sample is divided as reference and healthy data. Damage datasets are created from Y data (damaged condition). Damage features were extracted using errors from fitting autoregressive models with exogenous inputs to collected time histories. Autoregressive models were fitted to all the segmented data samples in reference, healthy and damage datasets.

Damage index is used to determine the damage occurred in a structure. Higher the damage, greater is damage index. Using this we can determine damage occurred or not. You also need to determine the severity of damage.

\[
\text{Damage index} = \text{Ratio of variance of prediction errors from healthy, reference and damage sets.}
\]

Several test cases are considered by giving different temperature for different elements and acceleration responses for each case are obtained. Similar way 30 sets of responses are obtained. Now the Mahalanobis distances between AR models of different combination of response sets are obtained. The peak
values of Mahalanobis distance from each combination are taken to obtain the Upper Control Limit (UCL).

\[ \text{UCL} = \text{Mean} + (3 \times \text{SD}) \]

In the numerical studies, vibration responses are generated through Finite element code in MATLAB. To illustrate the effectiveness of the proposed method of damage detection, the experimental data of a benchmark structure from Los Alamos National Laboratory (LANL) laboratory is considered. Experimental data include undamaged condition and damaged condition from a benchmark structure.

The structure tested is a three-story frame structure model as shown in Figure 3.

This configuration allows relative motion between the column and the floor to be detected. The nominal sensitivity of each accelerometer is 1 V/g. A 10 mV/lb force transducer is also mounted between the stinger and the base plate. This force transducer is used to measure the input to the base of the structure. A commercial data acquisition system controlled from a laptop PC is used to digitize the accelerometer and force transducer analog signals. The data sets that were analyzed in the feature extraction and statistical modeling portion of the study were the acceleration time histories. In each test case, three separate data sets were collected with the shaker input level at 2, 5 or 8 V.

RESULTS AND DISCUSSION
A. With Simply Supported Beam Girder
This is the graph plotted between sensor number and damage index. Graph includes 19 sensors and corresponding damage index is

![Figure 3: Basic Dimensions of the 3 Story Frame Structure](image)

![Figure 4: Floor Layout as Viewed from Above](image)
plotted. Here, the damage index is the Mahalanobis distance between the AR models. Here the UCL is found to be 4.6 and it is shown in Figure 5. Damage index for the tenth element is crossing the UCL.

And here also damage condition at fifth and fifteenth element shift beyond UCL and it is shown in the Figure 6.

In order to prove the method, the undamaged and damaged data are taken from LANL laboratory. Tests are done for different volt condition and damage condition and it is shown in the Figure 9. And the Figure 10

B. With Cantilever Beam Girder
Here the damage is simulated into cantilever beam girder by reducing the stiffness of tenth element. Here the UCL is found to be 3.96. Damage index for the tenth element is crossing the UCL and it is shown in the Figure 7.
represents damage at both locations and it is a case with volt2. Here 3rd and 4th sensors got damaged.

**Figure 10: Damage at both Locations-Brackets was Completely Removed-Volt2**

From the Figure 13 it is shown that the Intensity of damage is high at the nineteenth sensor. Which means that Damage is induced

This is the case with 5 V. Here damage condition is Single Damage-bolt were removed between Bracket and Plate and it is shown in the Figure 11.

**Figure 11: Single Damage-bolt were Removed Between Bracket and Plate -Volt5**

The Figure 12 gives the Damage at both locations 11th and 12th sensor got damaged.

**Figure 12: Damage at Both Locations-Brackets was Completely Removed-Volt5**

**Figure 13: Single Damage-bolt were Removed Between Bracket and Plate -Volt8**

**Figure 14: Damage at Both Locations-Brackets was Completely Removed-Volt8**
at the first floor of position A and Plate is damaged.

And for the Damage at both locations 19th and 20th sensor got damaged. This is shown in the Figure 14.

CONCLUSION

1. The main advantage is the sensitivity of the time-series coefficients to changes in characteristics of dynamic response. This advantage has resulted in the use of time-series models for damage identification purposes.

2. The application of time series analysis methods to Structural Health Monitoring (SHM) is a relatively new but promising approach. Time series methods are inherently suited to SHM where data is sampled regularly over a long period of time, which is typical for monitoring systems.

3. The use of model (AR) fitted using a set of data measured on the healthy structure can provide parametric information which can be useful for a further prognostic step. Most currently available damage detection methods are global in nature and global damage measures are not sensitive to minor damage and local damage. Furthermore, such methods involve finite element modeling and system identification methods, which can be computationally expensive. Here there is no need for the modeling.

4. The efficiency of the proposed methodology to identify the presence of damage is validated in this study, using a simply supported beam girder, cantilever beam girder and 3-storey bookshelf benchmark structure from LANL as numerical examples. From the results it is shown clearly, that the damage indicator has efficiently identified presence of the damage even from the noisy data and environmental variabilities.

FUTURE WORK

As an extension off the study on time series method, it is planned to propose more time series method and carry out experimental works to validate the results and conclusions obtained analytically.

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fitness function for (n) arx modelling”.


