Incremental Analysis for the Nonlinear Buckling Responses of the Reinforced Concrete and the Steel Spatial Arch Truss Structure Subjected to Displacement Dependent Loads

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***Abstract*—Recently, there are many studies of the nonlinear structural analysis. This paper presents the method to predict nonlinear response structures. A numerical approach is proposed for the nonlinear analysis structures problems using displacement method. The formulation is based on the rope stiffness matrix of the beam element, which is presented taking into account the nonlinearity of the materials and second-order effects due to node displacements. The tangent stiffness matrix of the element is obtained by the virtual work principal. In this numerical nonlinear study, the equilibrium of the structure is solved using iterative methods. The Simpson integration scheme is used to integrate the surface of the section. The system of equations is solved in the absolute coordinate. The method is implemented in Fortan 90. The numerical examples with bending fracture modes are presented. Considering the load of the structure, verification of the reliability of the developed software is done by comparing numerical result with experimental and analytical results.**

***Keywords*—concrete, nonlinear analysis, arch truss structure, increase load, simulation, load displacement curves**

I. Introduction

The nonlinear behavior of the reinforced concrete structures in the calculation is the problem that has been studied for very long time. In effect, as many authors have looked for representing cracks in an elastic material [1–3]. Some current methods are based on the nonlinear dimensioning analysis [4]. The study of the capacity of reinforced concrete structures requires realistic models on materials, finite element discretization, and the search of the nonlinear response [5–6]. Given the variety of the studies that have been carried out and the number of parameters are involved. Numerous research works was done on the post –beam structures in 2D and 3D [7] and their explanations. This will enable us to know the main characteristics of the nonlinear phenomena [8–10]. Some studies use nonlinear models [11–14] to study the steel spatial arch truss structure. In this work, the stiffness matrix is developed and the study is done by the integration procedure taking into account second-order effects due to node displacements. Each element contains two nodes with 6 degrees of freedom for each node. In this approach the element is only loaded at its ends, assumed short, so that the second order effects due to deformations in the intrinsic axis system are negligible, the torsional and shear stiffness are calculated by linear elasticity, normal stresses due to torsion are neglected (there is no effect of bi-moment). For each load step, the problem consists in searching increased nodes displacements. The paper is organized as follows: in the first place we give the laws of behavior of the various materials (concrete, steel), then; we describe the equilibrium of the structure. In the last section, verification of the reliability of theoretical developments and the developed software is done by comparing numerical results with experimental and analytical results published in the literature.

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II. Behavior of Materials

A. Concrete Behavior in Compression and Traction

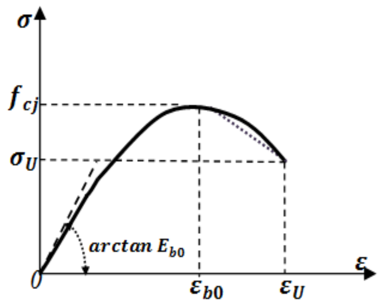


Fig. 1. Uniaxial constitutive law of the concrete compression.

For concrete in compression the Sargin law [3] is adopted, the tensile behavior will be modeled by the law of Grelat [3]. Concrete presents an elastic behavior at the beginning of loading then, nonlinear behavior and the descending branch beyond of the peak (the maximum) of the stress of the concrete.

The concrete compression behavior is given by the Sargin law as shown in Fig. 1.

$$\sigma = f_{cj} \frac{k_b.\bar{\varepsilon} + \left(k_b^{'} - 1\right)\bar{\varepsilon}^2}{1 + \left(k_b - 2\right)\bar{\varepsilon} + k_b^{'}\bar{\varepsilon}^2} \tag{1}$$

With $\bar{\varepsilon} = \frac{\varepsilon}{\varepsilon_{b0}}$, $k_b = \frac{E_{b0} \cdot \varepsilon_{b0}}{f_{cj}}$,

The behavior in the tensile is linear until the cracking of the concrete. Then, the different areas in cracked concrete located between two successive cracks of the bending contribute to the resistance in form decreasing branch and it cancels after the break of the steel.

The concrete behavior in traction is given by the Grelat law as shown in Fig. 2.

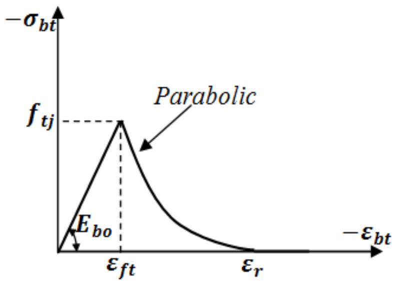


Fig. 2. Stress-strain behavior of tensile concrete.

$$\left|\varepsilon_{bt}\right| \le \varepsilon_{ft} : \sigma_{bt} = E_{b0} \cdot \varepsilon_{bt} \tag{2}$$

$$\varepsilon_{ft} < \left|\varepsilon_{bt}\right| \le \varepsilon_r : \sigma_{bt} = -f_{tj} \cdot \frac{\left(\varepsilon + \varepsilon_r\right)^2}{\left(\varepsilon_r - \varepsilon_{ft}\right)^2} \tag{3}$$

$$\left|\varepsilon_{bt}\right| > \varepsilon_r : \quad \sigma_{bt} = 0 \tag{4}$$

The behavior of reinforcing steel is given by the code for the reinforced concrete [8].

The elastoplastic model of steel is used:

$$\begin{cases} \sigma = E_a \varepsilon_S & \text{if} \quad \varepsilon_s \le \varepsilon_e \\ \sigma = f_e & \text{if} \quad \varepsilon_e \le \varepsilon_s \le \varepsilon_{su} \\ \sigma = 0 & \text{if} \quad \varepsilon_s \ge \varepsilon_{su} \end{cases} . \tag{5}$$

Note:

E_{b0} Elastic modulus of concrete

ε Longitudinal strain

ε_u Ultimate strain

f_{cj} Concrete compressive strength at *j* day

f_{tj} Concrete tensile strength at *j* day

ε_{b0} Peak of the strains corresponding t_0 f_{cj}

G Shear modulus

K_b and K_b' are the dimensionless parameters of the Sargin low.

$[\boldsymbol{K}]$ Stiffness matrix

$\{\Delta \boldsymbol{U}\}$ Vector of nodes displacements increase

$\{\Delta \boldsymbol{F}\}$ Vector nodes forces increase

$\{\Delta \boldsymbol{P}\}$ Vector of applied loads increase

III. Equilibrium of the Section

The section is defined by the series of trapezoidal table is shown in Fig. 3.

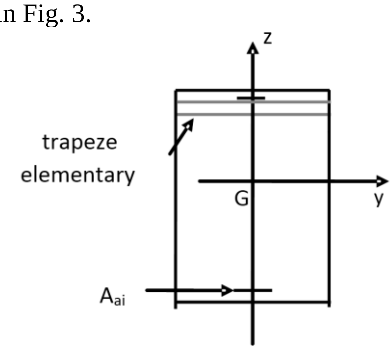


Fig. 3. Discretization of the section into trapezoidal tables.

The equilibrium of the beam element section is expressed by the following equation:

$$\begin{pmatrix} \Delta \boldsymbol{F}_{sn} \\ \Delta \boldsymbol{F}_{st} \end{pmatrix} = \left[\boldsymbol{K}_s\right] . \begin{pmatrix} \Delta \boldsymbol{\varepsilon}_n \\ \Delta \boldsymbol{\varepsilon}_t \end{pmatrix} \tag{6}$$

where $\Delta \boldsymbol{F}_{sn}$ and $\Delta \boldsymbol{F}_{st}$ are normal forces increase and shear forces increase. $[\boldsymbol{K}_s]$: the cord stiffness matrix of the section

The Structure's Stiffness Matrix:

In the axis system XYZ, one positions of the local coordinate system $x_0\, y_0\, z_0$ of the element associated with here initial position, at the increasing of the loading, $\boldsymbol{I}_0$ and j_0 nodes of the element are moved in *I* and *J*, respectively, then introduced the concept of intrinsic axis system, noted xyz.

Either an element of the structure, initially between the node i_0 and the node j_0 linked to the local axis system $x_0y_0z_0$, and OXYZ the absolute axis system connected to the structure.

For each loading step, the problem is to calculate the nodes displacement increase $\{\Delta \boldsymbol{U}\}$ by solving the following nonlinear system witch describe the structure equilibrium:

$$\Delta P = [\boldsymbol{K}]\ \{\Delta \boldsymbol{U}\} \tag{7}$$

The principle consists of the application one step of loading up to breaking. Calculation process and the research of the equilibrium structure is illustrated in Fig. 4 the developed procedure.

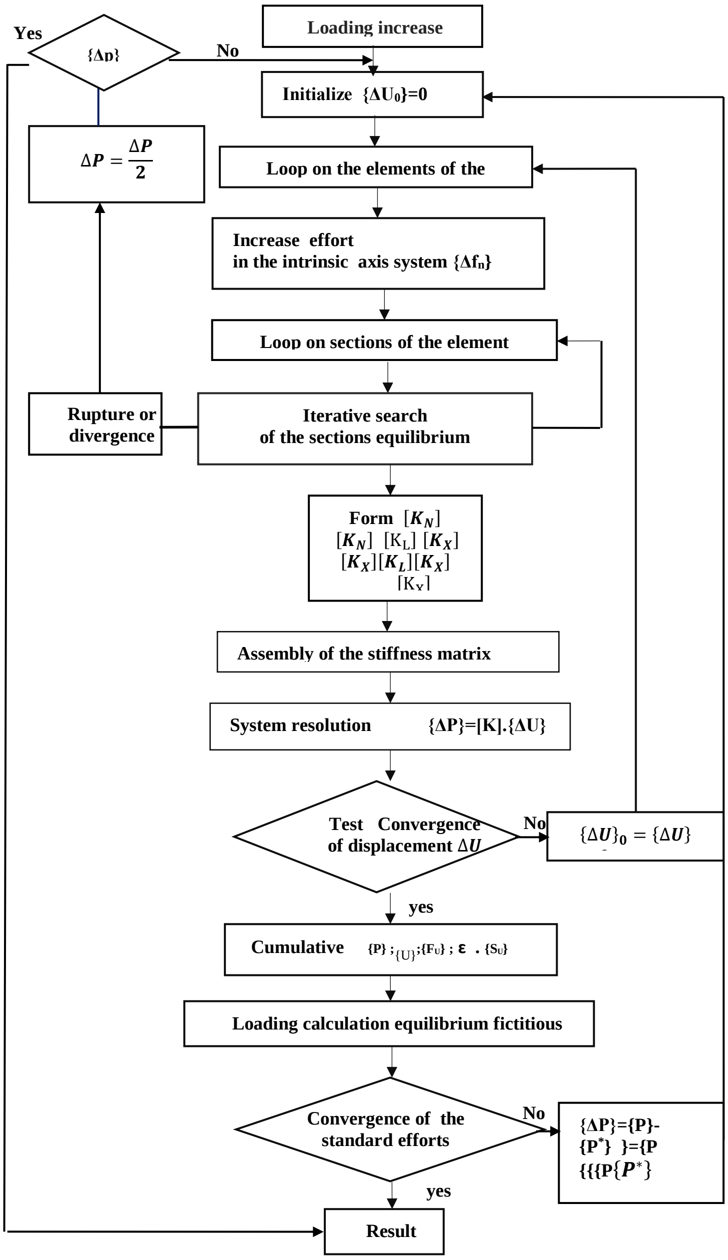


Fig. 4. Flowchart of the calculation process.

IV. Model Validation

Software was developed in FORTRAN language; the equilibrium of the structure in the three dimensions, taking into account the real behavior of materials.

- Numerical simulation and confrontations

The computer program was developed according to the Fortran 90, and the method is applicated for the porico and the arc structure.

A. The Portico of Vecchio

This test is performed by Vecchio and Emara in 1992. The portico is recessed at the feet, consists in first applying a total vertical load of 700 kN to the each column, this load was maintained throughout the test [10]. The lateral load (F) was applied to ruin portico. The resistance of concrete compression is taken equal to 30 MPa, and the longitudinal modulus of concrete is taken equal 26 GPa. The yield strength of the steel of is taken to be 418 MPa, and the plastic limit of the steel is taken to be 596 MPa. The longitudinal modulus of steel is taken as equal to 192.5 GPa. The dimensions of the chosen gantry, are referenced in the article [10] and are shown in Fig. 5.

Fig. 6 shows the evolution of the deflection according to the applied force *F*. The calculated curve approximates the experimental curve, the calculated maximum deflection is however lower of the experimental one; this is probably due to a numerical problem.

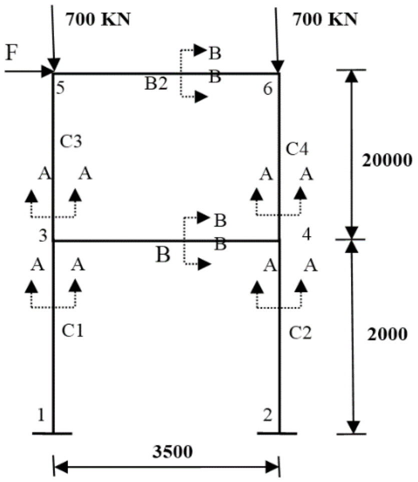
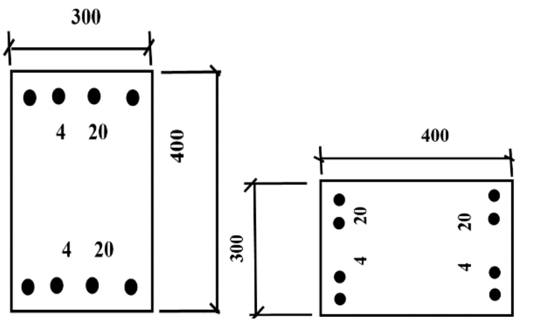


Fig. 5. The portico modeling of Vecchio and Emara.



(a)The cross section of the column (b)The cross section of the beam

Fig. 6. The cross section of the portico.

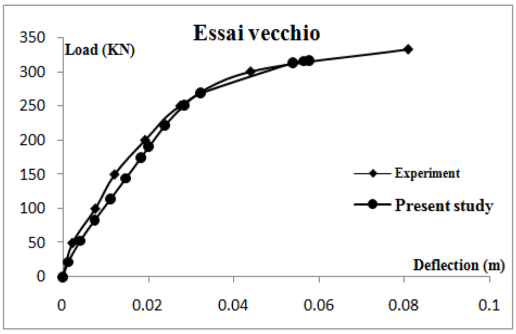


Fig. 7. Evolution lateral displacement according to lateral load.

B. *Arch Truss Structure*

Fig. 8 shows a 35-member truss structure of the plane arch shape subjected to a vertical concentrated load P [11]. This example has been analyzed both by Rosen and Schmit to examine the influence of geometric imperfections, and also by Kondoh and Atluri [12] to analyze the effects of local and global imperfections on the entire structure. The arch-truss structure is composed of 35 members, all of which have a circular cross section and an identical modulus of elasticity of: $E = 68.964 \times 10^6$ KN/m^2. The nodal coordinates of the arch-truss structure and the cross section of the members are listed in Tables I and II, respectively.

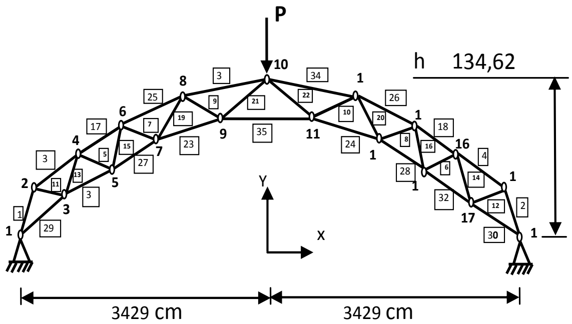


Fig. 8. Arc-structure.

TABLE I. COORDINATES OF EACH NODE OF THE ARCH-STRUT STRUCTURE FROM KONDOH AND ATLURI

Nodal Number	Coordinates X	Coordinates Y
19.1	±3429.0	0.00
18.2	±3048.0	50.65
17.3	±2667.0	34.75
16.4	±2286.0	83.82
15.5	±1905.0	65.30
14.6	±1524.0	110.85
13.7	±1143.0	87.99
12.8	± 762.0	128.50
11.9	±381.0	100.65
10	0.0	134.60

TABLE II. CROSS-SECTION AREAS OF EACH MEMBER OF THE ARCH-STRUT STRUCTURE FROM KONDOH AND ATLURI

Member's Number	Cross-section Area cm^2
1-10-35	51,61
11,12	64,52
13-16	83,87
17,18	96,77
19-22	103,23
23,24	161,29
25,26	193,55
27,28	258,06
29,32	290,32
33,34	309,68

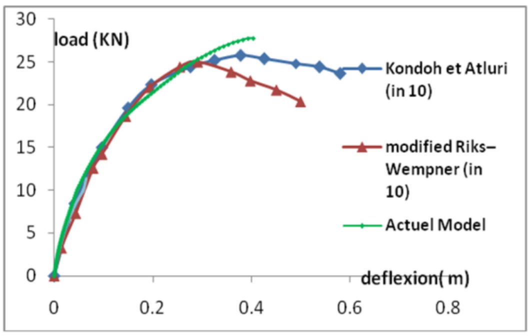


Fig. 9. Load-displacement relation for arch-truss structure, node 10.

- Results interpretation

Fig. 9 shows the load-displacement node 10. The limit load obtained in the present work is about 27.78 KN, a close agreement is obtained by comparing to the data obtained by modified Riks–Wempner [11], 25.11 KN, and by Kondoh and Atluri [12] 25.87 KN. The presented numerical example confirms that the results obtained by developed software are in agreement with the theoretical solution and example published by other authors.

V. Conclusion

The objective of this work was to define a modeling method for the simulation of the nonlinear behavior of reinforced concrete and the steel spatial arch truss structure under the increasing load for prediction breaking response.

The nonlinear governing equilibrium equation are obtained using displacements method.

The procedure uses two calculation steps, on the first step, the formation of the stiffness matrix of the section. The second step the formation of the stiffness matrix of the structure, the calculation scheme is presented in the Fortran program to have the solution of the nonlinear system of equations

The solution obtained by our calculation approximates the deflection measured in phase of the elastic behavior. The comparison of the result given by the program developed in this study gives us a good agreement for the analyzed example.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Youcef bouafia conducted the research; Youcef Bouafia, Mohand Said Kachi and Fatiha Iguetoulene analyzed the data; all authors had approved the final version

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