Int. J. Struct. & Civil Engg. Res. 2013

International Journal of Structural and Civil Engineering Research

ISSN 2319 – 6009 www.ijscer.com
Vol. 2, No. 4, November 2013

Research Paper

PREDICTING STRUCTURAL BEHAVIOR OF FIBRE REINFORCED LAMINATED COMPOSITE PLATES BY DIFFERENT FINITE ELEMENTS: A STUDY

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Successful prediction of structural behavior of Fibre Reinforced Laminated (FRL) composite plates using any Finite Element (FE) software depends largely on the choice of elements. However, verifying the suitability of a chosen element for a specific analysis and gauging the range of applicability of the element is difficult since the available studies on FE analyses for such FRL plates in literature rarely use commercial FE software. Consequently structural behavior of FRL composite plates using popular and commercially available FE software is studied. Two different elements are chosen: a three dimensional solid element and a shell element. Rigorous study is carried out by solving numerical problems involving plates with various thickness ratios, lamination lay-ups, boundary conditions, etc. To gauge the accuracy and range of applicability of such elements, the results of present analysis are compared with elasticity solutions, wherever possible. New results are also presented.

Keywords: Fibre Reinforced Laminated (FRL) composite plates, Finite Element (FE) software, Solid element, Shell element

INTRODUCTION

Structural modeling of a Fibre-Reinforced Laminated (FRL) composite plate is a challenging task in terms of both accuracy in predicting the structural parameters realistically and economy of computational time and memory. The models, available in literature, range from rigorous three dimensional elasticity methods to simple two dimensional models.

Elasticity solutions are undoubtedly accurate (Pagano, 1970; Pagano and Hatfield, 1972) and the most reliable method of analysis of a fibre reinforced laminated composite plate. However, trying out different geometries and various combinations of laminate lay-ups,

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boundary conditions and loadings is extremely difficult. This is because one has to go for individual analysis for each such combination, which in turn is extremely time- consuming and hence impracticable. In such situations, Finite Element Method (FEM) is a good and popular choice.

Study of the literature reveals that there are several ways in which the FEM can be implemented for analysis of a FRL composite (Ha, 1990; Sivakumaran *et al*., 1994; Zhang and Yang, 2009). Based on computational involvement, such an analysis comes under three broad categories: (i) Three dimensional finite element model; (ii) Layerwise theories; and (iii) Single layer plate theories.

Analysis using three dimensional finite elements (Liou and Sun, 1987) predicts the behavior of the plate realistically and accurately. However, as the Degrees of Freedom (DOF) are large, the analysis requires huge computing and sometimes the computational involvement become prohibitively high.

In order to reduce computational time and memory up to a certain extent while retaining the solution accuracy, layerwise theories are proposed by many researchers (Robbins and Reddy, 1993; Reddy, 1993). Unlike in three dimensional finite elements, the DOF here depends on the number of physical layers across the thickness in addition to in-plane dimensions. Therefore, if too many layers are present across the plate thickness, this model, too, becomes computationally inefficient.

Single Layer Theories (SLT) (Reddy, 1980; Reddy, 1984; Owen and Li, 1987; Topdar *et al.*, 2003; Chen and Zhen, 2008) are, by far, the most computation friendly for analysis of multilayered laminated composite plates as the DOF are defined only on the neutral surface; hence the DOF is independent of the thickness of the plate or number of physical layers across the depth. However, behavior of some of the structural parameters like transverse shear stresses may not be predicted realistically by SLT.

Almost each of the above studies on FRL composite plates by FEM uses computer codes based on the particular plate theory and finite element used for that specific analysis only. Studies on such plates using general purpose FE software are rare in literature.

For the purpose of analyzing FRL plates exhaustively for industrial use and research, it is impracticable in terms of time and effort to do detailed mathematical formulations and develop computer code to implement them. Very frequently, use of commercially available FE softwares is the only viable alternative in such cases. ANSYS is one such software which is frequently used in the present domain. However selecting the most appropriate element, from the element library of the software, for analysis of layered composite plates with a wide range of thickness and aspect ratios is critical for realistic and accurate prediction of the structural behavior of such plates under different loading and boundary conditions. Unfortunately, guidelines for choosing such elements based on the relevant geometry of the plate are rare in the literature. In this context, the work by Mokhtar *et al.* (2010) may be mentioned where a symmetric cross-ply laminate is analyzed by ANSYS using Shell 99 and Solid 46 elements. However, their study (Mokhtar *et al*., 2010) is

limited to a square plate where all the edges are simply supported. Moreover, only uniformly distributed load (*udl*) is considered for the analysis. The results of the sole numerical problem, which is solved by ANSYS, indicate that only a very thin plate (a/h= 100) is considered (Mokhtar *et al.*, 2010).

At this backdrop, the present study makes an effort to identify an appropriate element of ANSYS for static analysis of FRL composite plates within the elastic range. For this purpose, SHELL99 and SOLID5 elements are initially chosen. Their performances are rigorously studied for various geometry, lamination lay ups, loading and boundary conditions by solving variety of numerical problems. Consequently, their suitability for such type of analysis along with their range of applicability is commented upon. A number of new results are also presented which may benefit both the practicing engineers and researchers.

DETAILS OF ELEMENTS

In the present investigation, ANSYS 11 software is used. The particular elements chosen are SHELL99 and SOLID5. Brief details of these elements, as available from ANSYS 11 element library, are presented along with the relevant figures.

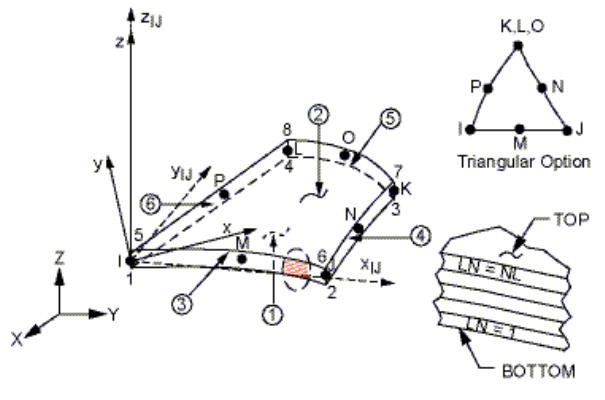
SHELL99 ELEMENT

This is a linear layered structural 3-D shell element and is defined by eight nodes. Each node has six degrees of freedom, i.e., translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes. SHELL99 allows up to 250 layers. It accommodates average or corner layer thicknesses, layer material direction angles, and orthotropic material properties as inputs. The element is said to be suitable for plates having thickness ratio (a/h) 10 or, higher. The geometry of SHELL99 element is shown in Figure 1, where,

LN = Layer Number

NL = Total Number of Layers

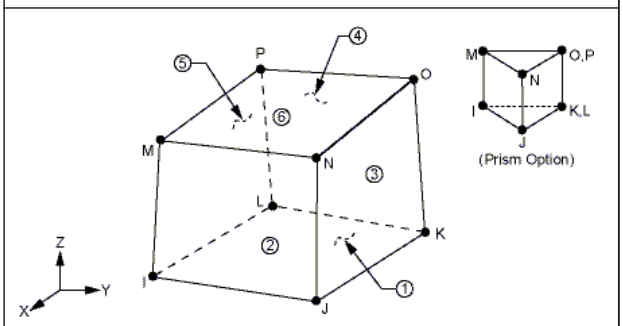
Figure 1: SHELL99 Geometry



SOLID5 ELEMENT

SOLID5 is a general 3-D solid element and it has a 3-D magnetic, thermal, electric, piezoelectric and structural field capability with limited coupling between the fields. The element has eight nodes with up to six degrees of freedom at each node. As the plates used here are ordinary laminated composite plates,

Figure 2: SOLID5 Geometry



the coupling terms are made zero during analysis. The geometry of SOLID5 element is shown in Figure 2.

RESULTS AND DISCUSSION

A number of numerical examples on laminated composite plates are presented. The examples, covering a wide range of features, are solved by using ANSYS 11 for static response. Initially, both the elements, viz., SHELL99 and SOLID5 are used for modelling and the results of analysis are compared with available exact solutions, wherever possible. This is done to assess the range of applicability of either element and to gain initial confidence on their performance. Consequently, new problems on static response of fibre reinforced laminated composite plates are solved. The coordinate system and lamination lay-up, used in each of the numerical examples, typically refer to Figures 3 to 4.

Figure 3: Plan View of a Typical Symmetric Laminated Plate

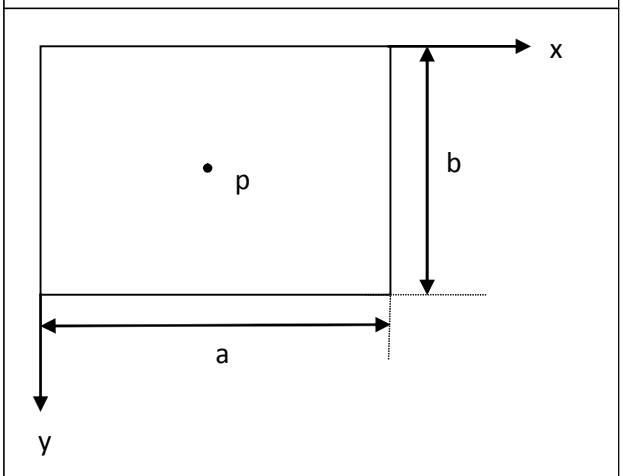
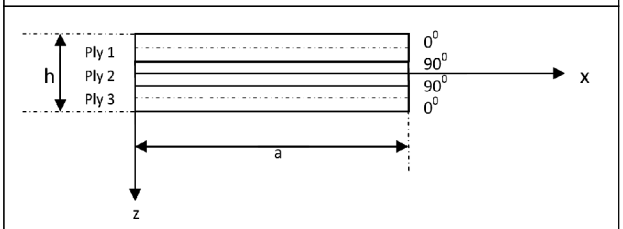


Figure 4: Elevation of a Typical Symmetric Laminated Plate



Example 1: This example carries out convergence studies for both SHELL99 and SOLID5 elements in terms of deflection and normal stress at the center of the plate. For this purpose, a three-ply square (b/a=1) laminated composite plate (0/90/0) with layers of equal thickness and subjected to sinusoidally distributed transverse load (*sdl*) $q = q_0 \sin(\pi x/a) \sin(\pi y/b)$ on its top surface is considered. All the edges are simply supported. The analysis is done using three different thickness ratios (a/h) of 100, 10 and 4 where, h is the total thickness of the plate. The lamina properties are as follows:

$E_1/E_2 = 25$; $E_2 = E_3$; $G_{12} = G_{13} = 0.5\,E_2$;

$G_{23} = 0.2\,E_2$; $v_{12} = v_{13} = v_{23} = 0.25$

Results, in non-dimensional form, are presented in Table 1. Very good convergence is obtained for SHELL99. The convergence, in general, is good for SOLID5; however, in a couple of cases it is observed that computation is not allowed beyond a certain mesh density. This is expected as explained earlier.

Example 2: To assess the range of applicability of the proposed finite element models for different plate geometries, a three-ply rectangular (b/a=3) cross-ply laminate (0/90/0) with varying thickness ratios are studied in this example. In each case, all the layers are of equal thickness. The plate is simply supported along all the edges; loading and material properties are same as in the previous example.

Table 1: Deflections and Normal Stresses in a Simply Supported (0/90/0) Square Plate Under Sinusoidal Load

$\frac{a}{h}$	$\overline{w}$				$\overline{\sigma}_x$			
	Present				Present			
	SHELL99		SOLID5		SHELL99		SOLID5	
	Mesh	Value	Mesh	Value	Mesh	Value	Mesh	Value
100	10x10	0.428	10x10	0.415	10x10	0.538	10x10	0.511
	14x14	0.429	14x14	0.424	14x14	0.536	14x14	0.525
	18x18	0.430	18x18	0.428	18x18	0.536	18x18	0.531
	22x22	0.431	22x22	0.429	—	—	22x22	0.533
	26x26	0.431	26x26	0.430	—	—	26x26	0.534
	—	—	30x30	0.430	—	—	30x30	0.534
10	10x10	0.658	10x10	0.624	10x10	0.513	10x10	0.550
	14x14	0.661	14x14	0.627	14x14	0.511	14x14	0.557
	18x18	0.663	18x18	0.630	18x18	0.512	18x18	0.561
	22x22	0.664	22x22	0.631	22x22	0.512	22x22	0.563
	26x26	0.665	26x26	0.631	—	—	26x26	0.564
	30x30	0.666	—	—	—	—		
	34x34	0.666	—	—	—	—		
4	10x10	1.747	10x10	1.700	10x10	0.437	10x10	0.717
	14x14	1.754	14x14	1.703	14x14	0.435	14x14	0.719
	18x18	1.760	18x18	1.706	18x18	0.435	18x18	0.720
	22x22	1.763	22x22	1.708	—	—	22x22	0.721
	26x26	1.764	26x26	1.709	—	—	26x26	0.721
	30x30	1.766			—	—	—	—
	34x34	1.766			—	—	—	—

Note: $\overline{w} = (wh^3E_2/q_0a^4)10^2$ where, $w = w$ (a/2, b/2).

$\overline{\sigma}_x = \acute{o}_x(a/2,b/2,h/2)(h^2/\ q_0a^2)$; $q_0 = q(a/2,b/2)$.

Deflections and stresses, in non-dimensional form, at important locations are found out for thickness ratios (*a*/*h*) 100, 20, 10 and 4 using SHELL99 and SOLID5 elements. The results are shown in Tables 2 to 5. To validate the present results with the established results, the relevant elasticity solutions are also presented in the tables.

The results indicate that SHELL99 predicts the values of deflections, normal stresses and in-plane shear stresses with very good to reasonably good accuracy for plates with thickness ratios of 20 and higher. However, results for transverse shear stresses are not satisfactory in general. SOLID5, on the other hand, seems to give a better prediction for only the normal stresses in thicker plates.

Table 2: Deflections and Stresses in a Simply Supported (0/90/0) Rectangular Plate (b/a=3) Under Sinusoidal Load (a/h = 100)

Reference / Parameter	Present				Elasticity (Pagano and Hatfield, 1972)
	SHELL99		SOLID5		
	Mesh	Value	Mesh	Value	
$\overline{w}$	10x10	0.503	10x10	0.481	0.508
	14x14	0.503	14x14	0.482	
	18x18	0.503	18x18	0.483	
$\overline{\sigma}_x$	10x10	0.623	10x10	0.590	0.624
	14x14	0.622	14x14	0.591	
	18x18	0.622	18x18	0.593	
$\overline{\tau}_{xz}$	10x10	0.329	10x10	0.0170	0.439
	14x14	0.328	14x14	0.0150	
	18x18	0.328	18x18	0.0130	
$\overline{\tau}_{xy}$	10x10	0.0082	10x10	0.0076	0.0083
	14x14	0.0083	14x14	0.0077	
	18x18	0.0083	18x18	0.0078	

Note: $\overline{\sigma}_x = \sigma_x(a/2,b/2,h/2)(h^2/\ q_0a^2)$; $q_0=q(a/2,b/2)$

$\overline{\tau}_{xz} = \tau_{xz}(0,b/2,0)(h/\ q_0a)$

$\overline{\tau}_{xy} = \tau_{xy}(0,0,h/2)(h^2/\ q_0a^2)$

$q_0=q\ (a/2,\ b/2)$

Table 3: Deflections and Stresses in a Simply Supported (0/90/0) Rectangular Plate (b/a=3) Under Sinusoidal Load (a/h = 20)

Reference / Parameter	Present				Elasticity (Pagano and Hatfield, 1972)
	SHELL99		SOLID5		
	Mesh	Value	Mesh	Value	
$\overline{w}$	26x26	0.575	14x14	0.538	0.610
	30x30	0.576	18x18	0.540	
	34x34	0.576	22x22	0.550	
$\overline{\sigma}_x$	10x10	0.622	14x14	0.604	0.650
	14x14	0.621	18x18	0.605	
	18x18	0.621	22x22	0.611	
$\overline{\tau}_{xz}$	10x10	0.329	14x14	0.0161	0.434
	14x14	0.328	18x18	0.0173	
	18x18	0.328	22x22	0.0180	
$\overline{\tau}_{xy}$	10x10	0.0084	14x14	0.0084	0.0093
	14x14	0.0083	18x18	0.0084	
	18x18	0.0083	22x22	0.0085	

It may be noted that for all the following examples, deflections and stresses are computed in non-dimensional form as in this example, if not mentioned otherwise specifically.

Example 3: In order to verify the applicability of the findings in the previous example to a square laminate, a four-ply square (b/a = 1) laminated composite plate (0/90/90/0) with layers of equal thickness is considered in this example. The plate is simply supported along all the edges and is subjected to a sinusoidally distributed transverse load $q = q_0 \sin(\pi x/a)$ $\sin(\pi y/b)$ at its top surface. The lamina properties are same as in Example 1.

The plate with thickness ratios 20, 10 and 4 is analyzed by SOLID5 for normal stresses and the results are presented in Table 6. Consequently, the same plate with thickness ratios 100 and 20 is analyzed by SHELL99 for deflections and in-plane shear stresses; the results are presented in Table 7. In each table, the relevant elasticity solutions are also presented for comparison. A close study of the above results confirms the validity of the findings of Example 2. Therefore, SHELL99

Table 4: Deflections and Stresses in a Simply Supported (0/90/0) Rectangular Plate (b/a=3) Under Sinusoidal Load (a/h = 10)

Reference / Parameter	Present				Elasticity (Pagano and Hatfield, 1972)
	SHELL99		SOLID5		
	Mesh	Value	Mesh	Value	
$\overline{w}$	30x30	0.800	10x10	0.715	0.919
	34x34	0.799	14x14	0.718	
	38x38	0.799	18x18	0.719	
$\overline{\sigma}_x$	10x10	0.621	10x10	0.630	0.725
	14x14	0.620	14x14	0.640	
	18x18	0.620	18x18	0.641	
$\overline{\tau}_{xz}$	10x10	0.329	10x10	0.0108	0.420
	14x14	0.328	14x14	0.0113	
	18x18	0.328	18x18	0.0119	
$\overline{\tau}_{xy}$	10x10	0.0103	10x10	0.0095	0.0123
	14x14	0.0104	14x14	0.0100	
	18x18	0.0104	18x18	0.0101	

may now be used with confidence for analysis of thin multi-layered plates both rectangular and square plates.

The FRL composite plates, as used in industrial applications, are mostly thin to moderately thick in nature. However, there is scarcity of results for thin plates in the existing literature.

Therefore, new results are presented for the present plate with thickness ratios 80, 60 and 40 using the SHELL99 element in Table 8.

Example 4: All the earlier problems consider the load on top of the plate as sinusoidally distributed and all the edges to be simply supported. To study the behavior of FRL composite plates under uniformly distributed load (*udl*) and varying boundary conditions, a square plate, having identical lamination lay-up and equal layer thicknesses as in Example 2, is studied under a *udl* q_0 at its top surface. The lamina properties are as in Example 1.

Three different sets of boundaries are used: (i) when all the edges are simply supported (SSSS); (ii) when two parallel edges are simple supported and the other two are clamped (SSCC) and iii) when two parallel edges are simple supported and the other two

Table 5: Deflections and Stresses in a Simply Supported (0/90/0) Rectangular plate (b/a=3) under Sinusoidal load (a/h = 4)

Reference / Parameter	Present				Elasticity (Pagano and Hatfield, 1972)
	SHELL99		SOLID5		
	Mesh	Value	Mesh	Value	
$\overline{w}$	38x38	2.353	10x10	2.035	2.820
	42x42	2.354	14x14	2.037	
	46x46	2.354	18x18	2.038	
$\overline{\sigma}_x$	10x10	0.612	10x10	0.871	1.100
	14x14	0.611	14x14	0.874	
	18x18	0.611	18x18	0.880	
$\overline{\tau}_{xz}$	10x10	0.326	10x10	0.0154	0.387
	14x14	0.325	14x14	0.0162	
	18x18	0.325	18x18	0.0172	
$\overline{\tau}_{xy}$	10x10	0.0203	10x10	0.0190	0.0281
	14x14	0.0204	14x14	0.0192	
	18x18	0.0204	18x18	0.0198	

Table 6: Normal Stresses in a Simply Supported (0/90/90/0) Square Plate Under Sinusoidal Load Using SOLID5 Element

$\frac{a}{h}$	20			10			4		
	Present		Elasticity (Pagano and Hatfield, 1972)	Present		Elasticity (Pagano and Hatfield, 1972)	Present		Elasticity (Pagano and Hatfield, 1972)
	Mesh	Value		Mesh	Value		Mesh	Value	
$\overline{\sigma}_x$	18x18	0.539	0.543	14x14	0.557	0.559	14x14	0.716	0.720
	22x22	0.540		18x18	0.561		18x18	0.720	
	26x26	0.541		22x22	0.563		22x22	0.722	

Table 7: Deflection and in-plane shear stresses in a simply supported (0/90/90/0) square plate under sinusoidal load using SHELL99 element

$\frac{a}{h}$	100			20		
	Present		Elasticity (Pagano and Hatfield, 1972)	Present		Elasticity (Pagano and Hatfield, 1972)
	Mesh	Value		Mesh	Value	
$\overline{w}$	18x18	0.430	0.438	18x18	0.487	0.517
	22x22	0.431		22x22	0.489	
	26x26	0.431		26x26	0.489	
$\overline{\tau}_{xy}$	10x10	0.0213	0.0216	10x10	0.0221	0.0230
	14x14	0.0212		14x14	0.0220	
	18x18	0.0212		18x18	0.220	

Table 8: Deflections and stresses in a simply supported (0/90/90/0) square plate under sinusoidal load using SHELL99 element

$\frac{a}{h}$	$\overline{w}$		$\overline{\sigma}_x$		$\overline{\tau}_{xy}$	
	Present		Present		Present	
	Mesh	Value	Mesh	Value	Mesh	Value
80	18x18	0.431	10x10	0.538	10x10	0.0213
	22x22	0.432	14x14	0.536	14x14	0.0212
	26x26	0.432	18x18	0.536	18x18	0.0212
60	18x18	0.434	10x10	0.537	10x10	0.0214
	22x22	0.435	14x14	0.535	14x14	0.0213
	26x26	0.435	18x18	0.535	18x18	0.0213
40	22x22	0.443	10x10	0.536	10x10	0.0215
	26x26	0.444	14x14	0.534	14x14	0.0214
	30x30	0.444	18x18	0.534	18x18	0.0214

Table 9: Deflections and Stresses in a (0/90/0) Square Plate Under Uniformly Distributed load for Different Boundary conditions (a/h=50)

Boundary	$\overline{w}$		$\overline{\sigma}_x$		$\overline{\tau}_{xy}$	
	Present		Present		Present	
	Mesh	Value	Mesh	Value	Mesh	Value
SSSS	4x4	0.681	10x10	0.808	10x10	0.0435
	6x6	0.680	12x12	0.807	12x12	0.0434
	8x8	0.680	14x14	0.807	14x14	0.0434
SSCC	10x10	0.544	18x18	0.648	14x14	0.0034
	14x14	0.544	22X22	0.646	18X18	0.0031
	18x18	0.544	26X26	0.646	22X22	0.0031
SSFF	10x10	0.681	18x18	0.772	18x18	0.0015
	14x14	0.681	22X22	0.771	22X22	0.0013
	18x18	0.681	26X26	0.771	26X26	0.0012

are free (SSFF). The plate is analyzed using SHELL99 for thickness ratio 50; as results in Example 2 indicates that this element is suitable for analysis of thin to very thin plates. Results are presented in Table 9. In absence of similar results in literature, the new results as presented in Table 9 may be used for comparison during similar studies in future.

CONCLUSION

Structural behavior of fibre reinforced laminated composite plates is studied using ANSYS 11. Two separate elements viz. SHELL99 and SOLID5 are used for the analysis. Numerical examples are carried out to study the performance and the range of applicability of these elements. For this purpose, symmetric laminated composite plates with various lamination lay-ups and different geometries are considered under different loading and boundary conditions. Results of analysis using both the elements are compared with the exact solutions, wherever possible. It is found that SOLID5 has only limited applicability for laminated composite plates. On the other hand, the performance of SHELL99 is encouraging for this category of structures in general, and for thin plates with thickness ratio 20 and higher in particular. However, as the results indicate, this element may not predict transverse shear stresses at the laminar interfaces reliably. Some new results are also presented using SHELL99 for the benefit of future researchers.

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