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Research Paper

ANALYSIS OF PROBABILISTIC PEAK ACCELERATION RESPONSE FOR RANDOM PEDESTRIAN LOADS

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Pedestrian loads that may cause excessive structural vibration involve some uncertain parameters such as walking frequency, step length, dynamic load factors and phases of harmonic components, which will lead to uncertainties of structural response and this issue need to be solved by probabilistic analysis. Considering that the traditional Monte Carlo simulation method for reliability analysis has rather low efficiency, an approach based on uniform design and response surface method for calculating the probabilistic structural response induced by pedestrian vertical loads is proposed to improve the efficiency of structural dynamic analysis with uncertainties. A few representative samples of time history of pedestrian loads are simulated using uniform design first, and then the corresponding peak acceleration response spectra are obtained by dynamic analysis on beam structures with different spans and damping ratios. The spectra which have a certain percentile are obtained by reliability analysis based on response surface method. Then the general formulae of peak acceleration response spectra, which can be used to calculate structural peak accelerations directly, are deduced from parametric analysis of damping ratio and span. Monte Carlo simulation is conducted to validate the precision of this method. The case study shows that compare to the results calculated by the proposed method, the formulae in two widely-used codes such as BS 5400-2:2006, overestimate the peak acceleration of structure with high frequency remarkably and it should be cautious when using them to obtain structural responses.

Keywords: Random pedestrian loads, Uniform design, Response spectrum, Peak acceleration

INTRODUCTION

With the development of building materials, structural form, construction technology and aesthetic standards, there is a trend that structures such as footbridges, gymnasium, stadium and airport passage, are designed to be lighter, slender and more flexible. Under pedestrian loads, these structures with light

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mass and small damping usually hardly undergo significant structural damage, but excessive vibrations that affect structural serviceability may occur due to inappropriate design. For example, in 1993, Fujino *et al.* reported that a cable-stayed pedestrian bridge in Japan presented excessive vibration in the congested condition. In 2000, during the opening day of Millennium Bridge in London, its vibration due to pedestrian was so excessive that the bridge needed to be closed on the opening day (Dallard, 2001; and Strogatz *et al*., 2005). After this incident, the issue of vibration serviceability of similar structures caused by pedestrian loads has increasingly aroused many scholars' and researchers' attention (Song, 2003 and 2005; Zivanovic, 2005, 2007a and b; Huang *et al*., 2007; Piccardo *et al*., 2008; and Fa, 2008). In these literatures, Zivanovic *et al.* (2005) summarized a comprehensive literature review on vibration serviceability of foot-bridges under human-induced excitations, including many aspects such as models of human-induced walking force, physical characteristics of footbridges, calculation methods of structural response, criterion of human comfort, etc.

In order to estimate vibration serviceability of these structures, pedestrian loads should be obtained to calculate structural vibration response at first, and then according to the criterion of human comfort, evaluation of human comfort in terms of response indexes such as peak acceleration, root mean square (RMS) acceleration, etc., can be conducted. Generally, there are two types of model of vertical pedestrian load: deterministic model and probabilistic model (Zivanovic, 2005). The deterministic force model, e.g., sinusoidal model, is simple and easy to be applied in subsequent dynamic analysis. As a result, deterministic sinusoidal models of vertical walking load are adopted in some codes and research. Based on a few deterministic time histories of vertical walking loads, Song (2003 and 2005) proposed a convenient method named response spectrum method, to calculate structural peak acceleration response. Silva *et al.* (2003 and 2007) used a deterministic force model to study vibrations of composite floor and footbridges caused by rhythmic human activities. Figueiredo *et al*. (2008) compared the structural acceleration responses under four different deterministic models of pedestrian vertical force. The results suggest that the change of location of pedestrian loads should be considered in the calculation of acceleration response.

In reality, however, the pedestrian load is a more complex narrow band random process rather than deterministic forces (Zivanovic *et al*., 2007b and 2011). Therefore, some probabilistic models of vertical walking loads also have been studied to obtain a more accurate structural response that can reflect the randomness property of loads and responses. There are also mainly two types of model: time- and frequency-domain models. For time-domain, Zivanovic *et al*. (2007a) took into account many variables in pedestrian loads such as walking frequency, step length, Dynamic Load Factor (DLF) and phrase of each harmonic component, and subsequently used Monte Carlo Simulation (MCS) to generate 2000 samples of pedestrian loads. They calculated the cumulative probability of RMS acceleration of footbridge by statistical analysis of the 2000 time histories of structural responses corresponding to the 2000 samples of pedestrian loads and then obtained the RMS acceleration with a certain percentile. Similar to the concept in Song (2005), Zivanovic *et al.* (2007b and 2011) and Wan *et al.* (2009) provided a few probabilistic design spectra for single walking scenario. In terms of frequency-domain analysis, Eriksson (1994) considered pedestrian loads as a stationary random process and obtained its auto-spectral density. Li *et al.* (2010) and Fan *et al.* (2010) studied the auto-spectral density of vertical walking loads and obtained the RMS acceleration responses.

From the above discussion, it is clear that the walking force is a narrow band random process and the deterministic force model may not reflect its randomness satisfactorily. Moreover, it may overestimate structural acceleration response significantly (Pimentel, 1997 and 2001) Therefore, calculating the structural vibration response by probabilistic force model is more reasonable. Obviously, using MCS can solve this issue very straightforwardly and simply. The conventional probabilistic calculation method using MCS can directly consider all stochastic variables of pedestrian load and be conducted directly, as in Zivanovic *et al.* (2007a). However, its efficiency is rather low as it generally needs covering a wide range of experimental datum (sample space) and conducting lots of experiments (simples). For example, a reliability analysis with failure probability P_f and relative error of simulation ε, the required sampling number of MCS N

$$N \geq \left(\frac{100\varepsilon}{P_f} \right)^2 \quad \text{...(1)}$$

Owing to the failure probability P_f of a real structure is often in the range of $10^{-3} \sim 10^{-4}$, it can be deduced from the above Equation (1) that MCS will lead to plenty of calculation effort. Although the increasing advances of computing power and speed, single calculation of issues such as dynamic analysis, finite element model, fluid dynamics model, etc., can make minutes to hours, if not longer. So analyses of these computer-based issues that require a large number of repeated calculations to obtain a reliable result could be difficult within a limited timeframe. Therefore, it can be seen that MCS is time-consuming and inefficient for these issues that the subsequent calculation of each sample requires much time, e.g., the dynamic analysis. The method of frequency-domain analysis has less calculation and is comparatively more efficient with respect to that of time-domain. However, the methods in literature limit to reflecting the stationary vibration response of structure rather than the real transient vibration response, which is excited by human walking loads in a relative limited time. In addition, the randomness of the excitement location, i.e., the location of foot, is not considered in this method. Consequently, the structure vibration responses based on the method may be different from the one of real responses.

In order to solve the above problem efficiently, the uniform experiment design (UD) method is introduced in this paper. UD can provide some representative samples, which can reduce the required number of samples significantly and cut down calculation work remarkably. In this paper, some representative samples of vertical walking loads were defined by UD, and then the peak acceleration responses are obtained by dynamic analysis in time-domain based on these samples of load. The peak acceleration response spectra with certain percentile were obtained by reliability analysis based on Response Surface Methodology (RSM). Some parametric analyses of damping ratio and span were conducted to provide general forms of response spectrum. The MCS was made to validate the precision of this method. Finally, a case study was presented to show how to use the spectra to calculate the related peak acceleration of structure induced by vertical pedestrian loads, together with a comparative analysis about methods in some related currently-used codes of practice to demonstrate that these methods may overestimate the structural response remarkably.

UD AND RSM

As mentioned in forgoing section, even though the computing power of computer has been improving remarkably, some engineering issues still require a large amount of time to obtain good accuracy. This deficiency may become an obstacle to deal with some issues such as dynamic analysis, finite element analysis and reliability analysis. In order to minimize the computational expense of running these computer analyses for reliability analysis that usually uses traditional MCS, statistical approximation techniques, e.g., experiment design and RSM are becoming widely used in engineering (Simpson, 2001a). An experiment design systematically selects a sequence of experiments to be performed, which is essential for effective experimentation (Simpson, 2001b) According to these representative experiments, the RSM can form an approximation of the relationship between response/output and a number of input parameters that is accurate enough to replace the original model. Then we can make use of this approximation to make subsequent analysis so that the computationally expensive simulation or calculation is no longer required, which can facilitate analysis and enhance the efficiency remarkably. A more extensive introduction about RSM is given by Box (2007).

UD method is one of the above experiment design methods. This method was proposed by Fan and Wang, which scatters experiment points uniformly in the range of experiment parameters, i.e., design space, and selects a sequence of representative experiment points to organize experiments (Fang, 1994 and 2001). Essentially, it is a type of fractional factorial design with an extra property of uniformity. Similar to orthogonal experimental design (OD), it carries out experiment runs according to a series of specifically designed tables, i.e., UD table. Compare to OD, UD has higher efficiency as it requires fewer sample points and obtain better coverage of design space. UD table is important for select some representative sample set and it is usually expressed as U_N (q^s), where U stands for uniform design, N is the number of experiment runs to conduct, q is the level number of each parameter and s is the number of total parameters that the table can contain at most. It is obvious that the remarkable character of UD is that its required number of experiment usually equals the number of parameter levers. This means that the number of experiment organized by UD is much less than that organized by OD or full factorial design. Take an experiment with s parameters and q levels for each parameter, for instance, UD only needs q experiment runs, while OD needs q^2 runs and full factorial design needs q^s runs. With respect to OD and full factorial design, UD can improve experiment efficiency significantly, especially for experiment involving more than 3 parameters or levels. Some successful applications are presented in (Fang, 1994, 2001, 2003 and 2008; Liang *et al*., 2001; Song *et al*., 2010 and 2012). Fang (1994) also provides many UD tables, which can be used directly for applications.

For an experiment with m variables, X_1, X_2, …, X_m, the procedures of UD are listed as follows:

1. Defining the range [X_{imin}, X_{imax}] (i = 1, 2, …, m) of each variable, where X_{imin} and X_{imax} are minimum and maximum value of the i^{th} parameter, respectively.
2. Dividing each variable into n levels, usually they are equally scattered,

$$X_{ij} = X_{i\min} + \frac{j-1}{n}\left(X_{i\max} - X_{i\min}\right) \qquad ...(2)$$

where j = 1, 2, …, n is the level number; X_{ij} is the j^{th} level of the i^{th} parameter. Levels also can be divided unequally, and the corresponding process can refer to Fang (1994).

3. Choosing a proper UD table to design the experiment

The selection of UD table is determined by the numbers of parameters and of levels. We can use appropriate UD table U_n (n^m) in Fang (1994) or we can construct UD table according to the methodology in Fang (1994 and 2003). After finishing all experiments, the corresponding response surface can be formed based on the input variable sets and the output.

Simpson *et al.* (2001) compared four design experiments by examples and the results show that the good design space coverage of UD tends to provide more accurate approximation globally even with a low sample size. Song *et al*. (2010, 2012) conducted a reliability analysis of compartment fire by adopting UD. The results of 24 sets of experiments run designed by UD present good agreement with the ones of 5,000 MCSs, which can show the efficiency of UD.

SIMULATION OF HUMAN-INDUCED WALKING FORCE BASED ON UD

In time-domain, vertical pedestrian waking loads are usually expressed as (Ebrahimpour *et al*., 1996; Zivanovic *et al*., 2005 and 2011)

$$F(t) = W\left[1 + \sum_{i=1}^{n} DLF_i \cos(2i\pi f_s t + \theta_i)\right] \qquad ...(3)$$

where, W is pedestrian weight; DLF_i is dynamic load factor of *ith* harmonic component; f_s is walking step frequency; θ_i is phase of *ith* harmonic component. According to Kerr's study (2001), the DLFs of higher harmonic are small and by the fifth harmonic the DLFs are about zero. This paper therefore considers only the first five harmonic components and the dynamic component of vertical walking loads could be expressed as

$$F(t) = W \cdot \sum_{i=1}^{5} DLF_i \cos(2i\pi f_s t + \theta_i) \qquad ...(4)$$

The duration and location of the above load model are also related to step length L_s, so the pedestrian walking loads reconstructed by the Equation (4) contain 13 parameters totally, i.e., pedestrian weight W, walking step frequency f_s, step length L_s, DLFs and phase θ_i of the first five harmonic component. According to Zivanovic *et al.* (2007a, 2011) and Kerr *et al*. (2001), these parameters and their probability distribution are shown in Table 1, which are represented as X_1 – X_{13} hereinafter for ease of referencing.

As the foregoing discussion, it is very straightforward to use MCS method to simulate many samples of vertical walking forces based on Equation (4) and distribution of each parameter in Table 1, as in Zivanovic *et al.* (2007a) and Fa *et al.* (2008). However, this method will cause a lot of calculation works for the subsequent dynamic response analysis. Therefore, UD method is introduced here to reduce the required number of samples and solve the issue efficiently.

The steps of UD for this issue are listed as follows.

(1) Defining the range [X_{imin}, X_{imax}] (i = 1,2,…,m) of each variable. In this paper, the range of each uniform parameter, i.e., the phase, is [0, 2π]; the range of each normal parameter is

$$X_{i\min} = \mu_i - 3\sigma_i \quad ...(5a)$$

$$X_{i\max} = \mu_i + 3\sigma_i \quad ...(5b)$$

where μ_i and σ_i are mean value and standard deviation of the parameter X_i respectively.

(2) Dividing each parameter into n levels equally,

$$X_{ij} = X_{i\min} + \frac{j-1}{n}\left(X_{i\max} - X_{i\min}\right) \quad ...(6)$$

where j=1,2,…,n is the level number, X_{ij} is the j^{th} level of the i^{th} parameter. In this paper, all parameters are divided equally into 30 levels in their own range. The levels of all parameters are shown in Table 2. It should be noted that in Table 2 some levels that are less than zero are replaced by zero as they are meaningless with negative values.

Table 1: Parameters Involved in Pedestrian Load Model and Their Probability Distributions

Parameters		Distribution	Mean	Standard Deviation
f_s/Hz	X_1	normal	1.87	0.186
L_s/m	X_2	normal	0.71	0.071
W/N	X_3	normal	640	82
DLF1	X_4	normal	μ_{DLF1^*}	0.16μ_{DLF1}
DLF2	X_5	normal	0.07	0.030
DLF3	X_6	normal	0.05	0.020
DLF4	X_7	normal	0.05	0.020
DLF5	X_8	normal	0.3	0.015
θ_i (i = 1 ~ 5)	X_9 ~ X_{13}	uniform in [0, 2π]	–	–

Note: μ_{DLF1} is relative to step frequency f_s, the relationship is μ_{DLF1} = –0.2649;f_s^3+1.3206;f_s^2–1.759f_s+0.7631→–0.2649f_s^3+1.3206 f_s^2–1.759f_s+0.7613, see Pimentel (2004).

Table 2: Levels of All Parameters

	X	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}
1	1.31	0.50	394	0.52	0	0	0	0	0.00	0.00	0.00	0.00	0.00
2	1.35	0.51	411	0.553	0	0	0	0	0.11	0.11	0.11	0.11	0.11
3	1.39	0.53	428	0.586	0	0	0	0	0.22	0.22	0.22	0.22	0.22
4	1.43	0.54	445	0.619	0	0.048	0.048	0	0.33	0.33	0.33	0.33	0.33
5	1.47	0.56	462	0.652	0.069	0.131	0.131	0	0.43	0.43	0.43	0.43	0.43
6	1.50	0.57	479	0.686	0.158	0.214	0.214	0.017	0.54	0.54	0.54	0.54	0.54
7	1.54	0.59	496	0.719	0.246	0.297	0.297	0.121	0.65	0.65	0.65	0.65	0.65
8	1.58	0.60	513	0.752	0.335	0.379	0.379	0.224	0.76	0.76	0.76	0.76	0.76
9	1.62	0.62	530	0.785	0.424	0.462	0.462	0.328	0.87	0.87	0.87	0.87	0.87
10	1.66	0.63	547	0.818	0.512	0.545	0.545	0.431	0.98	0.98	0.98	0.98	0.98
11	1.70	0.64	564	0.851	0.601	0.628	0.628	0.534	1.08	1.08	1.08	1.08	1.08
12	1.74	0.66	581	0.884	0.69	0.71	0.71	0.638	1.19	1.19	1.19	1.19	1.19
13	1.77	0.67	598	0.917	0.778	0.793	0.793	0.741	1.30	1.30	1.30	1.30	1.30
14	1.81	0.69	615	0.95	0.867	0.876	0.876	0.845	1.41	1.41	1.41	1.41	1.41
15	1.85	0.70	632	0.983	0.956	0.959	0.959	0.948	1.52	1.52	1.52	1.52	1.52
16	1.89	0.72	649	1.017	1.044	1.041	1.041	1.052	1.63	1.63	1.63	1.63	1.63
17	1.93	0.73	665	1.05	1.133	1.124	1.124	1.155	1.73	1.73	1.73	1.73	1.73
18	1.97	0.75	682	1.083	1.222	1.207	1.207	1.259	1.84	1.84	1.84	1.84	1.84
19	2.01	0.76	699	1.116	1.31	1.29	1.29	1.362	1.95	1.95	1.95	1.95	1.95
20	2.04	0.78	716	1.149	1.399	1.372	1.372	1.466	2.06	2.06	2.06	2.06	2.06
21	2.08	0.79	733	1.182	1.488	1.455	1.455	1.569	2.17	2.17	2.17	2.17	2.17
22	2.12	0.81	750	1.215	1.576	1.538	1.538	1.672	2.28	2.28	2.28	2.28	2.28
23	2.16	0.82	767	1.248	1.665	1.621	1.621	1.776	2.38	2.38	2.38	2.38	2.38
24	2.20	0.84	784	1.281	1.754	1.703	1.703	1.879	2.49	2.49	2.49	2.49	2.49
25	2.24	0.85	801	1.314	1.842	1.786	1.786	1.983	2.60	2.60	2.60	2.60	2.60
26	2.27	0.86	818	1.348	1.931	1.869	1.869	2.086	2.71	2.71	2.71	2.71	2.71
27	2.31	0.88	835	1.381	2.02	1.952	1.952	2.19	2.82	2.82	2.82	2.82	2.82
28	2.35	0.89	852	1.414	2.108	2.034	2.034	2.293	2.93	2.93	2.93	2.93	2.93
29	2.39	0.91	869	1.447	2.197	2.117	2.117	2.397	3.03	3.03	3.03	3.03	3.03
30	2.43	0.92	886	1.48	2.286	2.2	2.2	2.5	3.14	3.14	3.14	3.14	3.14

(3) Choosing a proper UD table to design the experiment

As mentioned before, the experiment involves 13 parameters and each parameter has 30 levels, the table U_{30}*(30^{13}) therefore is selected, which is presented in Table 3. The table can contain 13 parameters and each parameter has 30 levels. Once we have

Table 3: UD Table U30*(30^{13})

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}
1	1	2	5	6	8	15	17	19	20	21	22	27	28
2	2	4	10	12	16	30	3	7	9	11	13	23	25
3	3	6	15	18	24	14	20	26	29	1	4	19	22
4	4	8	20	24	1	29	6	14	18	22	26	15	19
5	5	10	25	30	9	13	23	2	7	12	17	11	16
6	6	12	30	5	17	28	9	21	27	2	8	7	13
7	7	14	4	11	25	12	26	9	16	23	30	3	10
8	8	16	9	17	2	27	12	28	5	13	21	30	7
9	9	18	14	23	10	11	29	16	25	3	12	26	4
10	10	20	19	29	18	26	15	4	14	24	3	22	1
11	11	22	24	4	26	10	1	23	3	14	25	18	29
12	12	24	29	10	3	25	18	11	23	4	16	14	26
13	13	26	3	16	11	9	4	30	12	25	7	10	23
14	14	28	8	22	19	24	21	18	1	15	29	6	20
15	15	30	13	28	27	8	7	6	21	5	20	2	17
16	16	1	18	3	4	23	24	25	10	26	11	29	14
17	17	3	23	9	12	7	10	13	30	16	2	25	11
18	18	5	28	15	20	22	27	1	19	6	24	21	8
19	19	7	2	21	28	6	13	20	8	27	15	17	5
20	20	9	7	27	5	21	30	8	28	17	6	13	2
21	21	11	12	2	13	5	16	27	17	7	28	9	30
22	22	13	17	8	21	20	2	15	6	28	19	5	27
23	23	15	22	14	29	4	19	3	26	18	10	1	24
24	24	17	27	20	6	19	5	22	15	8	1	28	21
25	25	19	1	26	14	3	22	10	4	29	23	24	18
26	26	21	6	1	22	18	8	29	24	19	14	20	15
27	27	23	11	7	30	2	25	17	13	9	5	16	12
28	28	25	16	13	7	17	11	5	2	30	27	12	9
29	29	27	21	19	15	1	28	24	22	20	18	8	6
30	30	29	26	25	23	16	14	12	11	10	9	4	3

defined the UD table, each experiment can be organized in the light of the table. For example, the eighth row of the table is {8, 16, 9 …}. The numbers in the row mean that the parameter set in the eighth experiment is the eighth level of X_1, the 16th level of X_2, the ninth level of X_3, and so on. Then time history of the eighth walking load can be obtained by substituting these 13 parameter defined by UD table into Equation (4). The time history of the eighth vertical walking load is shown in Figure 1. All 30 groups of parameter determined by the U_{30}*(30^{13}) are shown in Table 4.

Figure 1: Load History Defined by the Eighth Sets of UD Table

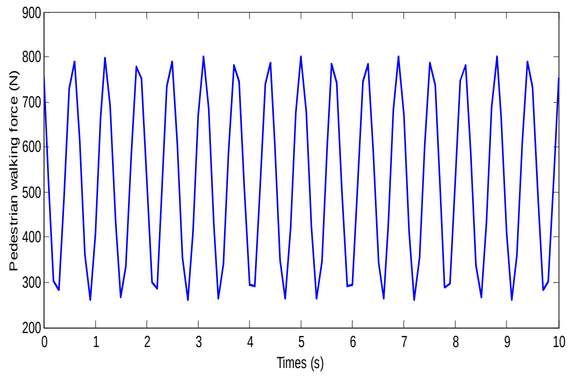


Table 4: All 30 Groups of Parameter Determined by U_{30}*(30^{13})

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}
1	1.31	0.51	462	0.087	0.067	0.064	0.073	0.066	0.76	1.73	2.06	2.27	2.92
2	1.35	0.54	547	0.125	0.160	0.015	0.031	0.053	1.63	0.22	0.87	1.30	2.60
3	1.39	0.57	632	0.168	0.061	0.093	0.000	0.041	2.49	2.06	3.03	0.32	2.27
4	1.43	0.60	716	0.218	0.154	0.044	0.077	0.028	0.00	0.54	1.84	2.71	1.95
5	1.47	0.63	801	0.274	0.054	0.000	0.036	0.016	0.87	2.38	0.65	1.73	1.63
6	1.50	0.66	886	0.131	0.148	0.073	0.000	0.004	1.73	0.87	2.82	0.76	1.30
7	1.54	0.69	445	0.185	0.048	0.023	0.081	0.000	2.60	2.71	1.63	3.14	0.97
8	1.58	0.72	530	0.245	0.141	0.102	0.040	0.075	0.11	1.19	0.43	2.17	0.65
9	1.62	0.75	615	0.312	0.042	0.052	0.000	0.063	0.97	3.03	2.60	1.19	0.32
10	1.66	0.78	699	0.386	0.135	0.002	0.085	0.050	1.84	1.52	1.41	0.22	0.00
11	1.70	0.81	784	0.176	0.036	0.081	0.044	0.038	2.71	0.00	0.22	2.60	3.03
12	1.74	0.83	869	0.246	0.129	0.031	0.002	0.025	0.22	1.84	2.38	1.63	2.71
13	1.77	0.86	428	0.322	0.030	0.110	0.089	0.013	1.08	0.32	1.19	0.65	2.38
14	1.81	0.89	513	0.404	0.123	0.060	0.048	0.001	1.95	2.17	0.00	3.03	2.06
15	1.85	0.92	598	0.493	0.023	0.011	0.007	0.000	2.82	0.65	2.17	2.06	1.73
16	1.89	0.50	682	0.213	0.117	0.089	0.093	0.072	0.32	2.49	0.97	1.08	1.41
17	1.93	0.53	767	0.297	0.017	0.040	0.052	0.059	1.19	0.97	3.14	0.11	1.08
18	1.97	0.56	852	0.387	0.110	0.000	0.011	0.047	2.06	2.82	1.95	2.49	0.76
19	2.00	0.59	411	0.481	0.011	0.069	0.098	0.035	2.92	1.30	0.76	1.52	0.43
20	2.04	0.61	496	0.579	0.104	0.019	0.056	0.022	0.43	3.14	2.92	0.54	0.11
21	2.08	0.64	581	0.239	0.005	0.098	0.015	0.010	1.30	1.63	1.73	2.92	3.14

Table 4 (Cont.)

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}
22	2.12	0.67	665	0.332	0.098	0.048	0.102	0.000	2.17	0.11	0.54	1.95	2.82
23	2.16	0.70	750	0.429	0.000	0.000	0.060	0.000	3.03	1.95	2.71	0.97	2.49
24	2.20	0.73	835	0.529	0.092	0.077	0.019	0.069	0.54	0.43	1.52	0.00	2.17
25	2.24	0.76	394	0.630	0.000	0.027	0.106	0.056	1.41	2.27	0.32	2.38	1.84
26	2.27	0.79	479	0.246	0.086	0.106	0.064	0.044	2.27	0.76	2.49	1.41	1.52
27	2.31	0.82	564	0.344	0.000	0.056	0.023	0.032	3.14	2.60	1.30	0.43	1.19
28	2.35	0.85	648	0.441	0.079	0.007	0.110	0.019	0.65	1.08	0.11	2.82	0.87
29	2.39	0.88	733	0.539	0.000	0.085	0.069	0.007	1.52	2.92	2.27	1.84	0.54
30	2.43	0.91	818	0.634	0.073	0.036	0.027	0.000	2.38	1.41	1.08	0.87	0.22

Peak Response Spectrum of Structure Induced by Vertical Loads from Walking

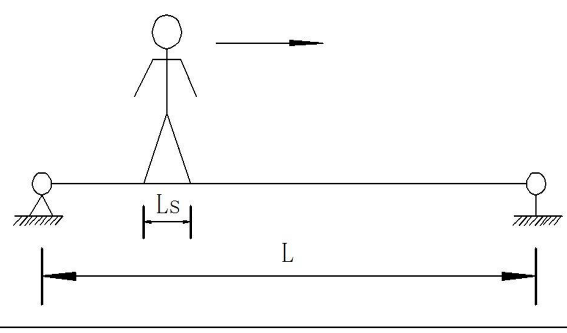
Once the vertical loads of human walking have been defined, the corresponding structural response, which can be used to evaluate the structural serviceability, can be calculated by structural dynamic analysis. Since the boundary conditions of structures hardly have substantial influence on the structural responses for this type of loads Song (2004) and Song and Jin (2005), only simply supported beam is analyzed here. The equation of motion of the structure under the excitation of single pedestrian is usually expressed their modal form as (Zivanovic *et al.*, 2007, 2007a and 2011; and Wan, 2009),

$$\ddot{y}(t) + 2\xi\omega_n\dot{y}(t) + \omega_n^2 y(t) = \frac{F(t)}{M}\sin\left(\frac{\pi v_p}{L}t\right) \quad ...(7)$$

where $y(t)$ is the modal vertical displacement of structure; ξ and ω are structural damping ratio and frequency of the mode, respectively; only considering the first mode, M is the generalized mass of the first mode and $F(t)$ is the vertical walking loads by single pedestrian defined in the foregoing section.

It should be noted that the location of loads defined by Equations (4) and (7) not only varies with time, but also the loads just excite on some certain points, instead of a continuing force on the beam like a vehicle force model in Equation (7) as shown in Figure 2. Therefore, the force model in Equation (7) may not reflect the actual human walking excitation accurately. In this paper, a force model that just excites on the locations of foot is proposed as in the following Equation (8), which can reflect the mechanism of walking excitation more accurately. For the beam-type structure shown in Figure 2, the structural response can be calculated according to the following equation of motion,

Figure 2: Structure and Excitation from Pedestrian Walking



$$\frac{\partial^2}{\partial x^2}\left[EI(x)\left(\frac{\partial^2 v(x,t)}{\partial x^2}+\alpha_1\frac{\partial^3 v(x,t)}{\partial x^2\partial t}\right)\right]+m\frac{\partial^2 v(x,t)}{\partial t^2}$$

$$+c\frac{\partial v(x,t)}{\partial t}=\sum_{i=1}^{N_s}\delta\left[x-(i-1)L_s\right]F(t) \qquad ...(8)$$

where $\mu(x, t)$ is the vertical displacement of beam; EI, m and c are bending stiffness, mass of unit length and damping of beam, respectively; N_s is total number of steps required to cross the beam; $F(t)$ is the vertical loads of walking obtained in foregoing section; α_1 is the ratio of Rayleigh damping to stiffness; $\delta(x)$ is defined as

$$\delta(x)=\begin{cases}0 & x\neq 0\\ 1 & x=0\end{cases} \qquad ...(9)$$

A common method to solve Equation (8) is mode superposition method to uncouple it to Single-Degree-of-Freedom (SDOF) systems as follows (Clough, 1993),

$$\ddot{Y}_k+2\xi_k\omega_k\dot{Y}_k+\omega_k^2Y_k=Q_k(t)/M_k \qquad ...(10)$$

where $\ddot{Y}_k,\dot{Y}_k,Y_k$, ξ_k and ω_k are modal (or generalized) acceleration, velocity, displacement, damping ratio and circular frequency for the kth mode, respectively; $M_k=\int_0^L\varphi_k^2(x)m dx$ is modal mass for the kth mode(in the initial calculation of this paper, M_1 is set as 1), where $\varphi_k(x)$ is k^{th} mode; $Q_k(t)$ is modal load for the k^{th} mode and can be calculated by

$$Q_k(t)=\int_0^L\sum_{i=1}^{N_s}\delta\left[x-(i-1)L_s\right]F(t)\varphi_k(x)$$

$$=\int_0^L F(t)\varphi_k(s\times L_s)dx \qquad ...(11)$$

here, s = 1, 2, …, is the sequence number of steps on the structure, so $s \times L_s$ is the location of each foot, where s can be defined by the following equation,

$$s=\text{int}(f_s t) \qquad t<t_d \qquad ...(12)$$

where t_d is duration of pedestrian walking on the structure; int(.) is the integral function towards zero. After the modal responses are obtained by solving the Equation (10) using step-by-step method, the structural acceleration response $\ddot{v}(x,t)$ can be expressed as

$$\ddot{v}(x,t)=\sum_1^n\varphi_k(x)\ddot{Y}_k(t) \qquad ...(13)$$

For the simply supported beam, the mode shape $\varphi_k(x)$ is

$$\varphi_k(x)=\sin\frac{k\pi}{L}x \qquad ...(14)$$

Only the response of the first mode is calculated here since the response of the first mode is dominant (Song *et al.* 2005; Zivanovic *et al.*, 2007 and 2011) and the second mode shape is zero at the mid-span. Therefore, the resultant acceleration response can be expressed as:

$$\ddot{v}(x,t)=\varphi_1(x)\ddot{Y}_1(t)=\sin(\frac{\pi x}{L})\ddot{Y}_1(t) \qquad ...(15)$$

Hence, it is apparent that the maximum structural acceleration response in the Eq. (15) is at the mid- span. So the peak response of structure with circular frequency ω_j could be deduced from the following equation,

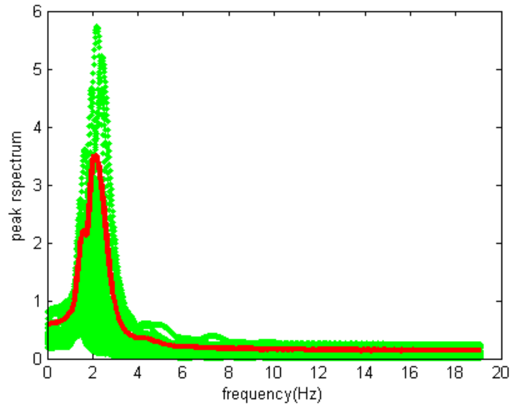
$$S_p(\omega_j)=\max\left(\left|\varphi_1(x)\ddot{Y}_1(t)\right|\right)$$

$$=\max\left(\left|\sin(\frac{\pi x}{L})\ddot{Y}_1(t)\right|\right)=\max\left(\left|\ddot{Y}_1(t)\right|\right) \qquad ...(16)$$

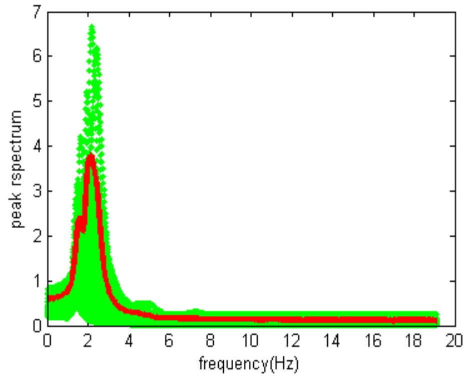
Therefore, the peak acceleration response spectrum of structure under walking loads can be obtained by calculating the peak response of structures with different frequency ω_j. Since response spectrum is also related to damping ratio of the first mode ξ_1 and structural span L (or duration of vibration), five different modal damping ratios ξ_1, i.e., 0.05, 0.02, 0.01, 0.005 and 0.002, the range of which is referred to the recommendation proposed by Bachman *et al.* (1995), and six different spans, i.e., 6 m, 9 m, 12 m, 18 m, 24 m and 32 m, are considered in calculation, respectively. Length of be confined to, only the peak acceleration response spectra of beam with damping ratio 0.05 are given in Figure 3 as the green background, which contains 30 sets of response. To be consistent with the response calculation proposed by Song *et al.* (2005), the values of response spectrum in the figures are divided by mean value of pedestrian weight.

Figure 3: Peak Acceleration Response Spectra of Simply Supported Beam With Damping Ratio 0.05 (Green Background is 30 Response Spectra, and The Red Heavy Line Curve is Response Spectrum With 95% Percentiles)

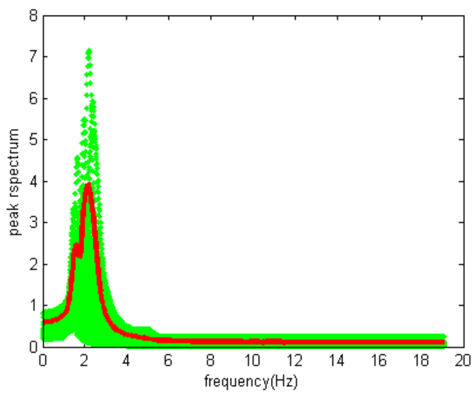
(a) Damping Ratio 0.05, Span 6 m



(b) Damping Ratio 0.05, Span 9 m



(c) Damping Ratio 0.05, Span 12 m



(d) Damping Ratio 0.05, Span 18 m

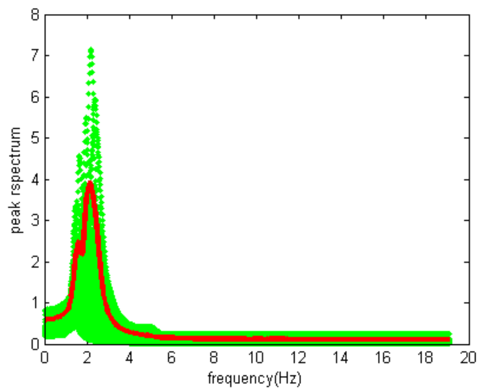
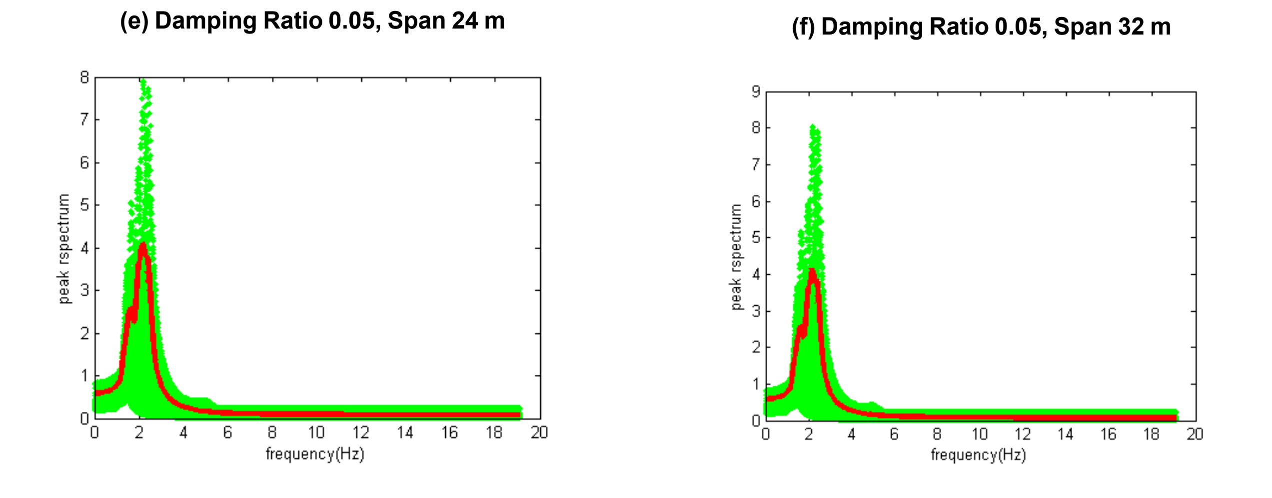


Figure 3 (Cont.)



Probabilistic Response Spectrum Analysis

It is obvious that for each structure with a certain frequency, 30 samples of walking load will produce 30 acceleration responses, and then a RS function between the response and the parameters of experiment can be obtained by regression analysis. This is a multiple regression, which can choose many functions, and in this paper the following linear fitting function is selected,

$$S_p(\omega_j) = \sum_{l=1}^{13} a_l X_l + b \qquad \text{...(17)}$$

where X_l (l =1, 2,.., 13) is the parameter as shown Table 1. The RS function of spectrum for all frequency ω_j can be obtained by regression analysis for each structural frequency ω_j point by point. For example, the parameters of the fitting function of RS of structure with span = 12 m, μ = 0.05 and natural frequency $\omega_j = 4\pi$ are shown in Table 5.

Once the RS function at each frequency ω_j is obtained, the response spectrum $S_{p\alpha}(\omega_j)$ with 95% percentile at every structural frequency ω_j can be deduced from the following equation,

$$S_{P\alpha}(\omega_j) = \mu_{p\omega_j} + 1.645\sigma_{p\omega_j} \qquad \text{...(18)}$$

where $\mu_{p\omega j}$ and $\sigma_{p\omega j}$ are mean and standard deviation of response spectrum at frequency ω_j, respectively, which can be calculated from the First-order Second-moment method,

$$\mu_{p\omega} = \sum_{l=1}^{13} a_l \mu_{X_l} + b \quad \sigma_{p\omega} = \sqrt{\sum_{l=1}^{13} a_l^2 \sigma_{X_l}^2} \qquad \text{... (19)}$$

here, μ_{X_l} and σ_{X_l} are mean and standard deviation of the l^{th} parameter X_l, respectively.

Table 5: Regression Parameters a_i and b (L= 12 m, ξ = 0.05, $\omega = 4\pi$ rad/s)

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	b
1.94	–7.28	0.001	7.20	2.68	2.36	7.52	–0.09	0.04	–0.20	0.09	0.17	0.11	0.00

Therefore, peak acceleration response spectra with 95% percentile for different damping ratios and different spans can be obtained, which are presented in Figure 3 as red heavy line curves and Figure 4.

For subsequent analysis, the regression analysis is made on response spectrum of structure with span = 12 m, and ξ = 0.01. The fitting results are expressed as Equation (20), and the fitting curves are shown in Figure 5.

Figure 4: Spectra for 6 Different Spans and 5 Different Ratios

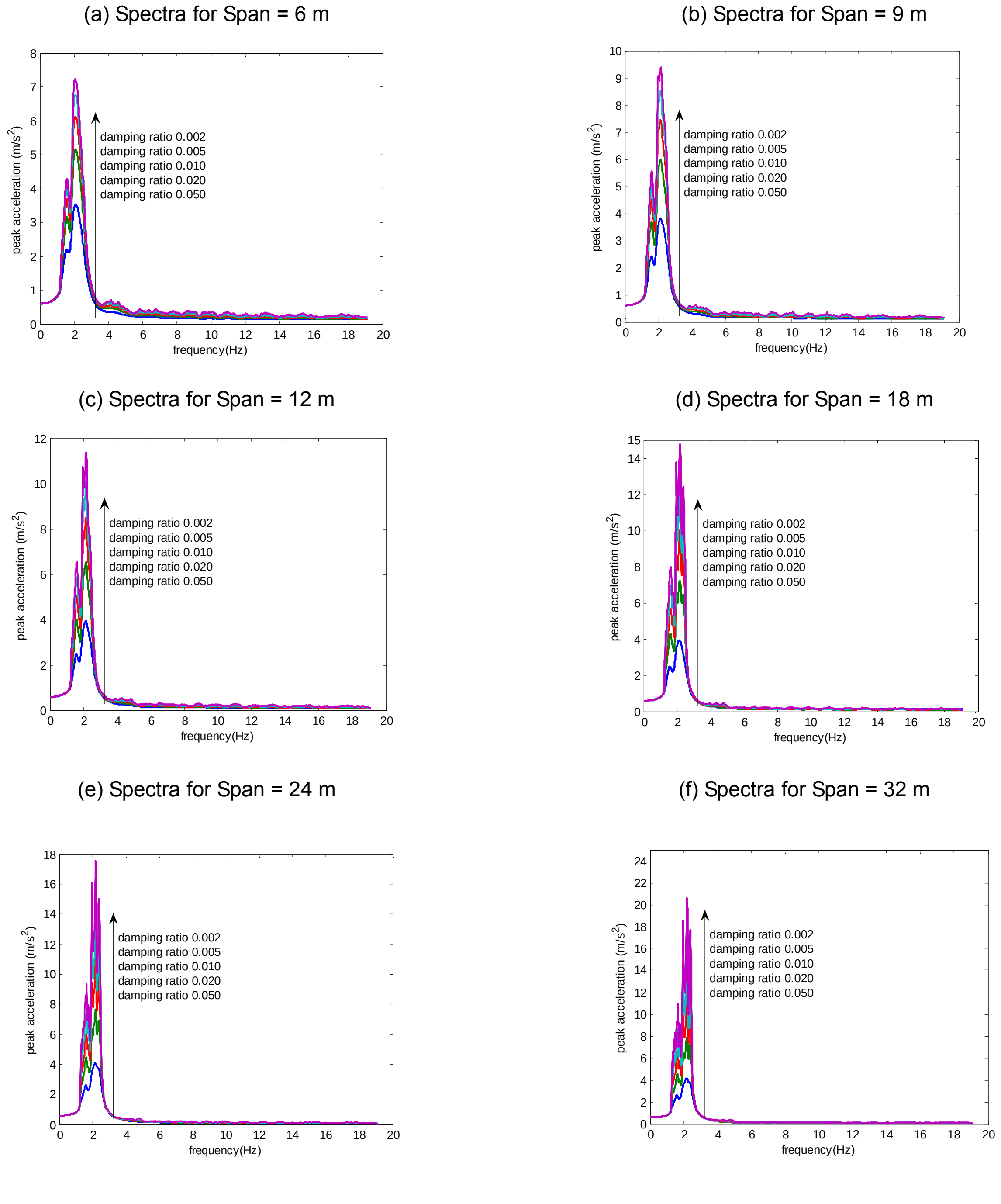
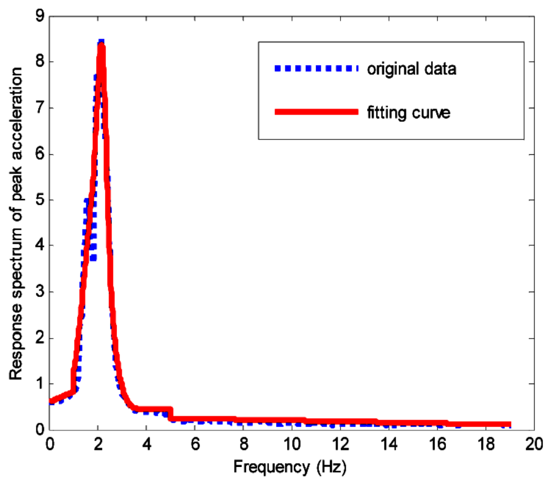


Figure 5: Peak Acceleration Response Spectrum of Structure with Span=12 m, ξ = 0.01



$$S_{P\alpha}(f_{st},\xi=0.01,L=12m)$$

$$=\begin{cases}0.581+0.204f_{st}\\ 4.477e^{-(\frac{f_{st}-1.967}{0.7060})^2}+3.670e^{-(\frac{f_{st}-2.210}{0.2571})^2}+0.455\\ 0.693-0.0286f_{st}\end{cases}$$

0 Hz < f_{st} <1 Hz

1 Hz ≤ f_{st} < 5 Hz

5 Hz ≤ f_{st} ≤ 19 Hz ...(20)

where f_{st} is structure frequency.

PARAMETRIC ANALYSIS

As can be seen in the results of response spectra in Figure 4, it is clear that all curves of spectrum have similar characteristics of shape although there are relative differences in numerical value. The characteristics of these spectra in three different structure frequency f_{st} intervals can be explained as follows:

(1) 0 Hz<f_{st}<1 Hz. The envelop curve for all fitting response spectrum in this interval is given as the Equation (21) since there is only slight difference between all spectra within this interval,

$$S_{P\alpha}(f_{st},\xi,L)=0.586+0.219f_{st} \quad ...(21)$$

It can be seen that the response spectrum in this interval only increases with structure frequency f_{st}.

(2) 1 Hz ≤ f_{st} < 5 Hz. Since this interval includes the range of walking frequency f_s, the response of structure may be magnified by potential resonance. From the Figure 4, it can be seen that all maximum values of peak acceleration response spectrum are located at about 2.2 ~ 2.3 Hz. Relative values of all response spectra are obtained on the base of the response spectrum of structure with span L = 12 m, ξ = 0.01, in other word, being divided by the corresponding spectrum value of of structure with span L = 12 m, ξ = –0.01. Then the sensitivity function $\phi(\xi, L)$ for ξ and L can be obtained by regression analysis for the all relative value,

$$\varphi(\xi,L)=\frac{0.8253+33.47\xi+0.06676L}{1+121.02\xi} \quad ...(22)$$

Therefore, the common function of peak response spectrum $S_{P\alpha}(f_{st}, \xi, L)$ for structure with frequency in this interval can be expressed as follows:

$$S_{P\alpha}(f_{st},\xi,L)=S_{P\alpha}(f_{st},\xi=0.01,L=12m)\times\varphi(\xi,L)$$

$$=\left(4.477e^{-(\frac{f_{st}-1.967}{0.7060})^2}+3.670e^{-(\frac{f_{st}-2.210}{0.2571})^2}+0.455\right)$$

$$\times\frac{0.8253+33.47\xi+0.06676L}{1+121.02\xi} \quad ...(23)$$

It is apparent that the response spectrum in this interval increases with decrease in î and increase in L.

(3) $5 \le f_{st} \le 19$Hz. The linear function of spectrum obtained by regression analysis for all response spectrums in this interval is as follow:

$$S_{P\alpha}(f_{st}, \xi, L) = 0.3766 - 1.236\xi - 0.0098 f_{st} - 0.0049L \quad ...(24)$$

From the Figure 4 and the above Equation (20) we can see that the response spectrum in this interval has little correlation to ω, f_{st} and L.

Therefore, the $S_{P\alpha}(f_{st}, \xi, L)$ for any f_{st}, ξ and L can be obtained by some simple procedures as follows: 1) Calculating structural properties such as f_{st} and M_1; 2) Selecting a formula from Equations (21), (23) or Equation (24) according to f_{st}, and then calculating $S_{P\alpha}(f_{st}, \xi, L)$ by the selected equation; 3) The actual peak acceleration response of the structure is $S_{P\alpha}(f_{st}, \xi, L) \times \mu_W / M_1$.

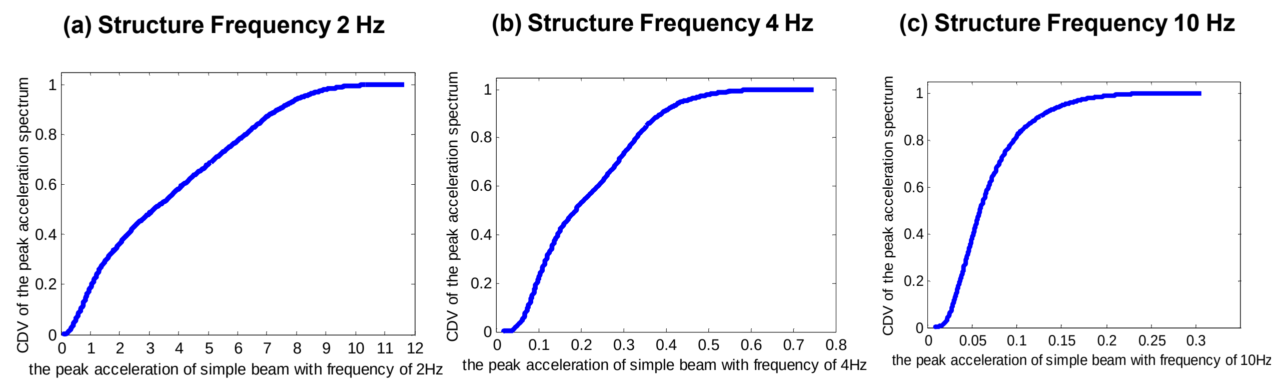
STRUCTURE RELIABILITY ANALYSIS BASED ON MONTE CARLO SIMULATION

In order to validate the precision of this method, probabilistic structural dynamic responses are also calculated by Monte Carlo simulation. 5000 MCSs are carried out on the simply supported beam with span=12 m, damping ratio = 0.01, structure frequency = 2 Hz, 4 Hz and 10 Hz, respectively. The calculation procedure is listed as follow:

1. Generate 5000 random samples for each parameter according to the probability distribution of each parameter in Table 1;
2. Obtain 5000 pedestrian loads by substituting the 5000 sets of parameters into the Equation (4);
3. Exert the 5000 loads on the structure respectively, and then obtain the corresponding 5000 peak acceleration responses by dynamic analysis in time-domain;
4. Calculate the Cumulative Distribution curve (CDV) of peak acceleration response by statistic analysis, then response value with 95% percentile can be obtained from the corresponding CDV easily.

The CDV of each structure is shown in the Figure 6. And for comparison, the acceleration responses with same percentile calculated by

Figure 6: CDV of Peak Acceleration Response Spectrum of Structure with Span = 12 m, ξ = 0.01



MCS and UD are shown in Table 6. It can be seen that the differences between the results of two methods are small, which means that the method proposed in this paper has a satisfied precision. The cause of the minor differences may be that for calculating response spectrum rapidly and easily, the linear fitting regression is adopted to construct RS, which will definitely lead to some differences. If a more appropriate fitting function is chose according to each RS at each frequency, the subsequent reliability analysis would have better results.

CASE STUDY

In order to show the procedure of calculating structural peak acceleration based on the response spectrum, a case study is presented here. The peak acceleration of the structure is obtained according to the given equations in this paper, i.e., Equations (21), (23) or Equation (24). And for comparison, the peak acceleration responses of the case are also evaluated by referring some standards such as BS 5400-2:2006 and Canadian Highway Bridge Design Code.

In the foregoing two codes, some procedures to calculate the peak acceleration response of beam structures with one, two or three spans excited by single pedestrian are provided. The procedures in these two codes are similar and hence only the latter one are explained here. In Canadian Highway Bridge Design Code, the peak acceleration of simply supported beam structure can be directly calculated by

$$a = 4\pi^2 f_1^2 w_s K\Psi \quad \text{... (25)}$$

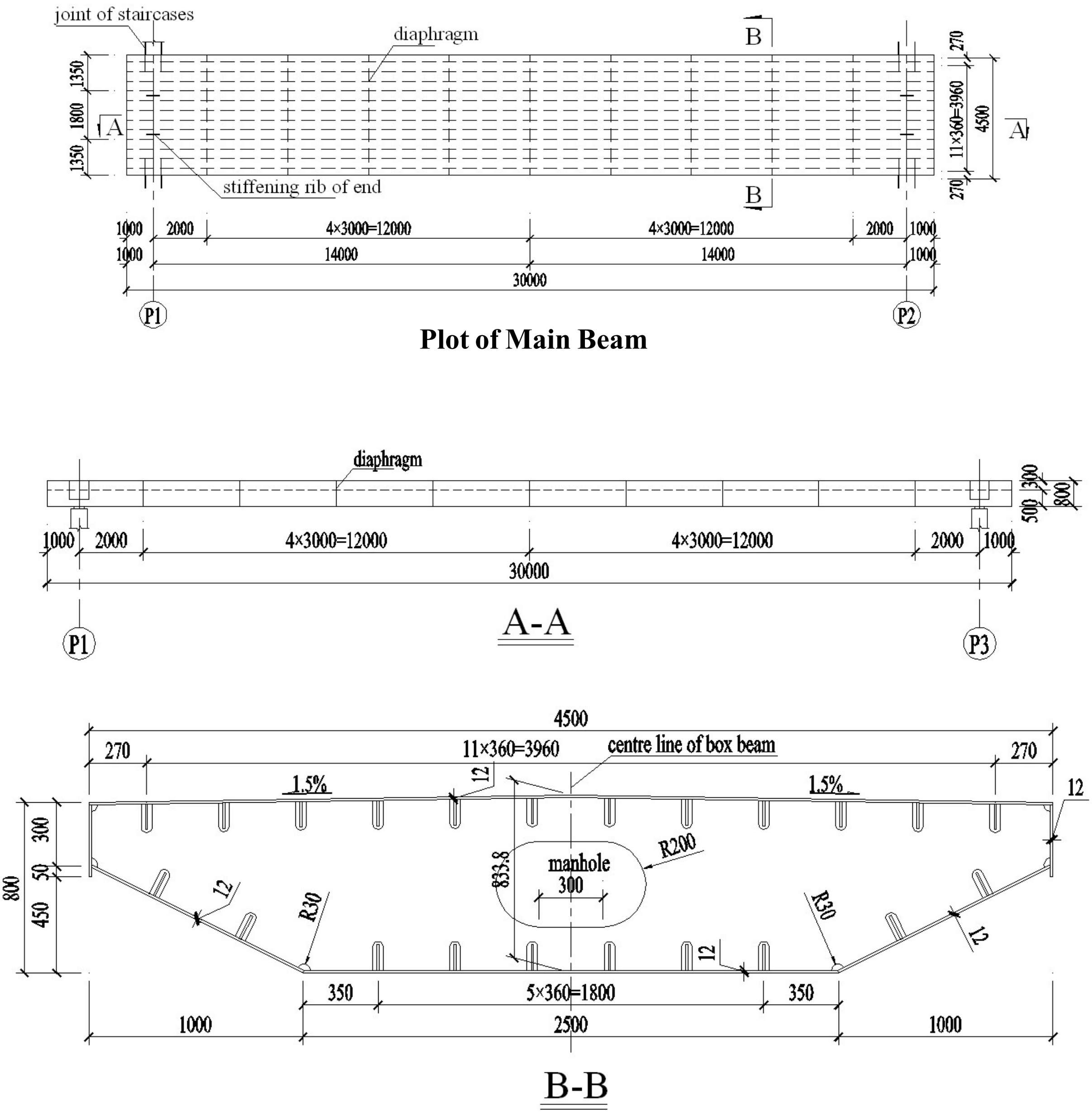
where, a= peak acceleration, m/s; f_1 is the first flexural frequency, which is less then 5Hz; w_s= maximum static superstructure deflection due to a vertical concentrated load applied at the mid-span; K is configuration factor, which is 1.0 for single span; Ψ is dynamic response factor, which is a function of damping and can be determined by the Figure C.3.4.4(f) in Commentary on CAN/CSA-S6-00.

The case is a simply supported footbridge across a street. The main beam of the footbridge is box girder with a span L= 28m, section A= 0.1450 m^2, and moment of inertia I is 0.017208 m^4. The material is steel with density of 7.9 × 10^3 kg/m^3 and the Young modulus 2.0 × 10^{11} Pa. The damping ratio the structure ξ= 0.005. The plan and two sections of the structure are shown in Figure 7. The procedures of predicting the peak acceleration

Table 6: Results of the Peak Acceleration Spectrum Based on Monte Carlo Simulation and the Peak Acceleration Spectrum Based on UD

Method	Structure Frequency		
	2 Hz	4 Hz	10 Hz
Monte Carlo Simulation	8.04	0.44	0.15
UD Method	7.77	0.39	0.17
Differences(%)	3.4	11.4	13.3

Figure 7: Plan and Two Sections of the Structure (All Dimensions are in mm)



response by the code and the method of response spectrum and their results are shown in Table 7.

From the Table 7, it can be seen that for this case, there is a large difference between the two acceleration responses and the results calculated from the code is much higher than the one obtained from the response method. In other words, the formula in the code overestimates the response of this structure significantly, which favors the statement proposed by Pimentel *et al*. This is because in the code, the frequency of vertical force by walking is assumed to coincide with the first

Table 7: Procedures of Calculating the Peak Acceleration Response by the Two Methods and Their Results

	Canadian Highway Bridge Design Code	Method of Response Spectrum
Evaluating formula	$a = 4\pi^2 f_1^2 w_s K\Psi$	$a = \lambda_1 S_{P\alpha}(f_1, \xi, L)$
Required Parameters	1. Structural mass of unit length $m = \rho A$ = 1146 kg/m 2. $f_1 = \frac{\pi}{2}\sqrt{\frac{EI}{mL^4}}$ = 3.47 Hz 3. $w_s = \frac{700L}{48EI}$ = 9.3018×10^{-5}m 4. K = 1 5. Ψ = 11.5	1. m = 1146 kg/m 2. f_1 = 3.47 Hz 3. The first generalize mass $M_1 = \int_0^L \varphi_1^2(x) m dx$ =16041 kg 4. The ratio of pedestrian weight to the generalized mass $\lambda_1 = 700 / M_1 =$ 0.0436 5. According Equation (19), $S_{P\alpha}(f_1, \xi, L)$ =0.8959
Peak acceleration	a = 0.5092m/s^2	a = 0.0391 m/s^2

natural frequency of structure with frequency less than 4 Hz and a constant DLF_1 of 0.257 is adopted for all frequency. However, as mentioned in foregoing section, the normal frequency of walking is about 1.6-2.4 Hz and DLFs vary remarkably for frequencies up to 5 Hz. It is obvious that the structure with frequency more than 2.4 Hz may resonate by the second harmonic. Therefore, the DLF_1 of 0.257 used for the first harmonic may be not appropriate for the second or higher harmonics, otherwise, it will overestimate the structural response as the DLFs of higher harmonic such as DLF_2, are much smaller than DLF_1, which can be seen from Table 1. Even a reduction factor 0.7 is introduced in calculating response of structure with frequency of 5 Hz, it still will overestimate the response significantly, which can be demonstrated by some other cases as shown in Table 8. In the Table 8, case 2 is the case above. In Case 1, 3 and 4, the structures have the same setup as case 2 except span length of each structure, which is shown in the Table 8, respectively. It can be seen that the code overestimates the peak response of structure with frequency more than 2.4 Hz remarkably and underestimate the peak response of structure with frequency less than 2.4 Hz a little, which is consistent with the results obtained by Pimentel *et al.* Therefore, it should be cautious when using the formula provided by these two codes to estimate the acceleration response of structure with frequency more than 2.5 Hz and some corrections are required to provide a more accurate prediction of structural acceleration response induced by vertical walking loads.

Table 8: Estimate of the Peak Acceleration by the Two Methods for the Four Cases

Case	Frequency (Hz)	Canadian Highway Bridge Design Code (m/s^2)	Method of Response Spectrum (m/s^2)
Case 1 L=24	4.7264	0.4646*	0.0374
Case 2 (L=28m)	3.4724	0.5092	0.0391
Case 3 (L=32m)	2.6586	0.4456	0.1745
Case 4 (L=36m)	2.1006	0.3961	0.5628

Note: *A reduction factor of 0.78 is introduced for this structure in case 1.

CONCLUSION

UD method is introduced to avoid using Monte Carlo method to generate lots of random samples and to improve the efficiency of solving this type of issue. Then acceleration response spectra with 95% percentile are obtained by reliability analysis based on response surface constructed by only a few representative simples of pedestrian loads defined by UD and the corresponding structural responses calculated by dynamic analysis. The Monte Carlo simulation validates the method has enough precision. It can be seen that this calculation system based on UD reduces the calculative work significantly and provides a method to simplify the probabilistic dynamic response analysis of structure under random incitement.

Once the structure parameters such as f_{st} and M_1 are given, the structural peak acceleration response could be calculated in a rather simple way by suing the general equations of peak acceleration response spectra in the paper rather than by time-consuming dynamic calculation, which is of practical application.

The case study shows that compare to the results calculated by the proposed method, the formulas in BS 5400-2:2006 and Canadian Highway Bridge Design Code provide a considerable overestimate of the peak acceleration of structure with frequency more than 2.5 Hz and using them to obtain responses of these structures should be cautious.

And for some structures with complicated shape, as long as the some dynamic characteristics such as frequency and mode shape could be determined, and the probabilistic structural acceleration response could be calculated by the proposed method based on UD and RSM.

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