

Research Paper

DYNAMICAL MODELING AND RESONANCE FREQUENCY ANALYSIS OF 3.6 M OPTICAL TELESCOPE PIER

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The pier is an important building block of large optical telescopes. There are several sources of vibrations all around the optical telescope. A large height makes it vulnerable to low frequency oscillations, both as surface waves, as well as bulk oscillations which may be transferred to the optical system. It mainly happens through the pier, since it is directly connected to the telescope. In order to avoid enhanced transfer of energy between pier and optical system, it is necessary to ensure that the resonance frequency of the pier and the telescope fixtures are fairly separated. The as-built structure of telescope pier was simulated using FEM analysis for finding the resonating modes. Various test procedures were defined and on-site testing was done using 3C geophones and piezoelectric sensors to closely observe response of the pier. The main mode in low frequency regime was found to occur at approximately 22 ± 2 Hz by both testing and simulations. Theoretical supports are also provided for observed modes. A proof of concept was demonstrated for impulse response based resonance frequency determination. It was also demonstrated that a low-cost piezoelectric sensor based test-bench can be used for finding the resonating modes, compared to expensive 3C geophones.

Keywords: Telescope Pier, 3C geophones, Piezoelectric sensors, Resonance frequency, FEM analysis

INTRODUCTION

A 3.6 m diameter optical telescope is being constructed at Devsthal, Uttarakhand (Sagar *et al.*, 2013). One important challenge is to make sure that the telescope is not prone to vibrations due to dynamics of various objects in the telescope assembly. The dominant frequencies induced in telescope building are

due to dynamics of telescope dome, unsteady wind loads, earthquake tremors and the dynamics of the telescope body itself. It is important to study the propagation of these vibrations into the optics of telescope leading to inaccuracies and jitter in beam-pointing. This necessitates the study of natural frequency of the Telescope Pier which is the

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Table 1: Material Properties used for Analysis

Properties: Concrete	Compressive Strength (Cubic)	Tensile Strength (Mean)	Flexural Strength	Elastic Modulus	Shear Modulus	Poisson Ratio	Damping Ratio
M10	14MPa	1.82MPa	2.2MPa	17GPa	7.83GPa	0.15	0.07
M25	32.5MPa	2.86MPa	3.5MPa	25GPa	11.74GPa	0.15	0.07

main support structure for the telescope.

TELESCOPE PIER

Pier is supposed to be the most stable support structure of the telescope. Hence its foundation is also laid deeper and stronger on bedrock rather than just soil (Pierre Bely, 2003). The pier has to be rigid with natural frequencies higher than 20 Hz in order to protect the telescope from seismic loads (Pierre Bely, 2003). The main telescope enclosures are prone to large wind loads due to their geographical location and large surface area. This causes a moment on the underlying foundation which may ultimately transfer vibrations to the pier and the telescope. Hence, it is important to have rigid pier foundation and good isolation with the telescope enclosure foundation to mitigate the effect of wind load on telescope and vibrations due to dome dynamics. Pier is generally made up of reinforced concrete due to its low cost and excellent damping properties (Pierre Bely, 2003).

At present, the telescope pier has been built and construction of dome framework is in progress. It is important to assess the compliance of as constructed telescope pier with as designed specifications. Dynamical testing and analysis provides a way to inspect key parameters of the pier. This paper will

encompass the Finite Element Method (FEM) based study of as built telescope pier as well as results obtained from on-site vibration testing of telescope pier using 3C geophones. Also, the piezoelectric sensor based low-cost test setup has been devised for vibrational testing, which is described in brief. A coarse estimate of the resonating modes is also determined analytically. A brief sensitivity analysis has also been done for pier properties.

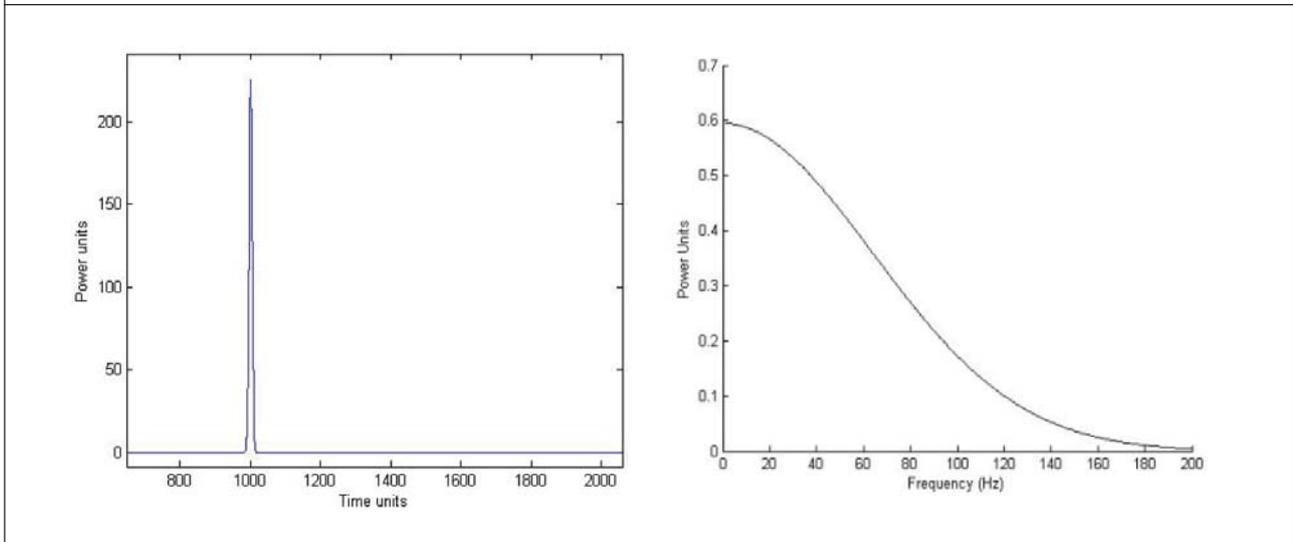
GENERAL METHODOLOGY

In dynamical analysis of pier, 3 modes are important to study, which are rocking mode, torsional mode and vertical mode. The torsional mode is not very significant in the present analysis due to high torsional rigidity of the structure. So mainly rocking mode, i.e., shear mode and vertical mode are manifested in the testing as well as FEM simulations.

Analysis of Impulse Response

An impulse can be thought of as a very narrow Gaussian function in time domain. Hence, infrequency domain it becomes a wide Gaussian function, as shown in Figure 1. When a system is excited with an impulse, the frequencies in the band covered by the frequency domain Gaussian are excited at once. But the frequencies which are not the natural modes of the system will attenuate

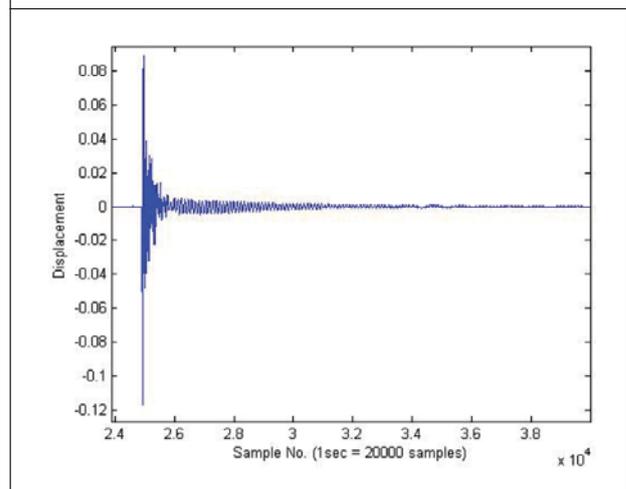
Figure 1: Impulse as Narrow Gaussian in Time Domain and its Fourier Transform (Frequency Domain)



below a defined threshold level faster compared to natural modes. Hence, after a sufficient time, only the natural mode frequencies will be left above a threshold level. To observe how different frequencies decay in time, we need a time-frequency representation of the signal. It is achieved by using wavelet decomposition of the signal into several scales of frequency.

A representative impulse response for the testing and for simulation is shown in Figure 2. As explained earlier, we take a time-window after approximately 11 dB attenuation from first peak till approximately 16 dB attenuation. This particular window has been selected since most of the other modes are attenuated by this time and some clear peaks can be observed in frequency domain. Also it has a moderate signal to noise ratio which is in the range 3 (at the end) to 8 (at the start). This window has been analyzed for energy distribution in the frequency domain (by FFT) as well as decay

Figure 2: A Representative Impulse Response



of frequency with time (by wavelet based decomposition into scales) for both testing and simulation. We have used Gaus4 wavelet for decomposing the signal into scales.

Analysis of Harmonic Response

The system is excited with sinusoidal forcing of various frequencies and the response is observed at various sensor locations. The

harmonic load excitation is achieved here by rotation of unbalanced (eccentric) mass mounted over a motor shaft. The frequency of excitation is varied by varying the angular speed of the eccentric mass. The motor is mounted on the position where we desire to apply loading. The frequency of the harmonic response is same as the frequency of harmonic loading. The maximum amplitudes obtained at each frequency are recorded, and are divided with forcing amplitude to generate transfer function of the system. The peaks in the transfer function denote the natural frequencies of the system.

FEM Analysis of Model

The Model

A 3D model of the as-built telescope pier was made using Solidworks CAD Package. FEM based simulations were performed on the model. The details about the dimensions and materials are as shown in the Figure 3. Wall thickness of pier = 1 m; inner diameter of pier = 5 m; top slab thickness = 1 m.

Impulse Response Analysis

A narrow ‘Gaussian’ type of loading was applied over specific regions which are: Top surface center, Bottom surface center, Top surface peripheral and Bottom surface peripheral. The top-bottom delay in the response was noted for wave velocity. The response was recorded at topsurface-center position and bottom-surface-center position for about 1.6 s after application of impact. The analysis of impulse response was done according to the method described earlier.

The loading function

$$F(t) = \frac{200}{\sqrt{\pi}} \times \exp(-40000 \times (t - 0.4)^2)$$

The results of impulse response analysis are as shown in Figures 4 and 5.

Harmonic Response Analysis

Harmonic loading was administered at the same 4 loading locations as for impact loading and the response was studied at the same sensor locations. The transfer function graphs were obtained in the frequency range 0-100 Hz. The results are shown in Figure 6.

Figure 3: (A) Pier Model As Constructed (B) Loaded Telescope Pier For Impact Testing (C) The Coordinate System Used

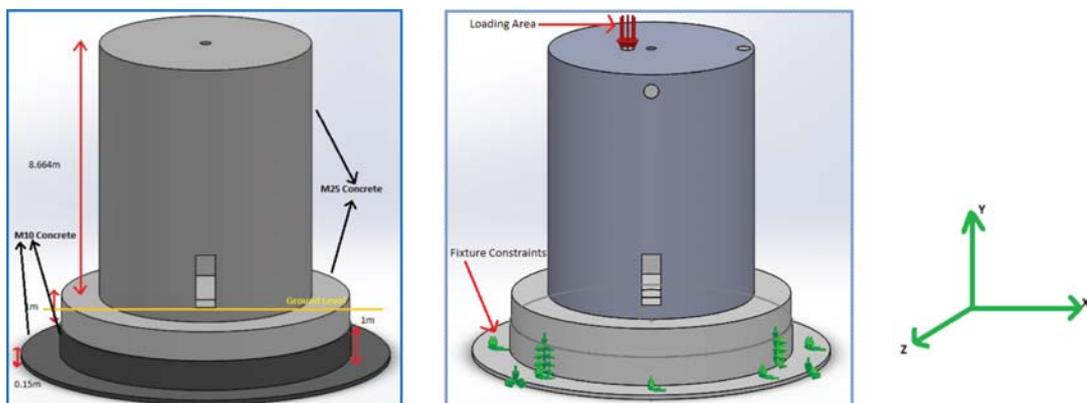


Figure 4: FFT of After-Impact Time-Domain Signal: a) Signal in X-Direction b) Signal in Z Direction c) Signal in Y Direction

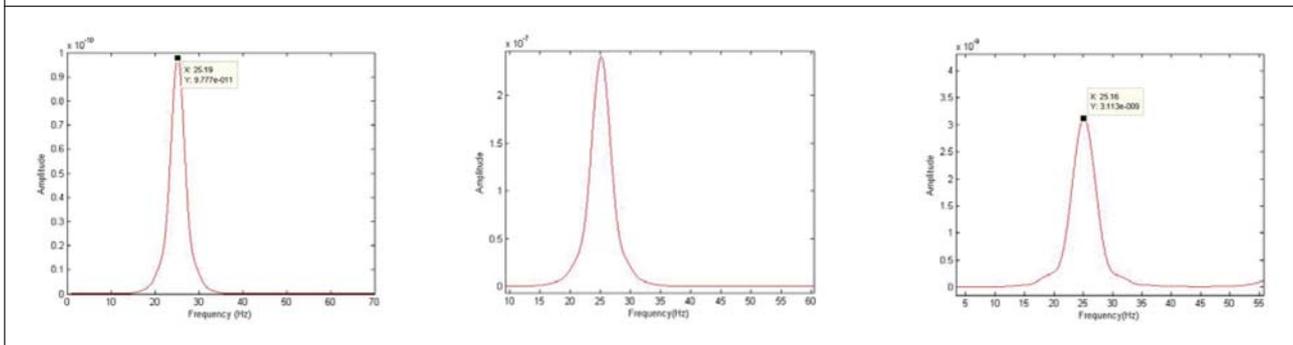
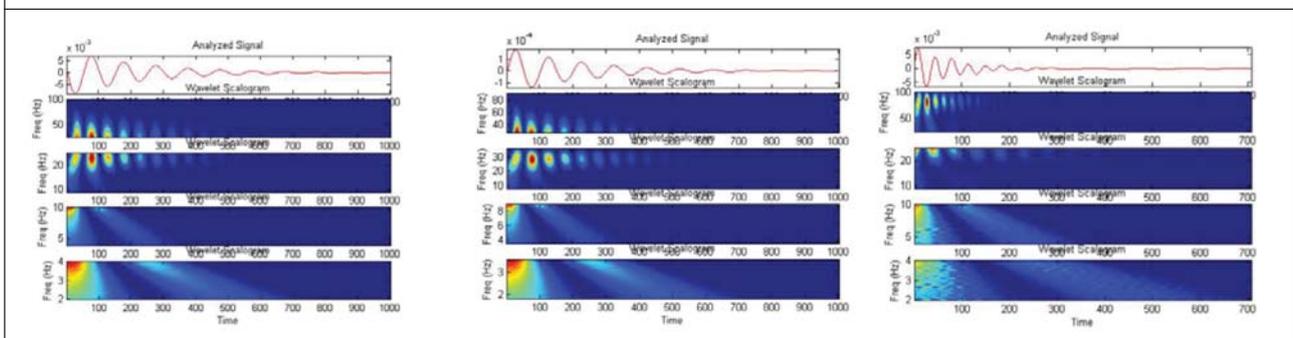
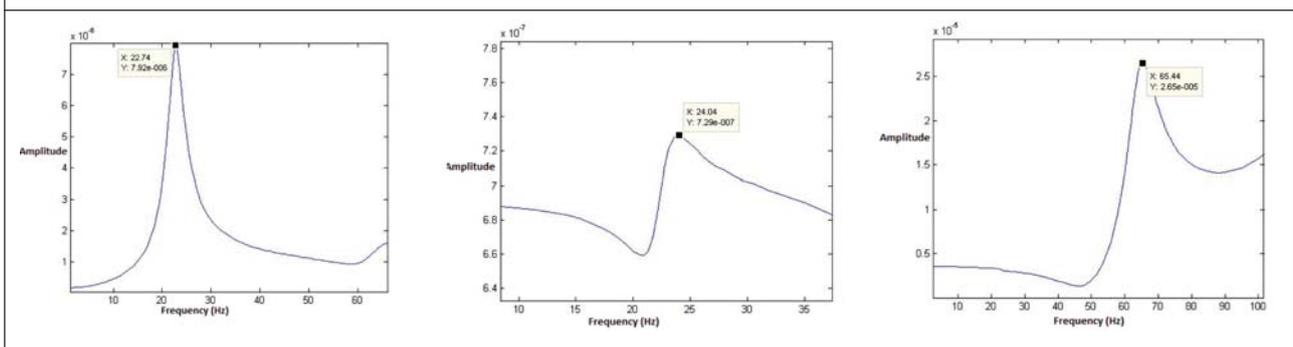


Figure 5: Time-Freq representation of After-Impact Time-Domain Signal: a) Signal in X-Direction b) Signal in Z Direction c) Signal in Y Direction



Note: Red: Highest Amplitude, Blue: Least amplitude

Figure 6: Harmonic Response in Simulation: a) X-Direction b) Z- Direction c) Y- Direction



On-site Testing using 3C Geophones

This was executed by applying impact loads at various locations on pier and simultaneously observing the response at fixed sensor locations. The impact load was provided by a

nylon attenuator coated hammer. The mass of the hammer used was 10 kg. Each 3-Component Geophone sensor measures response along 3 orthogonal directions X-Y-Z. The sensors were employed at Pier-top-surface-center and Pier-bottom-surface-

center. The impact was applied at 4 main locations which are top-surface-center, bottom-surface-center, top-surface-north (peripheral) and top-surface-east (peripheral). Also, the time taken for vibrations to travel from bottom surface to top surface was measured to give a measure of wave-velocity.

Impulse Response Analysis

The analysis of impulse response was done according to the method described earlier in methodology. The results are summarized in Figures 7, 8, and 9.

Testing Using Piezoelectric Transducers

Figure 7: FFT of After-Impact Time-Domain Signal for Testing:
 a) Signal in X-Direction b) in Z Direction c) in Y Direction

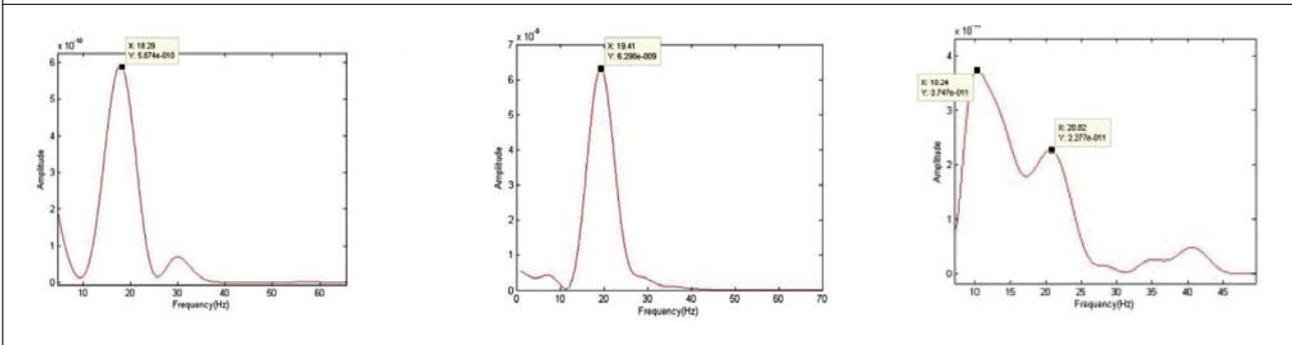


Figure 8: Time-Freq representation of After-Impact Time-Domain Signal for Testing:
 a) in X-Direction b) in Z Direction c) in Y Direction

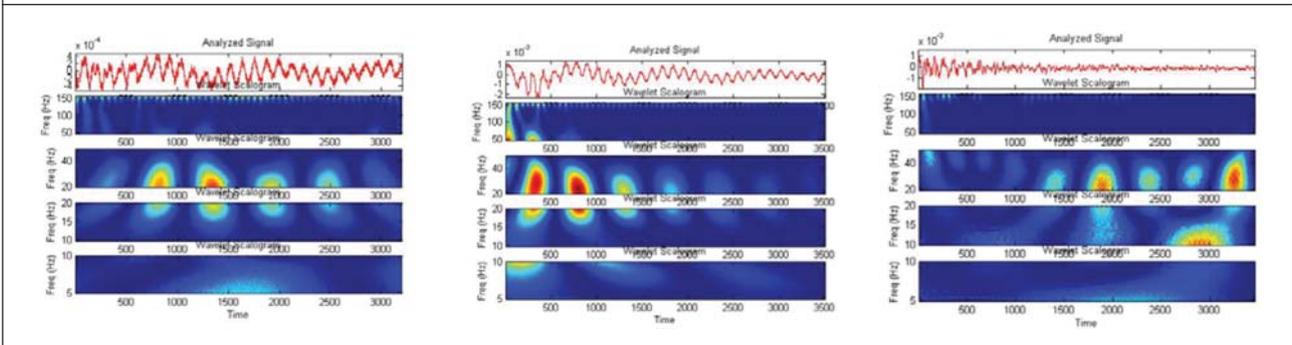
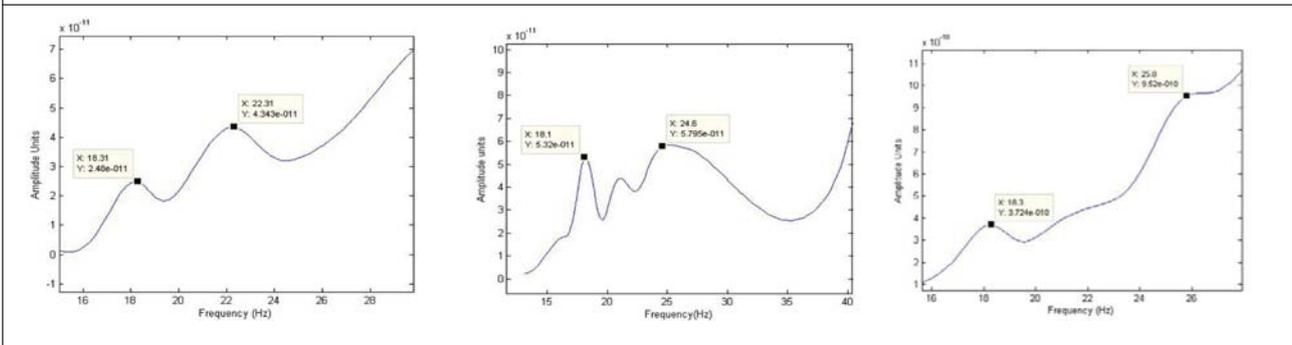


Figure 9: Harmonic Response in 3C-Geophone Testing:
 a) X-Direction b) Z- Direction c) Y- Direction



Test Setup

The Piezoelectric transducers generate voltage across its terminals when it is deformed due to a vibration or force. A piezoelectric ceramic disk diaphragm manufactured by Murata Manufacturing Co. Ltd. was used for vibration testing of pier. The block diagram and the sensor are shown in Figure 10. This is a family of low cost pressure transducers. These are active transducers and hence the sensitivity and linearity are not affected by input voltage fluctuations. The piezoelectric sensors have a flat frequency response curve up to around 100 Hz and the resonance frequency

is 2.8 kHz, thus it is possible to measure the response at low frequencies without significant distortion. The phase-distortions during noise filtering are eliminated by applying an IIR filter to a signal twice, once forwards and once backwards. The acquired data is analyzed in the computer using a python code. Impulse response test was performed using this setup. Only X and Y directions were studied using piezoelectric sensors since it was found that X and Z directions have similar responses. The impact was administered at Pier bottom surface center and the setup was also firmly fixed at a distance of about 30 cm from impact location.

Figure 10: a) Piezoelectric Sensor Block Diagram B) Setup Using 2 Piezo Sensors C) Interfacing Circuitry

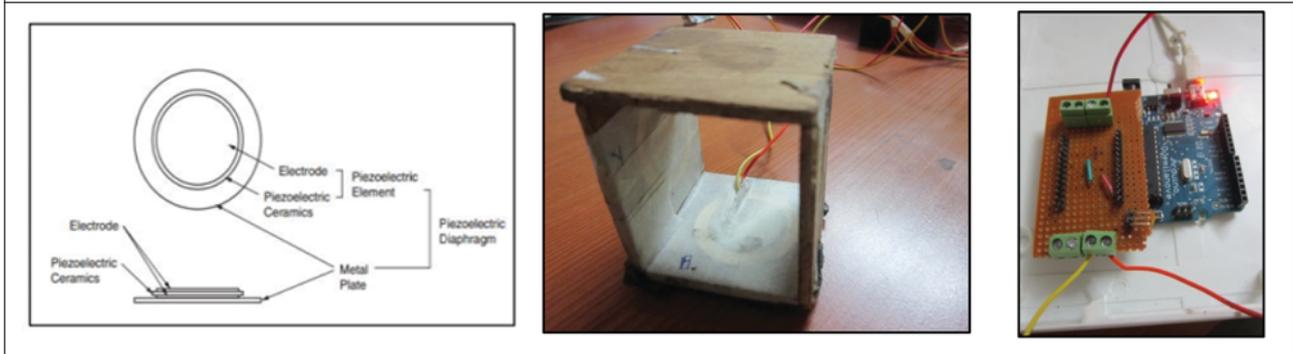
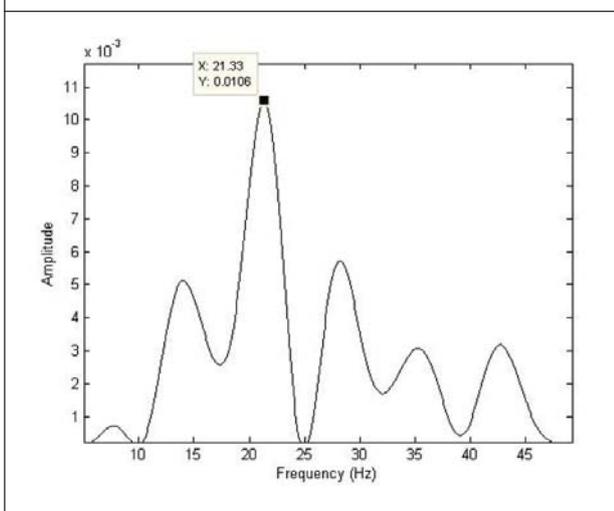


Figure 11: FFT of Y-Direction Response in Piezoelectric Sensor Based Impact



RESULTS

Analysis of the pier response is performed using wavelet based decomposition to eliminate sensor characteristics. We use sym8 scaling function for the decomposition and decompose the signal into 9 levels. From the decomposition, it is observed that the low frequency range of interest has significant energy contribution in the response and is mainly concentrated in 6th and 7th levels of decomposition for longer durations. FFT of 11 dB to 17 dB window is performed to compare the results with 3C geophones. The FFT for Y-direction signal is shown in Figure 11. The

various levels of decomposition are shown in Figure 12.

Comparison with Analytical Results For Shear Mode

From the time-frequency representation of both testing and simulation signals show that the 22-24 Hz signal survives for the longest times,

which in turn means that it is the principal mode of vibration for the pier. This is also expected from the analysis of time taken for the signal to travel from Top surface to bottom. Top-Bottom time delay = 0.0105 s; height of Pier = wavelength/4 = 8.66 m. Hence, velocity of the wave is 822.86 m/s. The frequency is thus calculated to be 23.9 Hz.

Figure 12: Wavelet Decomposition of Impact Response Obtained using Piezoelectric Setup

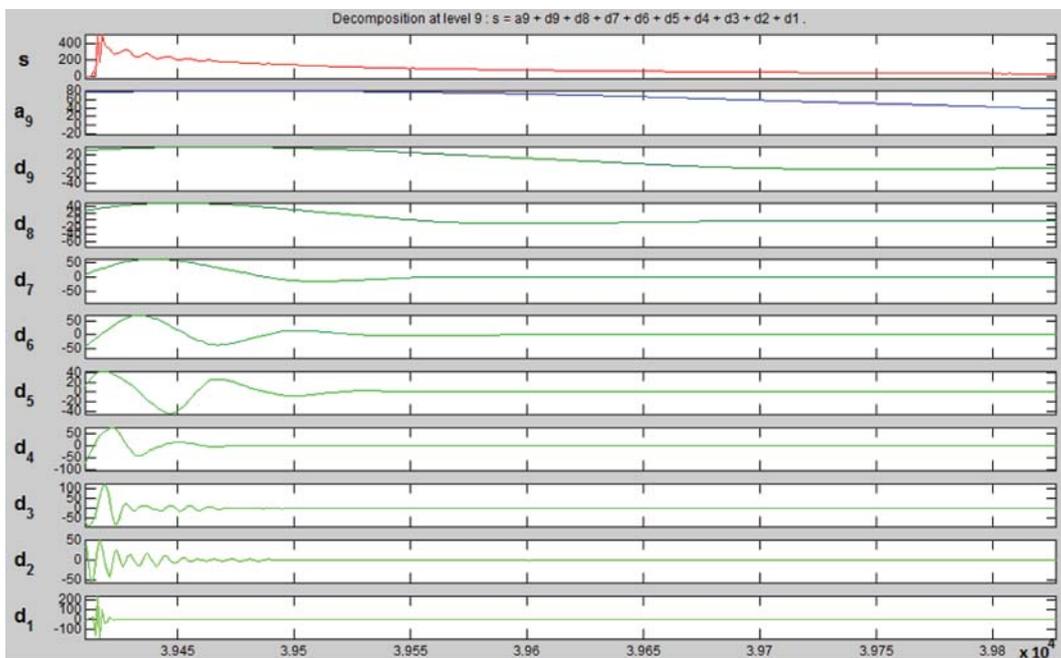
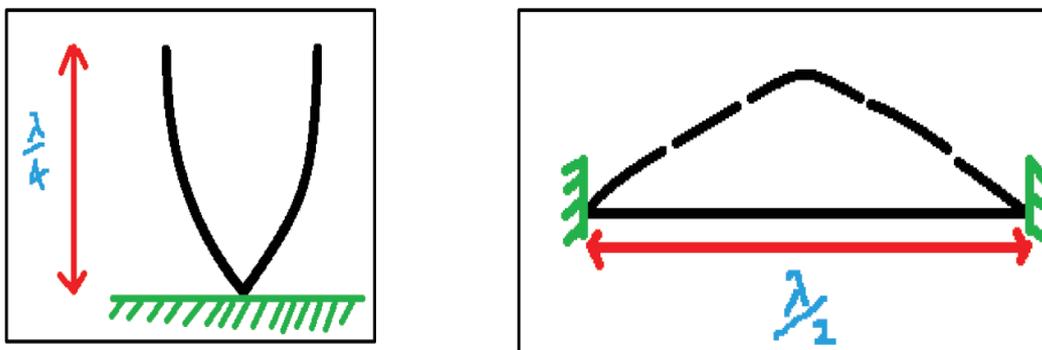


Figure 13: Illustrations of a) Shear Mode b) Vertical Mode of Vibration



For Vertical Mode

The circular base and top circular surface vibrates up and down. Hence the critical length is the diameter of the surface. Diameter of free surface is 5 m with 1 m additional thickness on both sides. Taking an end correction of 0.5 m in the radial direction, we get half of the wavelength = 6 m. Thus the calculated frequency is thus 66.7 Hz. This frequency is predicted by the simulations, though the energy content is two orders of magnitude lower than shear mode.

Improvements

It was observed that the simulation results are most sensitive to the material properties for M25 concrete. It also depends on the fixture constraints used to simulate the foundation. So it is important to simulate the structure for several values of elastic constant for M25 concrete within a range specified in Indian Standards for Concrete. These simulations were done and a graph of natural frequency in low frequency regime was plotted against Elasticity Modulus of M25 Concrete, as shown in Figure 14. It is important to simulate the

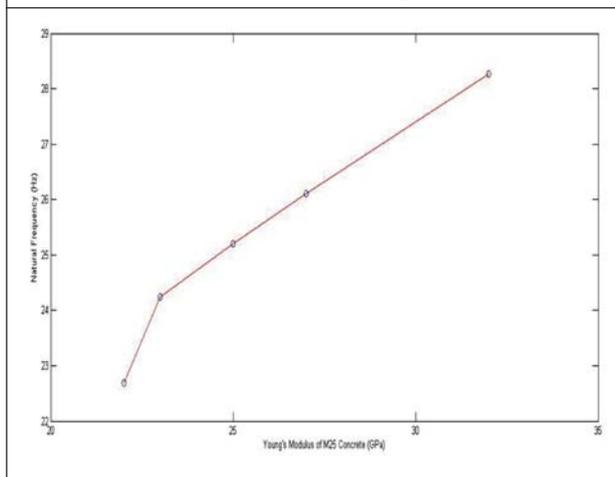
structure using practical soil properties and geometry as well as characteristics of bed-rock on which the foundation is laid. This will give results closer to the observed response and will also allow to parameterize the foundation in better way. Also the piezoelectric setup should be used with harmonic load testing as well to evaluate its performance in a complete manner.

CONCLUSION

Results were obtained for several conditions of testing and simulation, but only a few representative results are shown. Some important interpretations can be drawn from the simulations and experimental data. The natural frequency of the pier in low frequency domain is nearly 22 ± 2 Hz. This frequency is mainly contributed by shear modes in X (North) and Z (East) Directions.

The main mode of oscillation for the vertical mode (Y-Direction) is at 65 Hz but it shows peaks in frequency domain near 23 Hz because of strong coupling with the shear modes. There are some distinct 20 Hz modes observed in case of impact loading at peripheral points. That mode is appearing due to variation in material properties across the pier. Also, the low-frequency content observed with 3C geophone based testing; especially in Y-direction is mainly due to the geophone sensor having the resonance frequency of 4 ± 0.5 Hz. These are not observed in piezoelectric sensor based testing. By observing the time frequency behavior of the response signal in impact response testing, we can observe how various frequency contents of the signal decay in time. So it is easier to find out the resonating and non-

Figure 14: Sensitivity Analysis of Pier for M25 Concrete Young's Modulus



resonating modes. Thus a reliable test setup and procedure is developed for dynamical analysis of telescope pier.

A proof of concept is demonstrated for Impulse response based method to determine natural frequencies of large and multi-degree of freedom structures. Also, a proof of concept for the piezoelectric test setup is established. Usefulness of wavelet based study in finding the natural modes of a system is also described.

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