Int. J. Struct. & Civil Engg. Res. 2013

International Journal of Structural and Civil Engineering Research

ISSN 2319 – 6009 www.ijscer.com
Vol. 2, No. 4, November 2013

Research Paper

PLASTIC BUCKLING OF SSSS THIN RECTANGULAR PLATES SUBJECTED TO UNIAXIAL COMPRESSION USING TAYLOR-MACLAURIN SHAPE FUNCTION

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In this paper, a solution for the plastic buckling of a thin rectangular isotropic plate with four simply supported edges under uniform in-plane compression is presented. The plastic buckling equation was derived using a deformation theory of plasticity and a work principle. The plate analysis was carried out through a theoretical formulation based on Taylor-Maclaurin series and application of energy method. The approximate shape function for the plate boundary conditions using the Taylor-Maclaurin series was truncated at the fifth term. The shape function was substituted into the plastic buckling equation and the critical plastic buckling load was obtained. The plate buckling coefficient was determined for aspect ratios within the range of 0.1 and 1.0 at increments of 0.1. The results were compared with solutions from previous studies and the average percentage difference was 0.091%. This difference demonstrates that the Taylor-Maclaurin series shape function is a very good approximation of the exact values for the displacement function of the deformed SSSS plate.

Keywords: Critical buckling load, Deformation plasticity theory, Displacement function, In-plane compression, Taylor-Maclaurin series, Thin plate

INTRODUCTION

Based on the stress-strain relationship, buckling of plates may be classified as elastic buckling or plastic buckling. Elastic buckling is based on Hooke's law where it is assumed that the proportional limit of the plate material is greater than the buckling stress. In many practical cases, however, buckling may occur in the plastic range. The actual buckling load in the plastic range is always lower than the buckling load in the elastic range. Hence, it is necessary to carry out plastic buckling analysis

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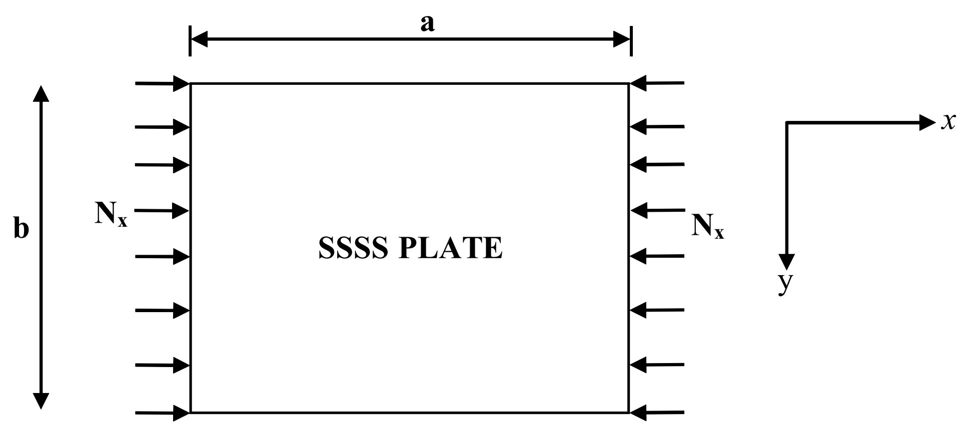
using plasticity theories so as to determine the accurate buckling load when buckling occurs in the plastic range.

The two commonly used plasticity theories in plate buckling are the deformation theory pioneered by Ilyushin (1947) and the flow (or incremental) theory developed by Handelman and Prager (1948). The deformation theory of plasticity is mathematically less consistent in comparison with the flow theory of plasticity. However, most researchers accept that the plastic buckling loads by deformation theory are always in better agreement with experimental results and that they have lower numerical values than those obtained from the flow theory. This is the well-known paradox of plate plastic buckling, and a universally accepted solution of the plastic buckling paradox has not yet been presented (Pride and Heimerl, 1949; Iskason and Pifko, 1969; Becque, 2010).

In finding solutions to plate buckling problems for both the elastic and plastic ranges, the use of Fourier series or trigonometric series in estimating the shape function of the deformed plate exists in literature. Irrespective of the plasticity theory used, some researchers used the numerical approach while others used the equilibrium and energy approaches in finding solutions to plastic buckling of plates. Studies by researchers such as Stowell (1948), Iyengar (1988), Shen (1990) and Wang *et al.* (2004) involved the use of trigonometric series. The use of Taylor's series in solving plate buckling problems has attracted very little attention.

To the best of the researchers' knowledge, the Taylor's series has not been used in the energy approach for analyzing the plastic buckling of SSSS plates. Therefore, the aim of this study is to use the Taylor-Maclaurin series to solve the plastic buckling problem of a thin rectangular isotropic plate with four simply supported edges subjected to uniaxial in-plane compressive loads. The problem definition is illustrated in Figure 1. The governing equation derived in the analysis is based on the deformation theory of plasticity using Stowell's approach.

Figure 1: SSSS Thin Rectangular Plate under Uniaxial In-plane Loading



MATHEMATICAL FORMULATION

Stowell (1948) expressed the differential equation of equilibrium for the plastic buckling of a thin, flat, rectangular plate under uniform compression in the *x*-direction as:

$$\left(\frac{1}{4}+\frac{3E_t}{4E_s}\right)\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}-\frac{N_x}{\bar{D}}\frac{\partial^2 w}{\partial x^2}=0 \qquad ...(1)$$

Where E_t is the tangent modulus, E_s is the secant modulus, N_x is the buckling load, $\bar{D}$ is the plastic flexural rigidity of the plate and *w* is the displacement in the *z*-axis. Transforming the $x-y$ coordinate system to $R-Q$ coordinate system, we have

$$R=\frac{x}{a};\ Q=\frac{y}{b}$$

It should be noted that *R* and *Q* are dimensionless parameters.

Eziefula (2013) applied a technique based on Ibearugbulem *et al.* (2013) where Equation (1) was transformed using the principle of conservation of work in a static continuum. Eziefula (2013) made N_x the subject of formula and obtained

$$N_x=\frac{\bar{D}p^2\int_0^1\int_0^1\left[\frac{H}{p^4}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)\frac{\partial^4 H}{\partial R^4}+\frac{2H}{p^2}\frac{\partial^4 H}{\partial R^2 \partial Q^2}+H\frac{\partial^4 H}{\partial Q^4}\right]\partial R\partial Q}{b^2\int_0^1\int_0^1 H\frac{\partial^2 H}{\partial R^2}\partial R\partial Q} \qquad ...(2)$$

where

$p = a/b$ ...(3)

$$\bar{D}=\frac{E_s t^3}{9} \qquad ...(4)$$

$w = AH$ ...(5)

p is the aspect ratio, *t* is the plate thickness, *a* and *b* are the length and width of the plate respectively, *H* is the plate buckling coefficient and *A* is amplitude of the shape function.

Ibearugbulem (2012) expanded the shape function using Taylor-Maclaurin series and obtained

$$w=w(x,y)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{F^{(m)}(x_0).F^{(n)}(y_0)}{m!n!}(x-x_0)^m.(y-y_0)^n \qquad ...(6)$$

where $F^{(m)}(x_0)$ is the m^{th} partial derivative of the function with respect to *x* and $F^{(n)}(y_0)$ is the n^{th} partial derivative of the function *w* and respect to *y*. *m!* and *n!* are the factorials of *m* and *n* respectively while x_0 and y_0 are the points of origin. He truncated the infinite power series at *m* = *n* = 4 and got

$$w=\sum_{m=0}^{4}\sum_{n=0}^{4}J_m K_n R^m.Q^n \qquad ...(7)$$

where

$$J_m=\frac{F^m(0)\times a^m}{m!} \qquad ...(8a)$$

$$K_n=\frac{F^n(0)\times b^n}{n!} \qquad ...(8b)$$

The boundary conditions for an SSSS plate are

$$w(R=0)=0;\ w''^R(R=0)=0 \qquad ...(9)$$

$$w(R=1)=0;\ w''^{R}(R=1)=0 \quad ...(10)$$

$$w(Q=0)=0;\ w''^{R}(Q=0)=0 \quad ...(11)$$

$$w(Q=1)=0;\ w''^{R}(Q=1)=0 \quad ...(12)$$

Substituting Equations (9) and (11) into Equation (7) gave

$J_0 = J_2 = 0; K_0 = K_2 = 0$

Substituting Equation (10) into Equation (7) and solving the resulting two equations simultaneously gave

$J_1 = J_4 ; J_3 = -2J_4$

Similarly, substituting Equation (12) into Equation (7) and solving the resulting two equations simultaneously gave

$K_1 = K_4 ; K_3 = -2K_4$

Substituting the values of J_0, J_1, J_2, J_3, J_4, K_0, K_1, K_2, K_3 and K_4 into Equation (7) gave

$$w=\left(J_4R-2J_4R^3+J_4R^4\right)\left(K_4Q-2K_4Q^3+K_4Q^4\right)$$

$$=J_4K_4\left[\left(R-2R^3+R^4\right)\left(Q-2Q^3+Q^4\right)\right] \quad ...(13)$$

From Equations (5), (7) and (13), we have

$$A=J_4K_4 \quad ...(14)$$

$$H=\left(R-2R^3+R^4\right)\left(Q-2Q^3+Q^4\right) \quad ...(15)$$

Partial derivatives of Equation (15) with respect to *R, Q* or both *R* and *Q* gave

$$H\frac{\partial^4 H}{\partial R^4}=24\left(R-2R^3+R^4\right)\left(Q-2Q^3+Q^4\right)^2 \quad ...(16)$$

$$H\frac{\partial^4 H}{\partial Q^4}=24\left(R-2R^3+R^4\right)^2\left(Q-2Q^3+Q^4\right) \quad ...(17)$$

$$H\frac{\partial^4 H}{\partial R^2\partial Q^2}=144\left(-R+R^2\right)\left(-Q+Q^2\right)\left(R-2R^3+R^4\right)\left(Q-2Q^3+Q^4\right) \quad ...(18)$$

$$H\frac{\partial^2 H}{\partial R^2}=12\left(-R+R^2\right)\left(R-2R^3+R^4\right)\left(Q-2Q^3+Q^4\right)^2 \quad ...(19)$$

Expanding and integrating Equations (16), (17), (18), and (19) partially with respect to *R* and Q in a closed domain respectively resulted in

$$\int_0^1\int_0^1 H\frac{\partial^4 H}{\partial R^4}\partial R\partial Q=0.23619 \quad ...(20)$$

$$\int_0^1\int_0^1 H\frac{\partial^4 H}{\partial Q^4}\partial R\partial Q=0.23619 \quad ...(21)$$

$$\int_0^1\int_0^1 2H\frac{\partial^4 H}{\partial R^2\partial Q^2}\partial R\partial Q=0.47183 \quad ...(22)$$

$$\int_0^1\int_0^1 H\frac{\partial^2 H}{\partial R^2}\partial R\partial Q=0.02390 \quad ...(23)$$

Substituting the values in Equations (20), (21), (22) and (23) into Equation (2) gave

$$N_x=\frac{\dfrac{\bar{D}}{b^2}\left[\dfrac{0.23619}{p^2}\left(\dfrac{1}{4}+\dfrac{3}{4}\dfrac{E_t}{E_s}\right)+0.47183+0.23619p^2\right]}{0.02390} \quad ...(24)$$

The plastic buckling load may be expressed as

$$N_x = H\frac{\pi^2\overline{D}}{b^2} \quad ...(25)$$

where

$$H = \left[\frac{1.00130}{p^2}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)+2.00027+1.00130\,p^2\right] \quad ...(26)$$

RESULTS AND DISCUSSION

The results from this study gave the equation of critical plastic buckling load as

$$N_{x,CR} = \frac{\pi^2\overline{D}}{b^2}\left[\frac{1.00130}{p^2}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)+2.00027+1.00130p^2\right] \quad ...(27)$$

From Iyengar (1988), the exact solution for the plastic buckling of an SSSS plate using Stowell's approach is

$$N_{x,CR} = \frac{\pi^2\overline{D}}{b^2}\left[\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)\left(\frac{m}{p}\right)^2+2n^2+n^4\left(\frac{p}{m}\right)^2\right] \quad ...(28)$$

In Equation (28), m and n are the buckling modes. For the first mode of buckling, $m = 1$. Also, since we are interested in finding the lowest value of N_x at which the plate buckles, n must be equal to one (Iyengar, 1988). Hence, Equation (28) may be simplified to

$$N_{x,CR} = \frac{\pi^2\overline{D}}{b^2}\left[\frac{1}{p^2}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)+2+p^2\right] \quad ...(29)$$

The factor, E_t/E_s is equal to one in elastic buckling but its value is always less than unity in plastic buckling. In this paper, the numerical value of E_t/E_s is taken to be equal to 0.9. Table 1 shows the values of H from this present study and Iyengar (1988) for different aspect ratios using $E_t/E_s = 0.9$.

Table 1: Values of H for Plastic Buckling of Uniaxially Compressed SSSS Thin Rectangular Plate

$p = a/b$	H from Present Study	H from Iyengar (1988)	Percentage Difference
0.1	94.6305	94.5100	0.1275
0.2	25.1954	25.1650	0.1208
0.3	12.3815	12.3678	0.1108
0.4	7.9490	7.9413	0.0970
0.5	5.9554	5.9500	0.0908
0.6	4.9335	4.9294	0.0832
0.7	4.3811	4.3778	0.0754
0.8	4.0883	4.0853	0.0733
0.9	3.9547	3.9520	0.0689
1.0	3.9277	3.9251	0.0662

From Table 1, the highest percentage difference is 0.1275% for *p* = 0.1, while the lowest percentage difference is 0.0662% for *p* = 1.0. The average percentage difference between the solution from this present study and Iyengar's solution is 0.091%. Iyengar's solution is an exact solution obtained from trigonometric series while the solution from the present study is an approximate solution based on Taylor-Maclaurin series. The solution from the present study is an upper bound solution. It can be observed that the closeness of the two solutions improves as the aspect ratio increases from 0.1 to 1.

CONCLUSION

In this study, plastic buckling analysis of a thin, flat, rectangular, isotropic SSSS plate was carried out using Stowell's plasticity theory and Taylor-Maclaurin series shape function. A work technique was applied to determine the plate buckling coefficient for different aspect ratios of the plate. The results showed that the solution is a very close approximation of the exact solution. Therefore, the Taylor-Maclaurin series is adequate for approximating the deformed shape of the SSSS plate in the plastic buckling analysis.

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