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Research Paper

FUNDAMENTALS OF DESCRIPTIVE GEOMETRY: APPLICATIONS FOR ARCHITECTURE AND ENGINEERING

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This paper aims to reflect on the issues that concern the fundamentals of descriptive geometry, in particular the roles it plays in research, analysis, and design in the fields of architecture and engineering. Its foundations, like its history, are legitimate and up-to-date for the representation of drawings, thus rendering specific the contribution made by the rules of the elements of projective geometry. The fundamentals of projective geometry are, in fact, made explicit by the operations defined by the properties of projection and section for the components of the relationships they describe by modelling, even in three-dimensions, the figures that can geometrically satisfy such combinations. The research undertaken with projective geometry is current/timely, because it refers to the need to finalize the project drawing for architecture, engineering, and the environment, as well as for its defined and codified method of representation that establishes the feasibility of the project.

Keywords: Descriptive geometry, Fundamentals, Architecture, Engineering

INTRODUCTION

Architecture, engineering, and the environment generate and form relationships amongst themselves; geometric properties are essential for satisfying these conditions. The principal and necessary reason for geometrically representing drawings is defined by the biunivocal correspondence attributable to the projection between entities; consequently, it is also possible to verify their measurements and dimensions. In addition, values that are intrinsic are also represented graphically, as they are specific and traceable for and in the graphic representation. Such representations attest to the method and procedure that belong to the logic of projective geometry. These alphanumeric values are expressed by geometric entities; the most widely used and recognized, and immediately quantifiable, are specified by the different scales of numerical ratio. Conversely, it is the relationship values that are determined by

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images understood as projections applied with the rule of orthogonal projection.

Perceptive geometric values obtained with the application of perspective whose image is fixed on the surface of the projection plane are also part of graphic representation. This plane is modelled and determined by a relationship that is set up and ensured by the geometric operation, for example with optimization for binocular vision. Another descriptive system that may be used for graphic research has the goal of representing the relationships formed due to the permanencies deduced from the environmental dimension of the territory or urban area. These belong to the historical design of the environment, or ground, as well as to its built history. This contribution is described graphically through concordances of geometric tracings necessary for recognizing what the geometric generators form, harmoniously coordinated in a drawing that defines systems from which one can derive lines, relationships, positions, measurements, and ratios.

Considering the codification of the double orthogonal projection (Monge) as an established and indisputable fact for drawing, descriptive geometry in its elements of projective geometry initiated a process of ordering through codification and formalization applied over a lengthy period, extending from the middle of the eighteenth through the entire nineteenth century. In recent times, drawing has become a pragmatic technique for describing built works. It has been disciplined and standardized (ISO–UNI) and is still used in a rigorous manner, particularly in the sphere of mechanical drawing. The understanding of descriptive geometry, as verified and optimized, derives from centuries of knowledge of drawing for building, from Villard de Honnecourt (XIII sec.) to Eugène Emmanuel Viollet-le-Duc (1814-1880), as demonstrated by the eloquent case of stereotomy whose principal goal was the cutting of wood and stones according to geometric principles. In the same way, the topic of statics, or graphic rational mechanics, is of significant importance and without doubt deserves an in-depth chapter of its own.

The word geometry derives from the Greek γεωμετρια, which is measurement of the ground. The system of measurement can also be recognized in environmental, territorial, or urban relationships; analogously, one could say a “topography”, or correlation between the design of the ground with architecture and waterways. These correspondences become measureable, as has been said, through the search for geometric generators that bring back to a unity of relationships and cognitions the content of the drawing of the ground, finding therein the set of relationships, codes, and characters of the historical built ground of the places in measurements.

DISCUSSION

The drawing, intended as descriptive geometry, is the *condicio sine qua non* for all graphic operations carried out in accordance with the rule of logic. On this topic, Aldo Rossi made an intelligent consideration in what was one of his final most important acts for architectural culture, and certainly among the most evolved for Italian architecture. Relative to the “*Terza Mostra Internazionale di Architettura*”, held in Venice in 1985, within the remarkable catalogue that accompanied the exhibition, Rossi inserted in his introductory essay the term "drawing" as a prefix and suffix, attributing it to all of the other disciplines related to architecture and engineering. This cultured and attentive observation demonstrated how drawing was without any doubt not only useful and necessary, but in particular also leads to the clarification of theoretical and constructive facts. As if to say that the geometric partitions of a built work transcend the architectural space and their constructive-structural dimensioning. Considering geometric value as a means and defining the thicknesses with regular geometric figures derived from the space of the compositional unit, we then identify exactly the points that in turn generate the geometric arrangement of further parts of the building. The beginning of this graphic/geometric search for the architecture is crucial, as it is reflected in what one can generate in the logical layout/tracing from the combinatorial system with the drawing of the ground. It should be noted, therefore, that there exists a oneness between geometric and constructive thought that gives rise to that which in the past was the intrinsic value of doing, as it was for measuring, composing, and drawing, or rather the intelligibility of things and facts.

Henceforth in this paper we will consider ideally the terms "architecture" and "engineering" as a single expression, since the ancients did not set forth differences or separations between them, because they did not exist. In the same way, the term drawing may be considered synonymous with research and project, or plan. In the past, in fact, it was conventional wisdom to unite and give strength to research and its implementation in the concept of the indissoluble unity of things, in the same way that projective geometric thought assumes the same value and meaning.

"To represent scientifically signifies acquiring and transmitting the knowledge of the form of a real or imagined entity and the rules that underlie this" (Catalano M G). With this assertion, appearing in an essay titled, "A theorem for the unification of the methods of the science of representation," by G M Catalano in the journal *Disegnare* n. 8/1994, the author expresses very well, and in brief, the concept of projective geometry. Drawing is therefore a science and it is thus because it consists of rules that projective geometry has defined in considering the concept of unity passed down from the experience of analytical geometry. The evolution that goes from Euclidean thought to that of projection, with the codification of double orthogonal projection, for the relationship of biunivocality of the entities projected onto two planes with the subsequent overturning of a plane coincident with the other, was first expressed by Gaspard Monge (1746-1818) in his treatise, "Géométrie descriptive". Geometric thought as it relates to descriptive geometry is not divisible or distinguishable *a priori* according to necessity or circumstances. It consists, rather, of a single attitude, that considers the whole experience codified as a unitary experience that can be reproduced and represented (Figures 1 and 2). In this way, we clearly obtain a useful, ordered sequence, with the single properties belonging to rational geometric thought (Figure 3). In fact, these are studied initially as rules and are distinguished in terms, considering obviously that they combine as unitary projective operations in

Figure 1: Terms of membership, rule: a point belongs to a straight line when the projections of the point belonging to the projections of thehomonymson the straight line

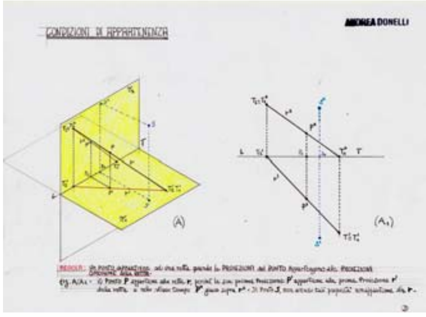


Figure 2: Orthogonal projections: the system of rotation of a plane of projection of the plane containing the segment. (segment A -B in true greatness)

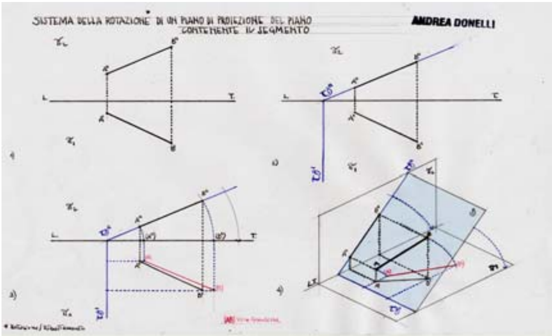
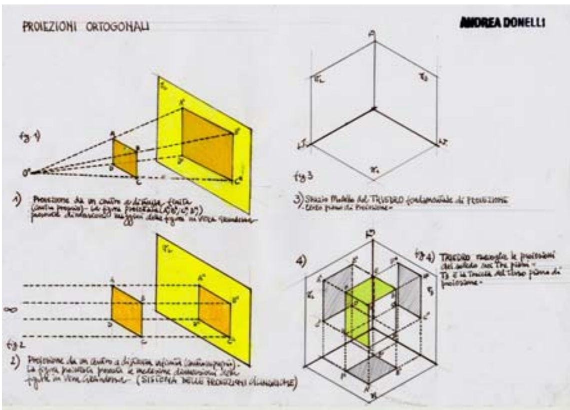


Figure 3: Graphic diagram of the basic structure of the projection system by finite distance and infinity



graphic renderings, in particular in the application of homology (Figures 4-8). The properties, therefore, are distinguished and determined in: definitions, concepts, or primitive entities, which are the point, the line, and the plane; theorems, corollaries, inverse theorems, postulates or axioms and, finally, sets, which by their definition are part of topological geometry. A pragmatic assumption related to the concept of unity for representation in descriptive geometry can be considered as a unified synthesis of graphical operations. This can also be demonstrated by means of a simple example. From a single basic shape drawn in plan and represented in double orthogonal projection with relative elevations, it is possible to integrate and obtain an axonometric projection and a perspective view, in the same way that it is always possible to elaborate from the same initial basic shape in projection a homological relationship, including the consequent overturning of the plane. In conclusion, in a single defined representation exist all of the operations of descriptive geometry (Figure 9). Traceable to these through the projections, in fact, are the relationships of the entities and the biunivocalities, which also describe the true size and forms, the intrinsic passages, and the accuracy of the graphic relationships. Through a further operation of projection that transforms the geometric figures, altering their appearance, which in any case between the original form and its projections remains, permitting one to pass from one figure to the next by means of a finite number of combinatorial operations of projection and sectioning; with these imagines, thus, homography is defined.

Figure 4, 5, 6: Graphic diagrams of the construction of homological relationship. The homology is the relation of correspondence between the points of two generic figures

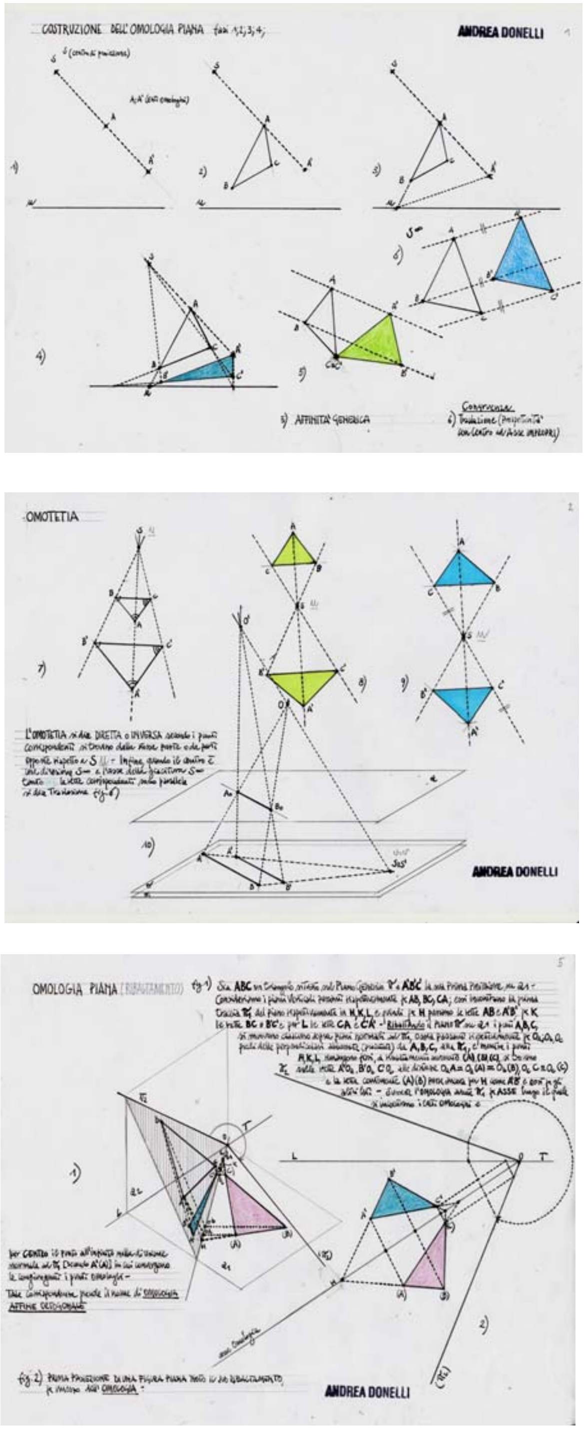


Figure 7: Scheme derived from the book : Migliari R., "Geometriade scrittiva", Cittàstudiedizioni

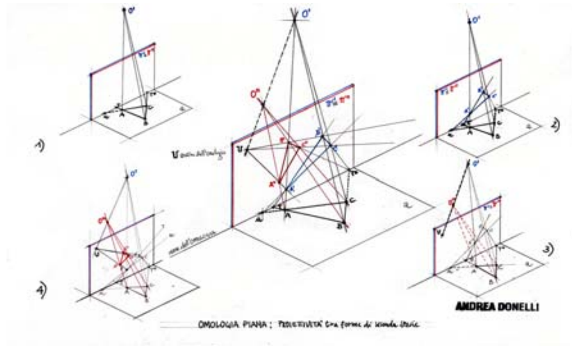


Figure 8: Drawing a simple holiday home, showing the application of homology with overturningplan

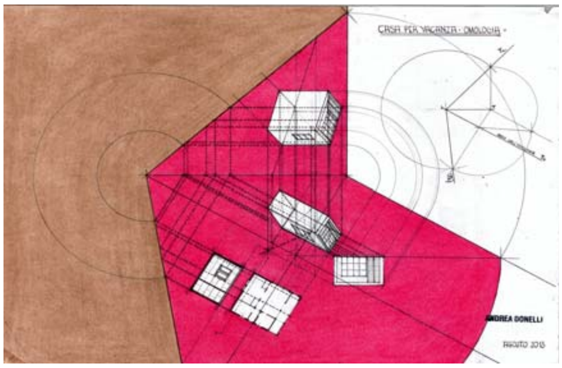
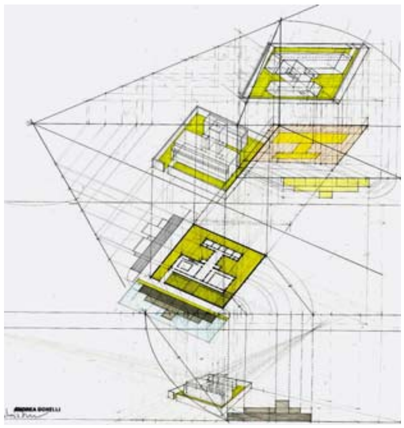


Figure 9: With a single representation defined there all the operations of descriptive geometry



As we have mentioned, the most important geometric entities involved in geometric operations are the values deduced from Euclidean experience, that is the point, the line, and the plane, to which further relate the directions of the lines and the position of the planes (Figures 10-13). This spatial dimension allows us to move from a finite and circumscribed condition to a larger dimension tending toward infinity by virtue of the operations of direction of the straight lines and of the position of the planes, guaranteeing the relationships of belonging. Girard Desargues (1591-1661) set forth these concepts, in terms of the direction of the straight line and the position of the plane, so that geometric thought was amplified beyond what was Euclidean space. The illustrative example is demonstrated by the biunivocal, or one-to-one, relationship of a dotted line r and its projection r', just as we obtain the image I'r from the direction of a line r. With these entities we have the demonstration that these are also the meeting points forming a part of the perspective. Keeping in mind that orthogonal projection originates from a point at infinity, unlike perspective whose centre is a proper point (Figure 14).

Piero della Francesca (1417-1492) long before Desargues' speculations and the codification of double orthogonal projection by Monge, thought in a utilitarian way about the concept of drawing in orthogonal projection. The use of perspective through the preparatory use of associated orthogonal projections according to the written contribution of Andrea Casale in *Disegnare* n. 12 /1996, shows us Piero aware of a concept that is controlled by the projective system and above all by the

Figure 10: Representation of a straight rule: two straight lines parallel to each other have the homonymsparallel projections

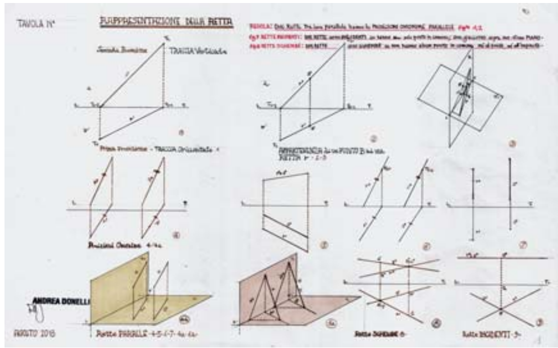


Figure 11: Representation of the plan: conditions for belonging

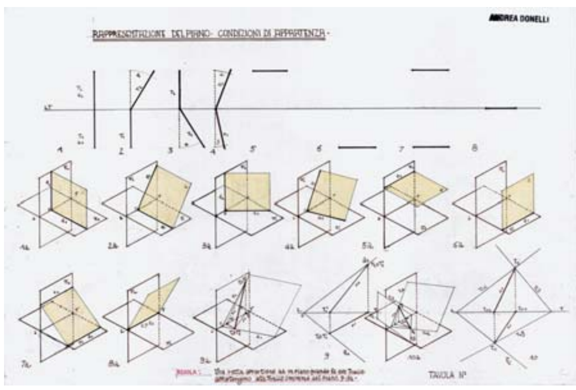


Figure 12: Representation of the plane

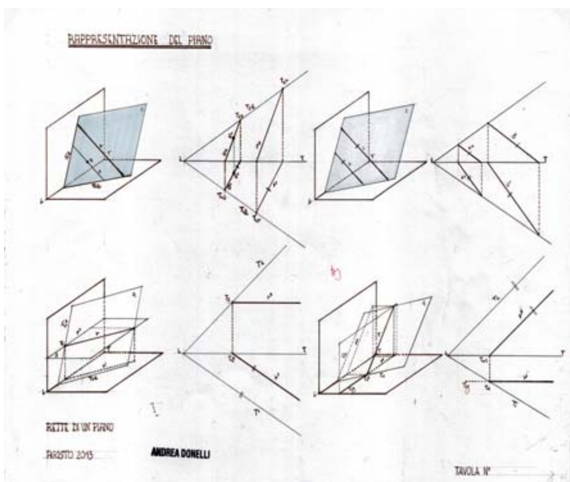


Figure 13: Applications: projecting planes and straight lines of a plane

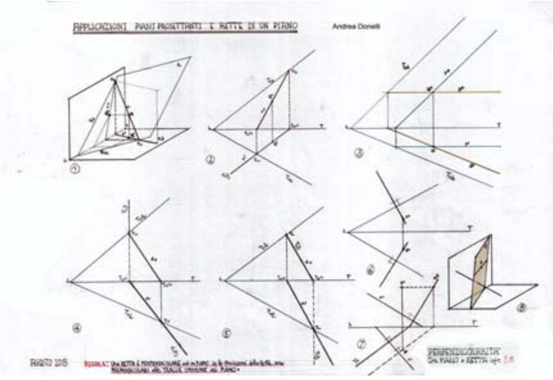
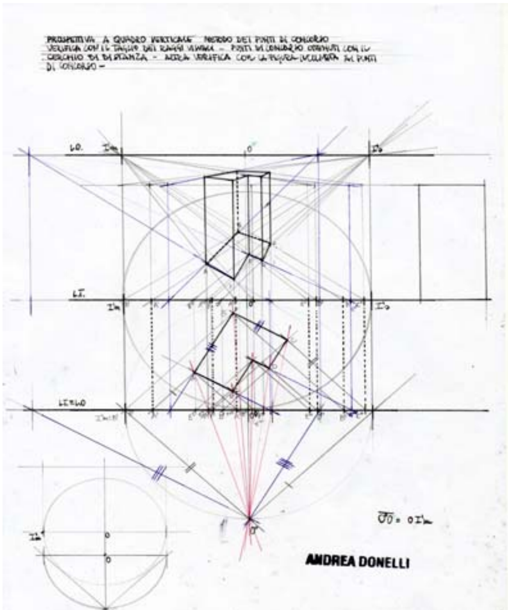
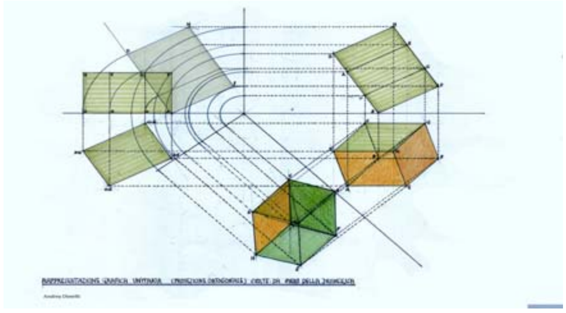


Figure 14: Graphic diagram of perspective in painting vertical point correlation method and verification exercise by other processes



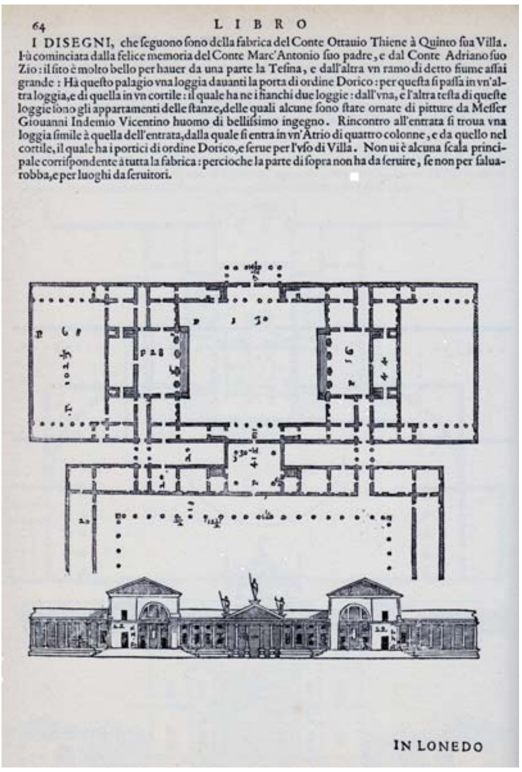
geometric value that he looks for in expressing experiences between measurement and perception (Figure 15). If the measurement is obtained by verifying the representation in orthogonal projection, the perception evoking other "measurements" is obtained from perspective representation .

Figure 15: Drawing orthographic projection deduced by Pierodella Francesca. See diagram presented in the journal "Disegnare" n. 12/1996 - pp. 15 -23 by Andrea Casale



Andrea Palladio (1508-1580), unmindful of Mongian double projection for obvious reasons of chronology, may have been aware of speculations about perspective from cultural sources, first of all from Piero della Francesca's treatise *"De prospectiva pingendi*" and then from studies by Albrecht Dürer (1471 – 1528). In his treatise, "I Quattro libri dell'architettura", Palladio drew plans and elevations using double orthogonal projection (Figure 16). Obviously, these representations are incorrect according to Monge's rules, since Palladio was not specifically aware of the relationship between the projection planes, much less how to arrive at the second image using the line running perpendicular to the ground line on the frontal plane. Consequently, his drawing is drawing of *utilitas* which, however, still guaranteed relationships of geometric origins, since the decision that lay with him did not involve, as it does in today's modern culture, having to juxtapose the conceptualism between subject and object. In fact, what occurred with Palladio is a simple consideration, even in the absence of perspective codifications that became explicit and more than ever crucial to clarifying and

Figure 16: House Villa of Andrea Palladio, drawing taken by facsimile relating to the Treaty of the "Four Books of Architecture"edizioniHoepli, Milano 1980. p.64.



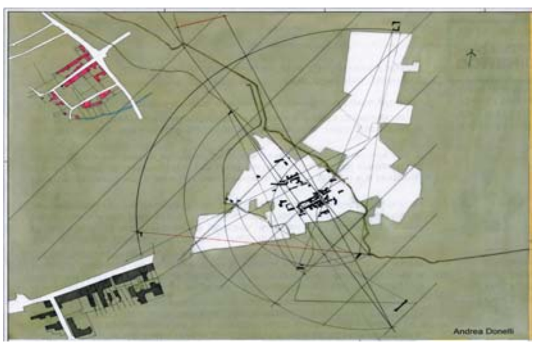
Note: How the concept of dual by projecting orthogonal coding is different from the Gaspard Monge. In fact, there are no distinct from the stakeout sign of hatching of the projections, and as the first projection (building plan), considered as the horizontal section is placed on the (floor of the framework and not as a first screening on which it is geometral.

recognizing the passage of integration between Euclidean geometry and projective geometry. In Palladio's day, the main concern was to bring theoretical dignity and scientific structure to knowledge. Palladio himself initially began working as a stonecutter, first in the workshop of Cavazza and later in the more prestigious *bottega* of "Pedemuro", where he learned the basics of drawing. However, he later expressed his potential at the Accademia del Trissino, where he completed classical, mathematical, and theoretical studies. Measurement, logic, the rationality of building, and understanding the raison d'être of architecture, as a practical activity, as scientific and theoretical knowledge, and as artistic expression were the factors that motivated Palladio to study the past, and this occurred, above all, through his survey drawings and redrawing of ancient architecture. Despite the difficulties and dangers of the time, Palladio travelled to Rome because of his interest in the ancient world and for the city as it was and as it had been, which he initially studied and then experienced first hand in order to understand its architecture, seizing the relative logic of know-how and observing that which is lasting, distinguishing it from the superfluous. In fact, this approach leads back to a measurable order, in representation, or rather it generates that which can be defined in the measure of reason, and therefore this stipulates consequential and reciprocal relationships between drawing (project) and construction, as is, indeed, the raison d'être of a principle of indissoluble unity.

The continual clarification of studies regarding descriptive geometry is attested to and improved upon by the graphic experience obtained through the operations of projection and section and, in particular, once again with the codification of the double orthogonal projection of Gaspard Monge. Monge's goal was three-dimensional representation, the realization of controlled models regarding figures. He carried out these speculations both analytically and graphically, clarifying with great attention the title of the study and the disciplinary expression; in fact, he refers to analytical descriptive geometry and graphical descriptive geometry. On Monge's fundamental contribution to analytical and, above all, graphical geometric thought, see in particular the Neapolitan school and Vincenzo Flauti (1782-1863). Flauti explained and added, according to the study conducted by professor Vito Cardone, some definitions that were not always spelled out clearly in Monge's writings. One particularly interesting example is his contribution to projection planes (Cardone V, 1998). Flauti had no other particular accolades, while very interesting is the debate set in motion by the magazine *XY dimensioni del disegno* with regard to the figure of Flauti, a Monge scholar. See on this subject the contributions of Riccardo Migliari and Vito Cardone listed in the bibliography.

A final consideration, as revealed by the content of the present paper, has to do with the concept of measurement obtained from the design of the ground in relationship to built form, or rather the search for geometric generators. The purpose of this research is to determine the relationship of measurements that occur between the historical design and the built work over time. In fact, it establishes precise points that clearly define geometry with the construction of built works, which implies an order derived from the measurement of the ground. Historically relevant to these ideas is the reading of maps of plans and cities by Opicino de Canistris, in the science of measurement, whose purpose is to juxtapose numbers and figures referring to the form of the ground that contribute to the magnitude of the architectural form. A further task undertaken with the search for geometric generators is to bring back these concepts to design as well: that is, to translate the dimensions that emerge from the relationships into another set of relationships for the new building. To this end we propose an example of research carried out in the locality of Vigasio, in the provinces of Verona in Italy. The drawing that describes the habitat of Vigasio reveals the search for connections between the parts derived from the drawing of the ground in relation to the historical, rural building. The research in question was conducted between December 2004 and January 2005, in part on the basis of work already carried out in 1984 with professor U Tubini at the IUAV, department of architecture in Venice, who proposed the analysis of the Piazza dei Signori in Verona and later with the students in the fifth-year Architectural Composition course, during the 2003-2004 academic year, in the degree course in Civil Engineering in Trento, on the *agri* and *urbis* form of the city of Trento. The theme of the paper concerning the graphic representation is the geometric generators deriving from the ancient systems that belong to the surface reading of the nature of the ground with their historical design. The circumferential arcs intercept a system of parallel and oblique lines obtained and rendered graphically by the recognisability of the Roman system of centuriation, obtaining the value attributed to the unit of the meter of 710. Other, more ample circumferential arcs intersect, with their radii, the lines belonging to the drawing of the ground, which in turn define the rural structures by tangency points. Additionally, starting from the point that defines the origin of the radius, one can draw subsequent sub-multiples of circumferential arcs which in turn define other tangential points with the tracing of the ground corresponding to the built structures, in the same way that we determine the entry point of the so-called "Leona" trench, the essential waterway of the Tartaro river, its main waterway. From this point arise other tracings of lines that are geometric generators which, passing through other known points, in turn form, with other lines, an intersection coincidental with the dimension of the historical or rural buildings (Figure 17). These results describe graphically how even an historical drawing of the ground interacts with the site, establishing a dimension that serves as the measurement and geometric construction of the same. From these considerations it is possible to determine the control of the built form within the geometric relationships that fix the ancient measurements of the ground of their division and subdivision with the superimposition of the measurements deduced from the cadastres: measurements that taken together determine distances, alignments, and lineaments useful for contextualising, even according to environmental criteria, the system of the design of the built work.

Figure 17: Historic town ofVigasio (Verona) Italy:Search for generating geometrical measure of historical design of the soil and geometric relationships defined by the intersection of the systems of the relationship between soil and historical buildings



CONCLUSION

The ways of describing graphically are rooted in the ground of our culture. As is evidenced in literature as well, in Plato's *Timaeus*, in particular in Chapter XXI, the explanation of the solid figures of geometric expression fixes a principle of regulation of geometry itself with nature, and consequently man. Analogously, in his treatise, "Writings", Le Corbusier argues that civilization is founded on geometry and that man lives, practically, by geometry alone. He continues, affirming that "The specificity of man is to establish what is at right angles with respect to him; this causes him to classify, order, and see clearly in front of himself". Again referring to civilization, he makes the point that through geometry man has found the way to measure space by using coordinates on three perpendicular axes. The task that Le Corbusier wisely suggests we take on derives from the fact that: "geometry is reflected in that work made by man that extends from the house to the site". In fact, broadening the study based on graphic geometric-mathematical relationships, one can observe that in addition to the established solid figures such as the cube, the cylinder, the sphere, and the round, pyramidal cone, generated by the rotation of a right-angled triangle around a cathetus, the contribution made by descriptive geometry takes the form of measurements which, in turn, generate logical relationships such that we define an indivisible unit according to a rationality that fixes universal graphic geometrical and mathematical codes applicable to search for an understanding of space for the project in architecture and engineering.

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